

Long-Range OASDI Projection Methodology

Intermediate Assumptions of the 2021 Trustees Report

August 2021

**Office of the Chief Actuary
Social Security Administration**

A. Flow Charts

Chart 1: Overview of Long-Range OASDI Projection Methodology

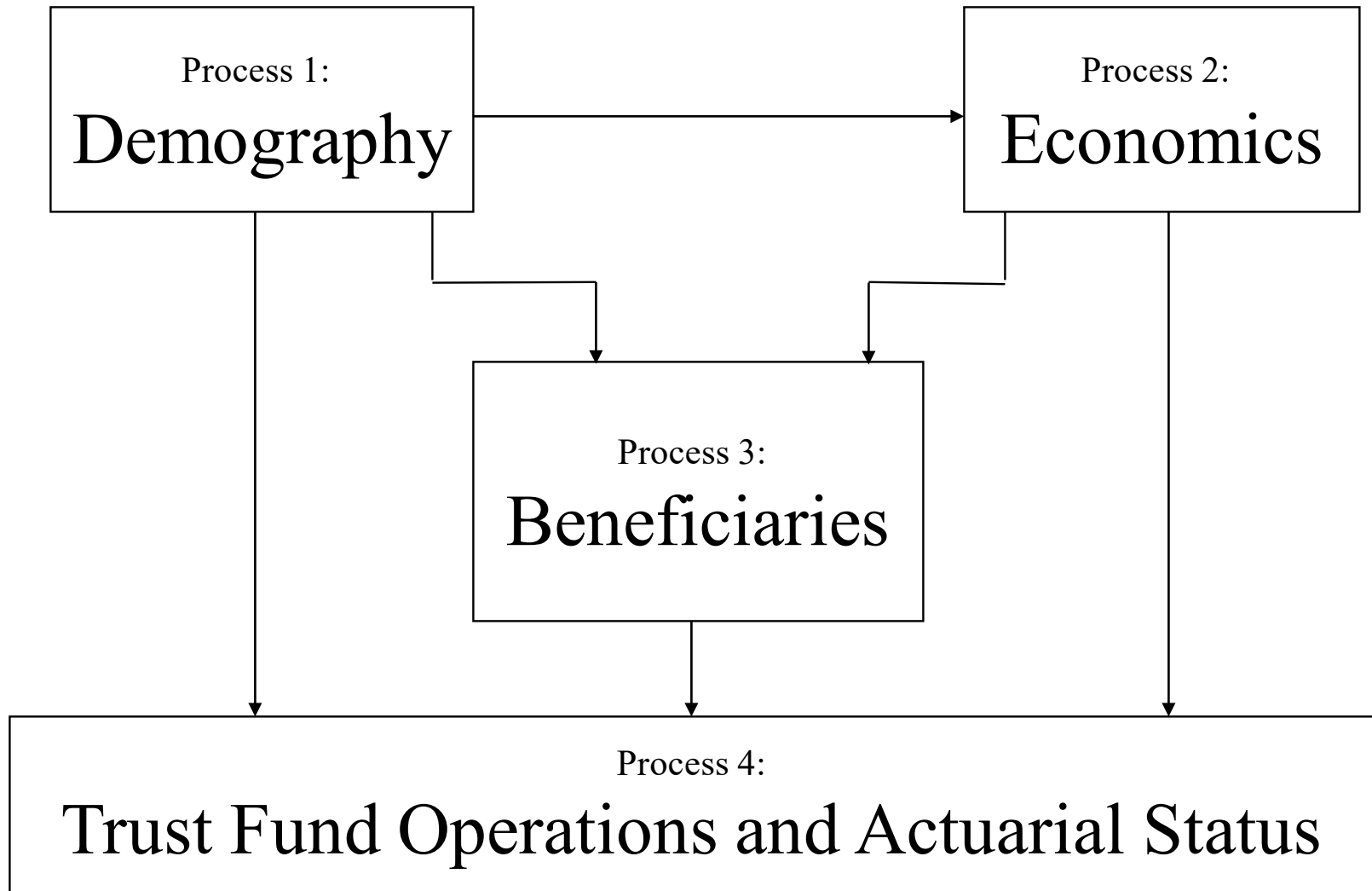


Chart 2: Demography – Process 1

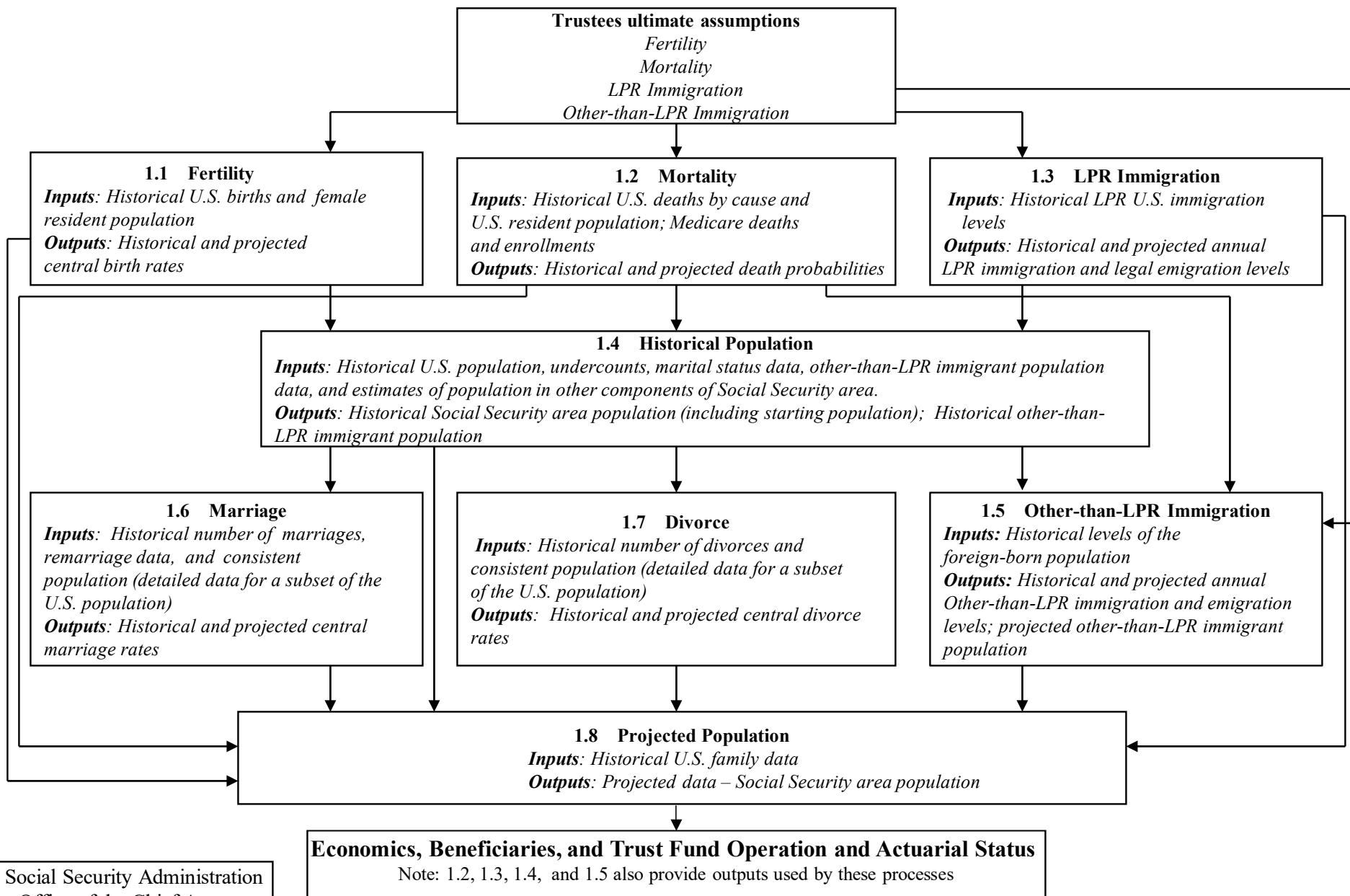


Chart 3: Economics – Process 2

Trustees Ultimate Economic Assumptions
Average real wage
Productivity
Average hours worked
Inflation
Full-employment unemployment rate
Ratio of OASDI taxable payroll to covered earnings

2.1 U. S. Employment
Inputs: Historical BLS data, population (Demography), disability prevalence rates (from prior TR), life expectancies (from Demography), & economic factors
Outputs: Historical & projected labor force participation & unemployment rates by age and sex

2.1 Real & Potential GDP, Nominal GDP
Inputs: Historical NIPA (BEA) data
Outputs: Real, potential, and nominal GDP

2.2 Class of Worker
Inputs: Historical BLS data, population (Demography), & economic factors
Outputs: Wage workers & self-employed by age and sex

2.2 U. S. Wages & Income
Inputs: Historical NIPA (BEA) data & employment (BLS) by sector
Outputs: Wages & SE income by sector

2.3 AWI Series
Inputs: Historical AWI, Wages
Outputs: Historical and Projected AWI

2.3 Covered Earnings
Inputs: Historical NIPA & covered earnings data by sector
Outputs: Historical & projected covered earnings by type of worker

2.3 Taxable Maximum
Inputs: Historical taxable maximum, AWI
Outputs: Historical & projected taxable max.

2.3 Covered Employment
Inputs: Historical U.S. & covered employment data, population including other-than-LPR immigrant population & DACA/DAPA immigrants (Demography)
Outputs: Covered worker rates, covered worker levels, & total employed (all by age, sex, type of worker)

2.3 Average Covered Earnings
Outputs: Projected average covered earnings by type of worker

2.4 Average Taxable Earnings
Inputs: Taxable earnings, covered workers
Outputs: Average taxable earnings

2.4 Taxable Earnings
Inputs: Historical distribution of earnings
Outputs: Taxable wages, taxable SE income & multi-employer refunds

Beneficiaries, Trust Fund Operations and Actuarial Status

2.4 Effective taxable payroll
Outputs: Effective taxable payroll

2.5 Revenues
Outputs: Short-range payroll tax revenue

Short-Range Trust Fund Operations

Social Security Administration
 Office of the Chief Actuary
 August 2021

Chart 4: Beneficiaries – Process 3

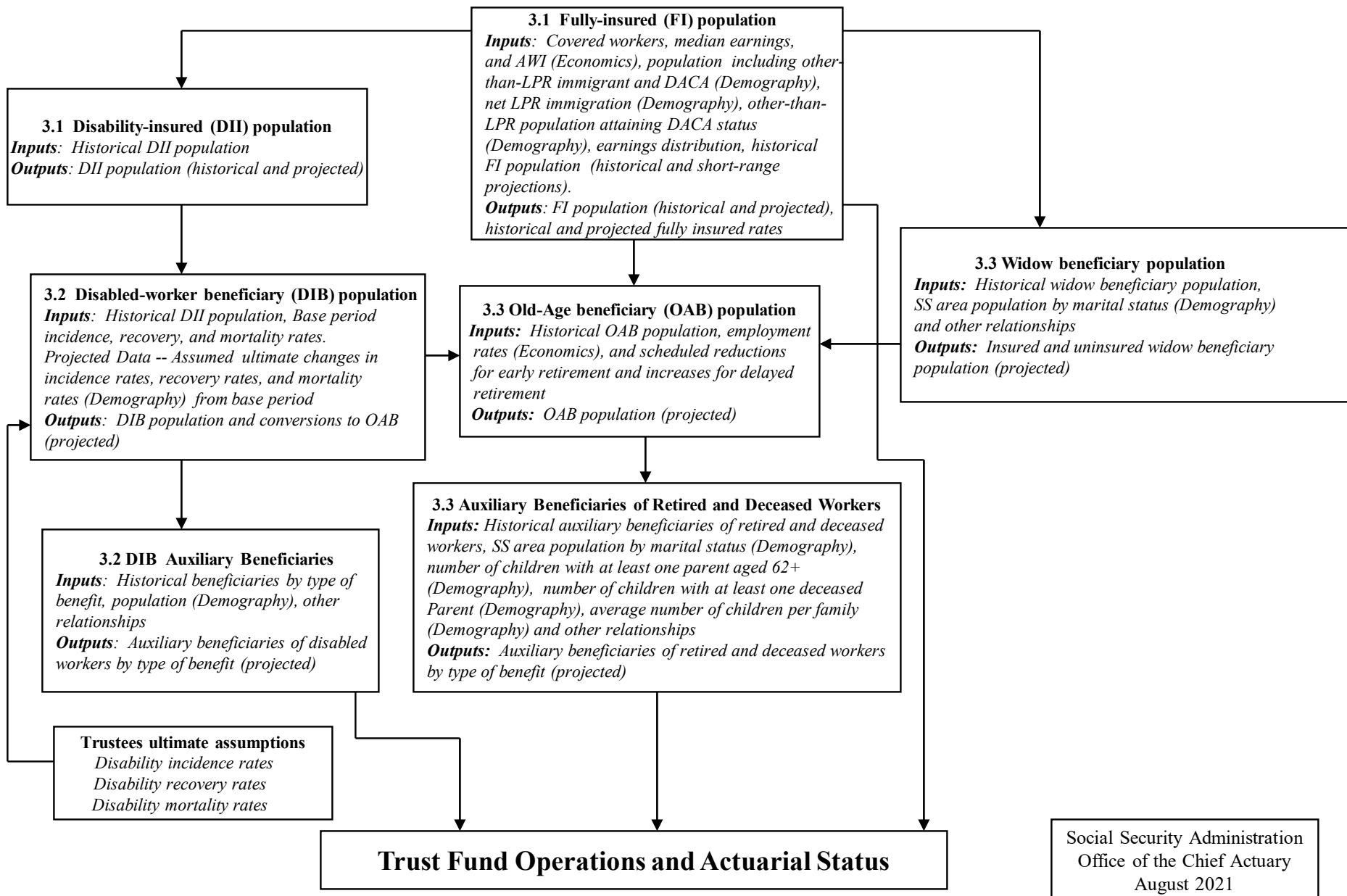
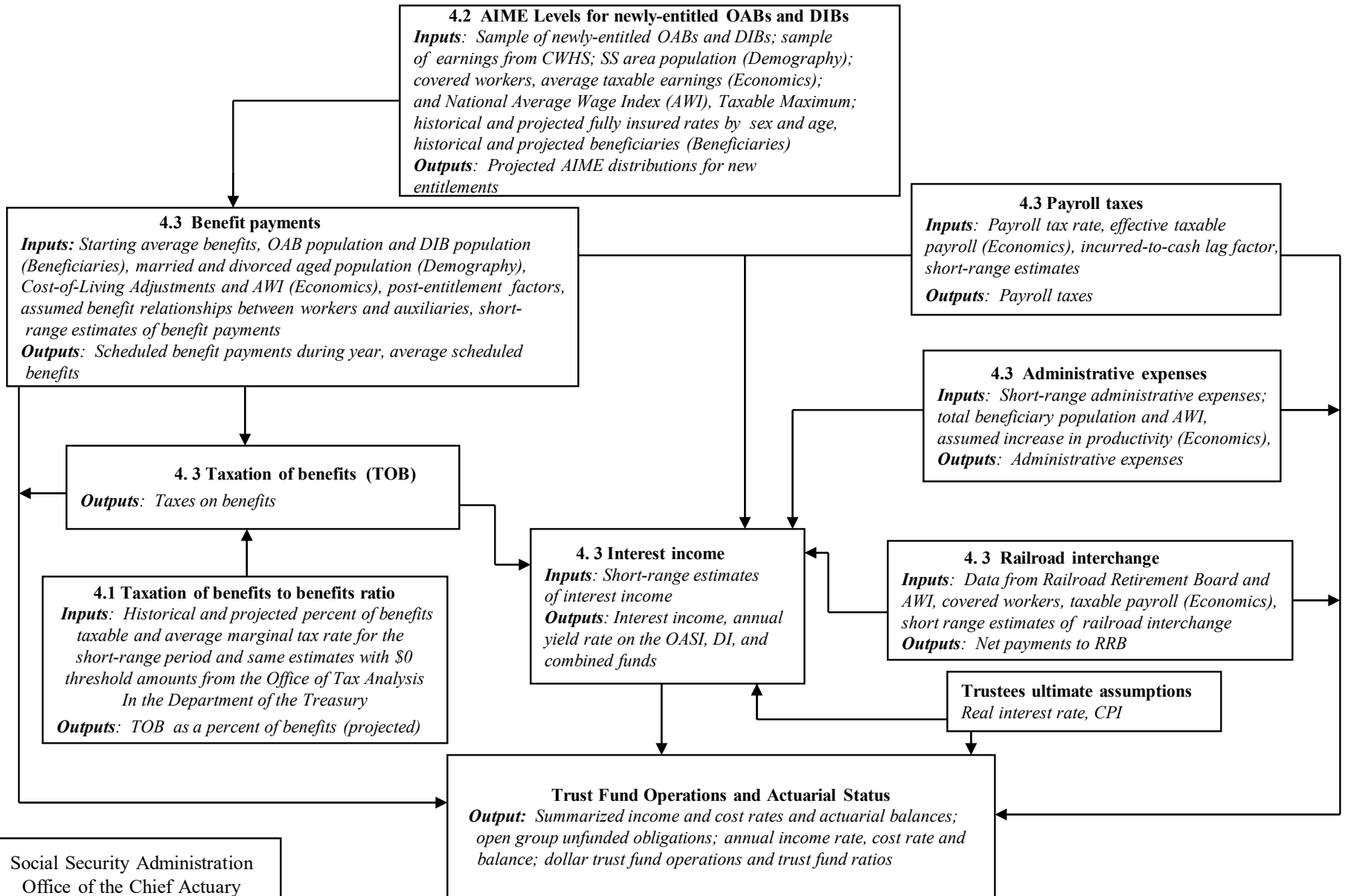


Chart 5: Trust Fund Operations and Actuarial Status – Process 4



B. Process Descriptions

The long-range programs used to make projections for the annual Trustees Report are grouped into four major processes. These include Demography, Economics, Beneficiaries, and Trust Fund Operations and Actuarial Status. Each major process consists of a number of subprocesses. Each subprocess is described in terms of three elements:

- This overview attempts to provide a general description of the purpose of each subprocess. This element introduces key projected variables used in the subprocess. Some variables are represented as being dependent in an equation, where the dependent variable is *defined* in terms of one or more independent variables. Independent variables may include previously calculated dependent variables or data provided from outside the subprocess. Other key variables are referenced by “(·)” following the variable name. This symbol indicates that the calculation of this variable cannot easily be communicated by an equation and, thus, requires a more complex discussion.
- Input Data – Data used in the subprocess are described. These data include those from other subprocesses, ultimate long-range assumptions provided by the Board of Trustees of the OASDI Trust Funds, data from other offices of the Social Security Administration, and data from outside the Social Security Administration (e.g., estimates of the U.S. population). Data description includes data source and data detail (e.g., define age detail of data). In addition, this element includes how often additional data are expected to be received.
- Development of Output – The key variables are described in greater detail, including the level of disaggregation of the data.

Process 1:

Demography

1. Demography

The primary purpose of the Demography Process is to provide estimates of the projected Social Security area population¹ for each year of the 75-year projection period in the Trustees Report (TR). For the 2021 report, the projection period covers the years 2021 through 2095. The Demography Process receives input data mainly from other government agencies, and provides output data to the Economics, Beneficiaries, and Trust Fund Operations and Actuarial Status processes.

The Demography Process is composed of eight subprocesses: FERTILITY, MORTALITY, LPR (Lawful Permanent Resident) IMMIGRATION, HISTORICAL POPULATION, OTHER-THAN-LPR IMMIGRATION, MARRIAGE, DIVORCE, and PROJECTED POPULATION. The following chart displays the key outputs of each subprocess:

Subprocess	Key Outputs
FERTILITY	<ul style="list-style-type: none"> • Birth rates, by age of mother
MORTALITY	<ul style="list-style-type: none"> • Probabilities of death, by age and sex
LPR IMMIGRATION	<ul style="list-style-type: none"> • LPR immigrants, by age and sex • Legal emigrants, by age and sex • Adjustments of status from other-than-LPR status to LPR status, by age and sex
HISTORICAL POPULATION	<ul style="list-style-type: none"> • Historical estimates of the Social Security area total population, by age, sex, and marital status • Historical estimates of the other-than-LPR population, by age and sex
OTHER-THAN-LPR IMMIGRATION	<ul style="list-style-type: none"> • Other-than-LPR immigrants, by age and sex • Other-than-LPR emigrants, by age and sex • Projected other-than-LPR populations, by age, sex, and type (never-authorizeds, nonimmigrants, visa-overstayers) • Historical estimates of the other-than-LPR population, by age, sex, and type
MARRIAGE	<ul style="list-style-type: none"> • Marriage rates, by age-of-husband crossed with age-of-wife
DIVORCE	<ul style="list-style-type: none"> • Divorce rates, by age-of-husband crossed with age-of-wife
PROJECTED POPULATION	<ul style="list-style-type: none"> • Projected total populations, by age, sex, and marital status

¹ The Social Security area population is composed of: (1) residents of the 50 States and the District of Columbia (adjusted for net census undercount); (2) civilian residents of Puerto Rico, the Virgin Islands, Guam, American Samoa, and the Northern Mariana Islands; (3) Federal civilian employees and persons in the U.S. Armed Forces abroad and their dependents; (4) non-citizens living abroad who are insured for Social Security benefits; and (5) all other U.S. citizens abroad.

1.1. FERTILITY

1.1.a. Overview

The National Center for Health Statistics (NCHS) collects data on annual numbers of births by single year of age of mother and the U.S. Census Bureau produces estimates of the female resident population by single year of age. Age-specific birth rates (b_x^z) for a given year z are defined as the ratio of (1) births (B_x^z) during the year to mothers at the specified age x to (2) the midyear female population (P_x^z) at that age. The total fertility rate (TFR^z) summarizes the age-specific fertility rates for a given year z . The total fertility rate for a given year z equals the sum of the age-specific birth rates for all ages x during the year. One can also interpret the total fertility rate as the average number of children that would be born to a woman if she were to survive her childbearing years and experience, at each age in her life, the age-specific birth rate of year z . There is also a cohort total fertility rate ($CTFR^t$). The cohort total fertility rate for a given cohort born in year t equals the sum of the age-specific birth rates for all ages x in each cohort's lifetime.

The FERTILITY subprocess combines the historical values of b_x^z and the ultimate $CTFR^z$ to develop projections of b_x^z . The primary equations of this subprocess are given below:

$$b_x^z = b_x^z(\cdot) \quad (1.1.1)$$

$$TFR^z = \sum_x b_x^z \quad (1.1.2)$$

$$CTFR^t = \sum_x b_x^{t+x} \quad (1.1.3)$$

1.1.b. Input Data

Trustees Assumptions -

1. Each year the Board of Trustees of the OASDI Trust Funds sets the assumed ultimate values for the cohort TFR. For the 2021 Trustees Report, the ultimate TFR is 2.00 and it is assumed to be attained starting with the cohort born in 2007.

Other input data -

2. From the NCHS, annual numbers of births by age of mother² (10-14, 15, 16, 17, ..., 48, 49-54) for years 1980-2019. In general, the NCHS provides an annual update including one additional year of final birth data. Previous historical years are only updated if the NCHS makes a historical revision to their data.
3. From the U.S. Census Bureau, estimates of the July 1st female resident population by single year of age for ages 14-49 for 1980-2019. In general, each year, Census provides updated data for years after the most recent decennial census.
4. From the NCHS, historical birth rates, by single year of age of mother (14-49) for the period 1917-79. No updates of these data are needed.

² Births at ages less than 14 are treated as having occurred at age 14, and births reported to mothers older than 49 are treated as having occurred at age 49.

- From the NCHS, birth rate data for the first, second, and third quarters of 2019 and 2020, by five-year age group (2020 data is provisional). In general, the NCHS provides data for a new year through the first quarter annually at the time the fertility projections need to be finalized. However, data through the third quarter was available for the 2021 TR.

1.1.c. *Development of Output*

Equation 1.1.1 - Age-specific birth rates

The FERTILITY subprocess produces the age-specific birth rates, by childbearing ages 14 through 49, for years 1941 through the end of the 75-year projection period. For historical years prior to 1980, age-specific birth rates come from the NCHS. For years 1980 through the remainder of the historical period, age-specific birth rates are calculated as: $b_x^z = \frac{B_x^z}{P_x^z}$, using birth data from the NCHS and estimates of the July 1st female resident population from the U.S. Census Bureau.

The age-specific birth rates are projected using a process that is consistent with both the observed trends in recent data and the assumed ultimate cohort total fertility rate. This process consists of the following steps:

- Calculate the age-specific birth rates, b_x^z , for each year during the period 1980-2019.
- For 2020, estimate b_x^{2020} values by taking the b_x^{2019} values and multiplying them by the weighted average (using 2019 final data for the weighting) of the first three quarters of 2020 NCHS five-year age-group data over the weighted average of the first three quarters of 2019 NCHS five-year age-group data. Ages in the exact middle of each age group (for instance, 27 is the exact middle of the 25-29 age group) only use that particular age-group ratio but other ages use a weighted average of the ratios for the age group they are in and the next closest age group.
- Calculate regression slopes and intercepts for each age series of b_x^z from 1975-2020.
- The starting values of the projection process are the b_x^z values from the last provisional historical data year (2020). Thus, these are denoted as b_x^{2020} .
- Calculate the ultimate year for each age. This is equal to $2020 + \text{age} - 13$.
- For the projection years (2021 and later), calculate preliminary b_x^z values by:
 - For ages under 20: Keeping the gap from the b_x^{2020} values and the regression lines constant until the ultimate year for each age. However, the value for each particular b_x^z must be greater than or equal to 90% of the prior year's value (b_x^{z-1}).
 - For ages 20 – 22: Linearly closing y percent of the gap from the b_x^{2020} values and the regression lines by the n^{th} projected year, where y is equal to $(x - 19) / (30 - 19)$ and n is equal to $x - 13$. Then, the value of b_x^z stays constant from year n and later.
 - For ages 23 – 29: Linearly closing y percent of the gap from the b_x^{2020} values and the regression lines by the 10th projected year, where y is equal to $(x - 19) / (30 - 19)$. Then, the value of b_x^z stays at a constant value away from the regression line until the ultimate year, at which point b_x^z stays constant for future years.
 - For ages 30 and older: Linearly closing the gap from the b_x^{2020} values to the regression lines in the 10th projected year (2030). The values of b_x^z then follow the regression line until the ultimate year for each age. The values of b_x^z stay constant for that year and onwards.
- Then, preliminary cohort total fertility rates, $CTFR_p^t$, are calculated as the sum of all b_x^z values for each cohort born in year t ($\sum_{x=14}^{49} b_x^{t+x}$).

8. Next, calculate the weight (w) to use on the ultimate cohort TFR of 2.00. Then, $1 - w$ will be the weight to apply to the preliminary cohort total fertility rates, $CTFR_p^t$.

$$w = \left(\frac{\sum_{x=a+1}^{49} b_x^{t+x}}{\sum_{x=14}^{49} b_x^{t+x}} \right)^{\max(a,15)-14} \quad \text{where } a \text{ is the age of each cohort in 2020.}$$

9. Final cohort total fertility rates, $CTFR^t$, are calculated as $w * 2.00 + (1 - w) * CTFR_p^t$.
10. Now, adjust projected rates (2021 and later), b_x^z , on a cohort basis so that the values of $CTFR^t$ from Step 9 are hit (multiply each projected b_x^z by $CTFR^t / CTFR_p^t$.)
11. After Step 10 is complete for all years, impacts from the COVID-19 pandemic are factored in. The 2021 rates in Step 10 are multiplied by a factor of 0.92 from ages 14 – 39, and this factor increases to 1 on linear grade-in basis from ages 40 – 45. For 2022, the rates in Step 10 are multiplied by a factor of 0.96 from ages 14 – 39, and this factor increases to 1.00 on linear grade-in basis from ages 40 – 45. For years 2024-26, the decreases in 2021 and 2022 are made up for on a cohort basis evenly over each year on a cohort-basis (i.e., add 1/3 of the sum of the decrease in 2021 and 2022 to the step 10 rates for each year 2024-26 for each cohort).

1.2. MORTALITY

1.2.a. Overview

The National Center for Health Statistics (NCHS) collects data on annual numbers of deaths and the U.S. Census Bureau produces estimates of the U.S. resident population. Central death rates (${}_yM_x$) are defined as the ratio of (1) the number of deaths occurring during the year to persons between exact ages x and $x+y$ to (2) the midyear population between exact ages x and $x+y$. For historical years prior to 1968, ${}_yM_x$ are calculated from NCHS and Census data by sex. For historical years beginning in 1968, the same data are used in the calculations for ages under 65, but data from the Centers for Medicare and Medicaid Services (CMS) are used for ages 65 and over. Based on death by cause data from the NCHS, the ${}_yM_x$ are distributed by cause of death for years 1979 and later.³

Over the last century, death rates have decreased substantially. The historical improvement in mortality can be quantified by calculating the percentage reductions in log linear regressions of central death rates (${}_yAA_x$). In order to project future ${}_yM_x$, the Board of Trustees of the OASDI Trust Funds assumes an ultimate annual percentage reduction (${}_yAA_x^u$) that will be realized during the projection period for each sex and cause of death.

The basic mortality outputs of the MORTALITY subprocess that are used in projecting the population are probabilities of death by age and sex (q_x). The probability that a person age x will die within one year (q_x) is calculated from the central death rates (the series of ${}_yM_x$).

Period life expectancy (e_x) is defined as the average number of years of life remaining for people who are age x and are assumed to experience the probabilities of death for a given year throughout their lifetime. It is a summary statistic of overall mortality for that year.

Age-adjusted death rates (*ADR*) are also used to summarize the mortality experience of a single year, making different years comparable to each other. Age-adjusted death rates are a weighted average of the ${}_yM_x$, where the weights used are the numbers of people, male and female combined, in the corresponding age groups of the standard population, the 2010 U.S. Census resident population (${}_ySP_x$). Thus, if the age-adjusted death rate for a particular year and sex is multiplied by the total 2010 U.S. Census resident population, the result gives the number of deaths that would have occurred for that sex in the 2010 U.S. Census resident population if the ${}_yM_x$ for that particular year and sex had been experienced. Age-sex-adjusted death rates (*ASDR*) are calculated to summarize death rates for both sexes combined. They are calculated as a weighted average of the ${}_yM_x$, where each weight is the number of people in the corresponding age and sex group of the 2010 U.S. Census resident population.

MORTALITY projects annual ${}_yM_x$, which are then used to calculate additional outputs. The equations for this subprocess are given below:

$${}_yM_x = {}_yM_x(\cdot) \quad (1.2.1)$$

$${}_yAA_x = {}_yAA_x(\cdot) \quad (1.2.2)$$

³ Data needed to project central death rates by cause of death were obtained from Vital Statistics tabulations for years since 1979. For the years 1979-98, adjustments were made to the distribution of the numbers of deaths by cause in order to reflect the revision in the cause of death coding that occurred in 1999, making the data for the years 1979-98 more comparable with the coding used for the years 1999 and later. The adjustments are comparability ratios created using published data from the National Center for Health Statistics and calculated using their methodology.

$$q_x = q_x(\cdot) \quad (1.2.3)$$

$$\dot{e}_x = \dot{e}_x(\cdot) \quad (1.2.4)$$

$$ADR_s^z = \frac{\sum_x {}_ySP_x \cdot {}_yM_{x,s}^z}{\sum_x {}_ySP_x} \quad (1.2.5)$$

$$ASDR^z = \frac{\sum_s \sum_x {}_ySP_{x,s} \cdot {}_yM_{x,s}^z}{\sum_s \sum_x {}_ySP_{x,s}} \quad (1.2.6)$$

where ${}_yM_{x,s}^z$ refers to the central death rate between exact age x and $x+y$, by sex, in year z ; ${}_ySP_x$ denotes the number of people in the standard population (male and female combined) who are between exact age x and $x+y$; and ${}_ySP_{x,s}$ denotes the number of people, by sex, in the standard population who are between exact age x and $x+y$.

1.2.b. Input Data

Trustees Assumptions -

1. Each year the Board of Trustees of the OASDI Trust Funds sets the assumed ultimate values for the ${}_yAA_x$ by sex, age group (less than 15, 15-49, 50-64, 65-84, and 85+), and cause of death (Cardiovascular Disease, Cancer, Accidents and Violence, Respiratory Disease, Dementia, and All Other). The annual percentage reductions reach their ultimate values in the 25th year of the 75-year projection period. The ultimate rates of reduction by sex, age group, and cause of death can be found in Appendix 1.2-1.

NCHS Data -

2. Annual numbers of registered deaths by sex and age group for the period 1900-67. These data are not updated. Registered deaths refer to deaths in the Death Registration area. Since 1933, the Death Registration area has included all of the U.S.
3. The monthly number of births, by sex, for years 1938-2019. These data are updated annually, when the NCHS provides an additional year of data.
4. The number of infant deaths, by age, sex, and age group (under 1 day, 1-2 days, 3-6 days, 7-27 days, 28 days-1 month, 2 months, 3 months, ..., 11 months, 1 year, 2 years, 3 years, and 4 years), for years 1939-2019. These data are updated annually, when the NCHS provides an additional year of data.
5. The population of states in the Death Registration area by age group (0-4, 5-9, 10-14, ..., 65-69, 70-74, and 75+) and sex, for years 1900-39. These data are not updated.
6. The number of registered deaths, by sex and age group (85-89, 90-94, and 95+), used for the years 1900-67. These data are not updated.
7. Starting values for q_x for infant and toddler age groups (under 1 day, 1-2 days, 3-6 days, 7-27 days, 28

days-1 month, 2 months, 3 months, ..., 11 months, 1 year, 2 years, 3 years, and 4 years) from 1939-41 decennial life tables. These data are not updated.

8. From the NCHS public use records, deaths by sex, single year of age, cause of death, and marital status from 1968-2019. Marital status is not available until 1979, and cause of death is only used for 1979 and later.

U.S. Census Bureau Data -

9. Estimates of the July 1 resident population by single year of age (0 through 100+) for years 1980-2019. Each year, Census provides an additional year of data and updated data for years after the most recent decennial census.
10. Standard Population by year, sex, and age: 1980, 1990, 2000, and 2010. Used to standardize death rates for comparison. Updated every decennial census.
11. U.S. resident population by sex, age group, and year 1900-39 (0, 1-4, 5-9, ..., 70-74, and 75+). These data are not updated.
12. The resident population at ages 75-79 and 80-84, by sex, for years 1900-40 (at ten-year intervals). These data are not updated.
13. The resident population, by sex and age group (0, 1-4, 5-9, ..., 80-84, and 85+), for 1940-59. These data are not updated.
14. Resident population by single year of age and sex for 1960-79 (ages 0 through 59 for 1960-69 and ages 0 through 84 for 1970-79). These data are not updated.
15. The United States and Armed Forces overseas (USAF) population, by sex and single year of age for 1960-69 (ages 60 through 85+ for 1960-67 and ages 60 through 84 for 1968-69). These data are not updated.
16. Population, by 5-year age group to split 85+ pre-1968. These data are not updated.
17. Estimates of the population by marital status, sex, and age from the American Community Survey (ACS) public use microdata sample (PUMS) files for years 2000-19. In general, an additional year of data is available each year.

CMS Data -

18. Annual numbers of deaths of all Medicare enrollees, by sex and single year of age (ages 65 and over), 1968-87. These data are not updated.
19. Annual numbers of deaths of all Medicare enrollees, by sex and single year of age (ages 65 and over), for the period 1988-2005. These data are not updated.
20. Annual numbers of deaths of Medicare enrollees who are also Social Security or Railroad Retirement Board beneficiaries, by sex and single year of age (ages 65 and over), for the period 1988-2005. These data are not updated.
21. Annual numbers of deaths of all Medicare enrollees who are also Social Security or Railroad Retirement Board beneficiaries, by sex and single year of age (ages 65 and over), for the period 2006-19. These data

are updated annually, when the CMS provides an additional year or years of preliminary data and replaces prior year (or years) preliminary data with final data.

22. Annual numbers all Medicare enrollees, by sex and single year of age (ages 65 and over), 1968-87. These data are not updated.
23. Annual numbers of all Medicare enrollees, by sex and single year of age (ages 65 and over), for the period 1988-2006. These data are not updated.
24. Annual numbers of Medicare enrollees who are also Social Security or Railroad Retirement Board beneficiaries, by sex and single year of age (ages 65 and over), for the period 1988-2006. These data are not updated.
25. Annual numbers of all Medicare enrollees who are also Social Security or Railroad Retirement Board beneficiaries, by sex and single year of age (ages 65 and over), for the period 2006-19. These data are updated annually, when the CMS provides an additional year (or years) of preliminary data and replaces prior year (or years) preliminary data with final data.
26. Factors for ratioing age 65 Medicare deaths between 1988 and 2005. These data are not updated.

Other Input Data -

27. Internally developed resident population by single year of age (85 – 100+) and sex, 1968-79. These data were developed from USAF population data and are not updated.
28. List of NCHS 113 cause of death codes found in PUMS data file mapped to the causes used in Trustees Report.

1.2.c. Development of Output

Equation 1.2.2 - Percentage Reductions in Log Linear Regressions of Central Death Rates (${}_yAA_x$)

The ${}_yAA_x$, by sex and cause, are calculated based on the decline in the ${}_yM_x$ for the period 2008 through 2019, and distributed by 21 age groups,⁴ 2 sexes, and 6 causes of death.⁵ The values are calculated as the complement of the exponential of the slope of the least-squares line through the logarithms of the ${}_yM_x$.

The assumed ultimate values for the central death rates (${}_yAA_x^u$), as set by the Board of Trustees of the OASDI Trust Funds, are assumed to be reached in the 25th year of the 75-year projection period. These ultimate values are specified by six causes of death for the following five age groups: under 15, 15-49, 50-64, 65-84, and 85 and older. Male and female values are assumed to be equal to each other.

The starting values of ${}_yAA_x$, by the 21 age groups, sex, and cause, are assumed to equal the percentage reductions in log linear regressions of ${}_yM_x$ for the period 2008-19 when that percentage reduction is non-negative. However, if that percentage reduction is negative, then the starting values are assumed to be 75 percent of the percentage reduction. The weights are 0.2, 0.4, 0.6, and 0.8 for the earliest four years of the 12 years and are 1.0 for all other years. Available Medicare preliminary data is used for overall levels with the last available NCHS data year cause of death percentages carrying forward. For each year after the last data year,

⁴ Age groups are: 0, 1-4, 5-9, 10-14, ..., 90-94, and 95+

⁵ The six causes of death are: Cardiovascular Disease, Cancer, Accidents and Violence, Respiratory Disease, Dementia, and All Other.

the ${}_yAA_x$ are calculated by transitioning from the starting values of ${}_yAA_x$ to the Trustees' assumed ultimate values, ${}_yAA_x^u$. This is accomplished by repeating the following steps for each historical year after the last data year and for the first 24 years of the projection:

1. The difference between the prior year's calculated ${}_yAA_x$ and the assumed ultimate ${}_yAA_x^u$ is calculated. Note that for the first year of this process, the starting values of ${}_yAA_x$, as defined above, are used instead of the prior year's ${}_yAA_x$.
2. The current year's ${}_yAA_x$ is the assumed ultimate ${}_yAA_x^u$ plus 80 percent of the difference calculated in step 1.

For the 25th year of the projection, the ${}_yAA_x$ are set equal to their assumed ultimate values, ${}_yAA_x^u$.

Equation 1.2.1 – Central Death Rates (${}_yM_x$)

Values of ${}_yM_x$ are determined for each historical and projected year by the 21 age groups, 2 sexes, and 6 causes of death. The starting year for the projections of the ${}_yM_x$ is 2019, and is the most recent data year in the historical period. However, instead of using the historical data for ${}_yM_x$ in this year as the starting point for mortality projections, starting ${}_yM_x$ values are calculated to be consistent with the trend inherent in the last 12 years of available data. Each starting value for the ${}_yM_x$, by sex and cause of death, is computed as the exponential of the value for the most recent year falling on a weighted least square line, where the logarithm of ${}_yM_x$ is regressed on year, over the last 12 years. The weights are 0.2, 0.4, 0.6, and 0.8 for the earliest four years of the 12 years and are 1.0 for all other years.

For years after 2019, ${}_yM_x$ are projected, by sex and cause of death, by applying the respective ${}_yAA_x$ to the prior year ${}_yM_x$.

The COVID-19 pandemic impacts death rates. Factors were incorporated to account for these impacts. For ages 15 and older, death rates were increased for years 2020-23 (by 16.4% in 2020, 15% in 2021, 4% in 2022, and 1% in 2023). For ages 0 through 14, death rates were decreased in years 2020-22 (by 14% in 2020, 10% in 2021 in 2% for 2022).

Equation 1.2.3 – Probabilities of death (q_x)

In order to project population by age and sex, probabilities of death are applied to determine the projected number of deaths that will occur in the population. These probabilities, denoted as q_x , reflect the probability a person age x will die within one year, where x refers to age last birthday as of the beginning of each year. For each year in the historical and projection period, separate q_x series are estimated by sex.

Different methods of projecting q_x are used for age 0, for ages 1 through 4, for ages 5 through 94, and for ages 95 and above. The following descriptions provide a brief discussion of these different methods. Additional detail is provided in Actuarial Study number 120. This study, titled Life Tables for the United States Social Security area 1900-2100, can be found at:

<http://www.ssa.gov/OACT/NOTES/s2000s.html>. (Choose study number 120.)

- *Values for q_x at Age 0:* During the first year of life, mortality starts at an extremely high level, which becomes progressively lower. This is unlike mortality at other ages, which does not change very much within a single year of age. Thus, it is particularly important at age 0 to estimate accurately the pattern of mortality throughout the year of age, as described above, for the calculation of q_0 . For the period 1940 through the last historical year, q_0 is calculated directly from tabulations of births by month and from tabulations of deaths at ages 0, 1-2, 3-6, 7-28 days, 1 month, 2 months, ..., and 11 months. After the last

historical year, q_0 is calculated from ${}_1M_0$, assuming that the ratio of q_0 to ${}_1M_0$ measured for the last historical year would remain constant thereafter.

- *Values for q_x at Ages 1 – 4:* For the period 1940 through the last year of historical data, probabilities of death at each age 1 through 4 (q_x , $x = 1, 2, 3, 4$) are calculated from tabulations of births by year and from tabulations of deaths at ages 1, 2, 3, and 4 years. After the last historical year, each q_x (where $x = 1, 2, 3, 4$) is calculated from ${}_4M_1$ assuming that the ratio of q_x to ${}_4M_1$ measured for the last historical year would remain constant thereafter.
- *Values for q_x at Ages 5 – 94:* Probabilities of death for these ages are calculated from the projected central death rates, ${}_5M_x$. As mentioned above, the calculations are discussed in detail in Actuarial Study number 120.
- *Values for q_x at Ages 95+:* It has been observed that the mortality rates of women, though lower than those of men, tend to increase faster with advancing age than those of men. An analysis of Social Security charter Old-Age Insurance beneficiaries has shown that at the very old ages mortality increases about five percent per year of age for men and about six percent per year for women. For men, probabilities of death at each ages 95 and older are calculated as follows:

$$q_x = q_{x-1} \cdot \left(\frac{q_{94}}{q_{93}} \cdot \frac{99-x}{5} + 1.05 \cdot \frac{x-94}{5} \right) \quad x = 95, 96, 97, 98, 99$$

$$q_x = 1.05 \cdot q_{x-1} \quad x = 100, 101, 102, \dots$$

For women, the same formulas are used, except that 1.06 is substituted for 1.05. The larger rate of growth in female mortality would eventually, at a very high age, cause female mortality to be higher than male mortality. At the point where this crossover would occur, female mortality is set equal to male mortality.

The values of q_x used in projecting the population are based on age last birthday and are calculated by sex for ${}_{1/2}q_0$ (neonatal) and for q_x , where x represents age last birthday for ages 0 through 100 (with 100 representing the age group 100 and older). Because life table values of probabilities of death are based on exact ages, values for q_x representing age last birthday are derived as follows:

$${}_{1/2}q_0 = 1 - L_0 / l_0 \quad \text{for neonatal}$$

$$q_x = 1 - L_{x+1} / L_x \quad \text{for ages 0 to 99}$$

$$q_{100} = 1 - T_{101} / T_{100} \quad \text{for age group 100 and older}$$

See Actuarial Study number 120 for the definitions of the life table terms. This study can be found at: <http://www.ssa.gov/OACT/NOTES/s2000s.html>. (Choose study number 120; then section IV.A in the table of contents.)

In addition, probabilities of death are broken down further into marital status. Historical data indicate that differential in mortality by marital status is significant. To reflect this, future relative differences in death rates by marital status are projected to be the same as observed during calendar years 2015-19. These rates were developed by:

1. Taking the five-year age-group deaths from the public use NCHS data by marital status and dividing by the equivalent population sums from the ACS to get preliminary five-year age-group death rates by marital status.
2. Adjusting the older age groups death rates for consistency with prior age groups.

3. Adjusting the older age groups death rates further so that all marital statuses gradually reach the same value at the 95+ age group.
4. Converting these five-year age-group death rates to single year of age by using a modified H.S. Beers method of interpolation.
5. Converting these death rates to probabilities of death.

Equation 1.2.4 – Life expectancy

Actuarial Study number 120 presents background information on the calculation of life expectancy, e_x , from the probabilities of death (q_x). This study can be found at: <http://www.ssa.gov/OACT/NOTES/s2000s.html>. (Choose study number 120; then IV.A in the table of contents.)

Appendix: 1.2-1

The Board of Trustees of the OASDI Trust Funds sets the ultimate rates of mortality reduction by age group and cause of death. For comparison purposes, rates are also presented for two historical periods. Note that although the male and female ultimate rates are the same, the historical rates differ.

Average Annual Rates of Reduction in Central Death Rates by Age Group, Sex, and Cause

	Historical		Alternative II*		Historical		Alternative II*	
	1979 to 2019	2009 to 2019	2020 TR	2021 TR	1979 to 2019	2009 to 2019	2020 TR	2021 TR
			2044 to 2094	2045 to 2095			2044 to 2094	2045 to 2095
Under Age 15	Male				Female			
Cardiovascular Disease	1.94	2.24	2.3	1.9	1.68	1.63	2.3	1.9
Cancer	2.35	1.69	1.5	1.5	1.98	1.50	1.5	1.5
Accidents and Violence	2.39	0.26	1.0	1.0	2.07	-0.08	1.0	1.0
Respiratory Disease	2.28	2.07	2.0	2.0	2.46	2.63	2.0	2.0
Dementia	3.07	3.93	N/A	0.1	1.80	-1.78	N/A	0.1
Other**	2.25	1.81	1.7	1.7	2.14	1.70	1.7	1.7
Resulting Total ***	2.28	1.54	1.53	1.51	2.12	1.44	1.56	1.54
Ages 15 - 49	Male				Female			
Cardiovascular Disease	1.78	0.75	1.5	1.3	1.19	0.42	1.5	1.3
Cancer	1.88	2.25	1.5	1.5	1.58	1.77	1.5	1.5
Accidents and Violence	0.28	-2.44	0.7	0.7	-0.21	-2.35	0.7	0.7
Respiratory Disease	0.44	1.91	0.5	0.5	-0.47	2.08	0.5	0.5
Dementia	1.06	0.50	N/A	0.1	0.96	1.84	N/A	0.1
Other**	0.14	0.31	0.8	0.8	-0.09	-0.06	0.8	0.8
Resulting Total ***	0.72	-0.75	0.85	0.82	0.49	-0.21	0.92	0.89
Ages 50 - 64	Male				Female			
Cardiovascular Disease	2.36	0.19	2.2	1.5	1.96	0.07	2.2	1.5
Cancer	1.55	2.05	1.5	1.5	1.22	1.39	1.5	1.5
Accidents and Violence	-0.39	-3.34	0.5	0.5	-0.68	-2.73	0.5	0.5
Respiratory Disease	0.45	-0.33	0.7	0.7	-1.17	-0.75	0.7	0.7
Dementia	-2.51	-2.79	N/A	0.1	-3.23	-3.52	N/A	0.1
Other**	-0.34	-0.65	0.6	0.6	-0.31	-0.83	0.6	0.6
Resulting Total ***	1.23	0.03	1.05	0.95	0.82	0.03	1.05	0.98
Ages 65 - 84	Male				Female			
Cardiovascular Disease	2.73	1.32	2.2	1.9	2.61	1.70	2.2	1.9
Cancer	1.07	2.19	0.9	0.9	0.35	1.85	0.9	0.9
Accidents and Violence	0.25	-1.78	0.5	0.5	-0.13	-1.57	0.5	0.5
Respiratory Disease	0.49	1.51	0.3	0.3	-1.82	1.07	0.3	0.3
Dementia	-6.65	-1.89	N/A	0.1	-7.73	-2.38	N/A	0.1
Other**	-0.29	-0.63	0.3	0.3	-0.44	0.16	0.3	0.3
Resulting Total ***	1.37	0.93	0.79	0.74	0.81	0.93	0.73	0.68
Ages 85 and older	Male				Female			
Cardiovascular Disease	1.67	1.02	1.2	1.5	1.87	1.25	1.2	1.5
Cancer	0.01	0.84	0.5	0.5	-0.29	0.14	0.5	0.5
Accidents and Violence	-0.82	-1.79	0.3	0.3	-1.18	-2.15	0.3	0.3
Respiratory Disease	-0.45	1.59	0.2	0.2	-1.57	0.52	0.2	0.2
Dementia	-9.48	-2.14	N/A	0.1	-10.32	-2.35	N/A	0.1
Other**	-0.83	0.07	0.2	0.3	-0.78	0.75	0.2	0.3
Resulting Total ***	0.35	0.39	0.53	0.58	0.23	0.17	0.50	0.54
Total	Male				Female			
Cardiovascular Disease	2.27	1.00			2.18	1.27		
Cancer	1.04	1.88			0.62	1.45		
Accidents and Violence	0.12	-2.40			-0.27	-2.16		
Respiratory Disease	0.20	1.34			-1.54	0.70		
Dementia	-7.71	-2.06			-8.77	-2.37		
Other**	-0.20	-0.25			-0.25	0.20		
Resulting Total ***	1.02	0.45	0.75	0.74	0.62	0.45	0.71	0.69

* Alternative 1 is 1/3 times Alternative 2; Alternative 3 is 2.0 times Alternative 2.

** Other includes dementia in the 2020 TR but not in the 2021 TR.

***Resulting total represents average annual percent reduction in age-adjusted death rates for the last 50 years of the 75-year projection period.

1.3. LPR IMMIGRATION

1.3.a. Overview

LPR immigration is defined as those persons who have been admitted into the United States and been granted lawful permanent resident status. Legal emigration consists of LPR immigrants and U.S. citizens who depart the Social Security area population to reside elsewhere.

For each year z of the projection period, the LPR IMMIGRATION subprocess produces estimates of LPR immigration (L^z) and legal emigration (E^z), by age and sex, based on assumptions set by the Trustees for each category. In addition, the LPR IMMIGRATION subprocess disaggregates the estimates of L^z into those who have been admitted into the United States during the year (NEW^z) and those who adjusted from the other-than-LPR population to LPR status (AOS^z).

Each fiscal year,⁶ the Department of Homeland Security (DHS) collects data on the number of persons granted LPR status by age, sex, and class of admission. The U.S Census Bureau provided OCACT with an unpublished estimate of the annual number of legal emigrants, by sex and age, based on the change between the 1980 and 1990 censuses. These historical data are used as a basis for developing age-sex distributions that are applied to the Trustees' aggregate immigration assumptions to produce annual LPR immigration and annual legal emigration estimates by age and sex.

The primary equations of LPR IMMIGRATION, by age (x) and sex (s), for each year (z) of the 75-year projection period are summarized below:

$$NEW_{x,s}^z = NEW_{x,s}^z(\cdot) \quad (1.3.1)$$

$$AOS_{x,s}^z = AOS_{x,s}^z(\cdot) \quad (1.3.2)$$

$$L_{x,s}^z = NEW_{x,s}^z + AOS_{x,s}^z \quad (1.3.3)$$

$$E_{x,s}^z = E_{x,s}^z(\cdot) \quad (1.3.4)$$

$$NL_{x,s}^z = L_{x,s}^z - E_{x,s}^z \quad (1.3.5)$$

where $NL_{x,s}^z$ are the number of net LPR immigrants by age (x) and sex (s) for year z .

1.3.b. Input Data

Trustees Assumptions -

1. Each year the Board of Trustees of the OASDI Trust Funds specifies the total annual assumed values for LPR immigration and legal emigration. For the 2021 Trustees Report, the ultimate values for LPR immigration and legal emigration are 1,050,000 and 262,500, respectively (both reached in 2026).

Department of Homeland Security -

2. Historical LPR immigration by fiscal year (1941-73), 5-year age group (0-4, 5-9, ..., and 80-84), and sex. These data will not be updated.
3. Legalizations due to Immigration Reform and Control Act of 1986 (IRCA) by type (pre-1982s and SAWs), single year of age (0-99 and unknown age), sex (including unknown) and month for the years 1989-2011. These data will not be updated.

⁶ The federal fiscal year begins on October 1 of the previous calendar year and ends on September 30 of the specified calendar year.

4. Historical LPR immigration by fiscal year (1973-2019), single year of age (0 through 99, and unknown age), sex (including unknown), and class of admission (New Arrival, Adjustment of Status, Refugee, and Asylee). These data are updated annually, with the DHS providing an additional year of data each year.
5. Total adjustments of status for the years 1966 to 1972 (OCACT further estimates total adjustments of status for 1963-65). These data will not be updated.

U.S. Census Bureau –

6. Unpublished estimates of annual legal emigration by five-year age groups (0-4, 5-9, ..., and 80-84) and sex for 1990 based on the change between the 1980 and 1990 censuses. These data are updated occasionally (based on having new data from an outside source and on OCACT resource time constraints).

Other input data -

7. Legal emigration conversion factors. These estimates were developed internally by five-year age groups (0-4, 5-9, ..., and 80-84) and sex to reflect the fact that the estimated number of people leaving the United States is not equivalent to the number of people leaving the Social Security area. These data are updated when annual legal emigration estimates are updated (see above).

1.3.c. Development of Output

Equations 1.3.1 and 1.3.2 – LPR Immigration

The Trustees specify the aggregate number of LPR immigrants for each year of the 75-year projection period. In order to incorporate the numbers of new immigrants into the Social Security area population projections, the total level of new immigrants is disaggregated by age and sex.

There are two ways for an immigrant to be admitted into the U.S. for lawful permanent residence:

- (1) New arrivals, such as persons living abroad who are granted an LPR visa and then enter the U.S. through a port of entry. Refugees and asylees that are granted LPR status are also treated as new arrivals in the OCACT model.
- (2) Adjustments of status, who are people already residing in the U.S. as other-than-LPR immigrants and have an application for adjustment to LPR status approved by the DHS.

The DHS provides data on LPR immigrants by sex, single year of age, classification of admission, and fiscal year of entry. The 10 most recent years of data are used to calculate separate age-sex distributions for both new arrivals and adjustments of status by taking the following steps:

1. Refugee and Asylee LPR admissions are subtracted from the DHS adjustment of status data and added into the new arrival category.
2. The data are converted from fiscal year data to calendar year data.
3. For each class of admission (new arrivals and adjustments of status), the historical data for the last 10 years (from 2010-19) are combined to create an average age-sex distribution.

$NEW_{x,s}^z$, the expected number of new arrival LPR immigrants by age (x) and sex (s) for each year (z), is calculated by applying the age-sex distribution for new arrivals to the Trustees' assumed level of new arrivals.

The Trustees' assumed number of adjustments of status is multiplied by the age-sex distribution of adjustments of status to calculate $AOS_{x,s}^z$.

Equation 1.3.4 – Legal Emigration

The Trustees specify the aggregate amount of legal emigration for each year of the projection period. This is done by assuming a ratio of legal emigration to LPR immigration. For the 2021 Trustees Report, the ratio is assumed to be 25 percent.

In order to produce the number of emigrants from the Social Security area population, the total level of emigrants is disaggregated by age and sex. The disaggregation is based on a distribution of emigrants, by sex and five-year age groups, provided to OCACT in unpublished estimates by Census that are based on changes between the 1980 and 1990 censuses. Since the emigration numbers estimated by Census are for all people leaving the United States, they are adjusted downward by a series of conversion factors so the data correspond to the number of people leaving the Social Security area population.

For each sex (s), the Beers formula is used to interpolate and distribute each five-year age group into a single year of age (x) distribution, $EDIST_{x,s}$. For each projection year, this distribution is used to distribute the assumed level of total legal emigrants by age and sex using the following equation:

$$E_{x,s}^z = .25 * \left(\sum_{s=m}^f \sum_{x=0}^{100} L_{x,s}^z \right) \cdot EDIST_{x,s}$$

1.4. HISTORICAL POPULATION

1.4.a. Overview

The HISTORICAL subprocess provides estimates of the Social Security area population for each year in the period December 31, 1940, through December 31, 2018. The Social Security area population consists of:

- U.S. resident population and armed forces overseas *plus*
- Net census undercount *plus*
- Civilian residents of Puerto Rico, the Virgin Islands, Guam, the Northern Mariana Islands, and American Samoa *plus*
- Federal civilian employees overseas *plus*
- Dependents of armed forces and federal civilian employees overseas *plus*
- Residual beneficiaries living abroad *plus*
- Other citizens overseas

The U.S. Census Bureau collects population data by age, sex, and marital status (and by other characteristics) every ten years for the decennial census. Generally, each subsequent year, the Census Bureau publishes a “post-censal” population estimate. This subprocess combines these census and post-censal estimates, along with the estimates of the other components of the Social Security area population listed above, and components of change described in sections 1.1 to 1.3, to develop historical estimates of the total Social Security area population ($P_{x,s}^z$) and other-than-LPR population ($O_{x,s}^z$). Combining the total populations by single year of age and sex with an estimated marital status matrix provides the total Social Security area historical population by single year of age, sex, and marital status ($P_{x,s,m}^z$). These estimates are then used as the basis for the PROJECTED POPULATION subprocess described in section 1.8. The primary equations for this subprocess are given below:

$$P_{x,s}^z = P_{x,s}^z(\cdot) \quad (1.4.1)$$

$$P_{x,s,m}^z = P_{x,s,m}^z(\cdot) \quad (1.4.2)$$

$$O_{x,s}^z = O_{x,s}^z(\cdot) \quad (1.4.3)$$

1.4.b. *Input Data*

Long-Range OASDI Projection Data -

Demography

1. Probabilities of death from the MORTALITY subprocess, by age last birthday and sex, for years 1941-2019. These data are updated every year.
2. The number of new arrival LPR immigrants plus adjustments of status LPR immigrants by age and sex for years 1941-2018. These data are from the LPR IMMIGRATION subprocess and are updated each year.
3. The number of legal emigrants by age and sex for years 1941-2018. These data are from the LPR IMMIGRATION subprocess and are updated each year.
4. The number of adjustments of status by age and sex for years 1941-2018. These data are from the LPR IMMIGRATION subprocess and are updated each year.
5. The number of other-than-LPR immigrants legalized under the Immigration Reform and Control Act of 1986 (IRCA) from the LPR IMMIGRATION subprocess. These data are reproduced each year and updated if new data is available.
6. Birth rates by single year of age of mother (14-49) for the years 1941-2019 from the FERTILITY subprocess. These data are updated each year.

U.S. Census Bureau Data -

7. Estimates of U.S resident population and Armed Forces overseas population as of each July 1 (1940-79) by sex and single-year of age through 84, and for the group aged 85 and older. These data are generally not updated.
8. Estimates of the U.S. resident population for each decennial census (April 1) 1970-2010 by sex and single year of age 0 through 85+. New decennial census estimates come out about every ten years.
9. Estimates of total U.S. resident population and total U.S. resident population plus Armed Forces overseas population for each January of each decennial census year from 1990 and 2010. New decennial census estimates come out about every ten years.
10. Estimates of U.S resident population, and U.S. resident plus Armed Forces overseas population as of each July 1 (1980-2019) by sex and single year of age 0 through 99, and ages 100 and older. Generally, the U.S. Census Bureau restates the data back to the most recent decennial census and includes one additional year of data.
11. Estimates of U.S resident population, and U.S. resident population plus Armed Forces overseas population as of each January 1 (1981-2019) by sex and single-year of age 0 through 99, and ages 100 and older. Generally, the U.S. Census Bureau restates the data back to the most recent decennial census (April) and includes one additional year of data.
12. Estimates of the population by marital status, sex, and age from the American Community Survey (ACS) public use microdata sample (PUMS) files for years 2000-19. In general, an additional year of data is available each year.

13. Undercount factors by single year of age (0-85+) and sex, estimated using post-censal survey or demographic analysis data. These data are updated after each decennial census.
14. The total annual population estimates for Puerto Rico, Virgin Islands, Guam, Northern Mariana Islands, and American Samoa for years 1951-2019. For each Trustees Report, an additional data year is downloaded from the U.S. Census Bureau's international database. Historical data back to 1951 is also obtained if any changes have occurred.
15. Decennial census population estimates, by varying degree of age detail and sex, for decennial censuses from 1950-2000 for territories and components outside the 50 states, D.C., and armed forces overseas. Most data are aggregated into 18 age groups for each sex, though single year of age data is available for young ages in the territories for 1960 and 1970 and for all ages starting in 1980. New estimates are added as they become available.
16. July populations of the territories by single year of age and sex from 2000-19. An additional year of data is available each year.
17. From the ACS PUMS, number of existing marriages from 2000-19 by age of husband crossed with age of wife. In addition, starting with the 2012 ACS, number of existing marriages by age of each spouse is available for same-sex couples. Final grids for same-sex couples are adjusted based on reported same-sex marriages from the states through 2014. Generally, an additional year of data is available each year.
18. From ACS PUMS, flows of foreign-born ins and Cuban ins by single year of age, sex, and year of entry used to produce total other-than-LPR estimates for January 1, 2013-19.
19. From decennial census PUMS (via the University of Minnesota's IPUMS website), number of existing marriages for decennial census years 1940-2000 by age group of husband crossed with age group of wife. New estimates are added as they become available.
20. From decennial census PUMS (via the University of Minnesota's IPUMS website), estimates of the population by marital status, age, and sex for decennial census years 1940-2000. New estimates are added as they become available.
21. Estimates of net immigration by age and sex of the U.S. resident population plus armed forces overseas (USAF) population from April 1, 2000, through July 1, 2019. An additional year of data is available each year.
22. Total Americans overseas estimate based on international data sources and estimates of federal employees and military in Iraq and Afghanistan. The data from the various international sources are derived from different years but center around the year 2003. Additional data will be updated as they become available.
23. Total Civilian population in Alaska and Hawaii for years 1940-49. These data are not updated.

Other input data -

24. From the Department of State, old historical total estimates of outside area populations (federal employees overseas, overseas dependents of federal employees and military, and other Americans overseas) for various years between 1951 and 1990.
25. The SSA Annual Statistical Supplement provides estimates of the total number of OASDI beneficiaries

living abroad as of December 31, for most years 1957-2019. Age group data is also available for 0 – 17, 18 – 54, and 55 – 61 starting in 1997. For each Trustees Report, an additional year of data is available. Age group data is updated for each decennial census year.

26. From the National Center for Health Statistics (NCHS), births by month and sex of child, for years 1931-2019. Each year, NCHS provides another year of data.
27. From the NCHS, National Survey of Family Growth (NSFG) public-use data that help split up the population eligible for same-sex marriage into marital statuses starting in 2013. These survey data are from 2011-15. This data is updated as they become available.
28. From the NCHS, National Survey of Family Growth (NSFG) public-use data that split up the total population, by sex, into the population eligible for same-sex marriage and the population eligible for opposite-sex marriage. These survey data are from 2011-15 and 2015-17. This data is updated as they become available.
29. From the Department of Homeland Security (DHS), the number of unauthorized immigrants and nonimmigrants from January 1, 2005-12. In addition, other estimates from the Unauthorized Immigrant Population Reports and emails from DHS are used to estimate January 1, 2012, through January 1, 2019, unauthorized immigrants and nonimmigrants including an adjustment for shift in reference date, undercount of nonimmigrants, legally resident immigrants, foreign-born flows from 1980 and later of LPR immigrants, mortality, and emigration.
30. From the Office of Personnel Management (OPM), total estimates of the number of federal employees overseas from July 1, 1998-2013. These estimates are updated as they become available on the OPM website.
31. From the OPM, the number of federal employees overseas by single year of age and sex from a subset of the OPM data source above. Years 1980-2019 are available. These estimates are updated as they become available.
32. From the Department of Defense, total numbers of armed forces in Puerto Rico, the Virgin Islands, Guam, and American Samoa each decennial census year starting in 1990. These data are updated as they become available.
33. Using 2015 TR death rates and historical populations, an assumed December 31, 1940, 85+ distribution. These data are not updated.
34. Assumed January total populations added to the Social Security for years 1951, 1957, and 1961 when new territories were added to the Social Security area. These data are not updated.
35. Armed forces counts, by age group and sex, from 1940-57. These data are not updated.

1.4.c. *Development of Output*

Equation 1.4.1 – Historical Population by age and sex ($P_{x,s}^z$)

The Census Bureau's estimate of the residents of the 50 States, D.C., and U.S. Armed Forces overseas is used as a basis for calculating $P_{x,s}^z$. The base estimate is adjusted for net census undercount and increased for other U.S. citizens living abroad (including residents of US territories) and for non-citizens living abroad who are insured for Social Security benefits.

The estimates of the number of residents of the fifty States and D.C. and Armed Forces overseas, as of July 1 of each year, by sex for single years of age through 84, and for the group aged 85 or older, are obtained from the Census Bureau. January 1 and April 1 estimates by sex for single years of age through 84, and for the group aged 85 or older for selected years starting in 1990 and 1970, respectively, are also obtained from the Census Bureau. Adjustments for net census undercount are estimated using post-censal survey data from the Census Bureau. Population counts over age 65 after the last Census year are modified to be consistent with OCACT mortality and Census USAF net immigration data. The numbers of persons in the other components of the Social Security area as of July 1 are estimated by sex for single years of age through 84, and for the group aged 85 or older, from data of varying detail. Numbers of people residing in Puerto Rico, the Virgin Islands, Guam, American Samoa, and the Northern Mariana Islands are estimated from data obtained from the Census Bureau. Numbers of Federal civilian employees overseas are based on estimates from the Office of Personnel Management (OPM). Dependents of Federal civilian employees and Armed Forces overseas are based on the stock of Federal civilian employees from OPM and the stock of armed forces overseas from the Census Bureau. Other citizens overseas covered by Social Security are also based on estimates compiled by the Census Bureau. The overlap among the components, believed to be small, is ignored.

The first step of the process is to estimate $P_{x,s}^z$ as of December 31st for certain “tab years” (1940, 1950, 1956, 1960, each December prior to a decennial Census year after [1969 through 2009], and the last year of historical data [2018 for the 2021 Trustees Report]). For ages 0-84, $P_{x,s}^z$ for each tab year, is set equal to an undercount adjustment plus other component populations plus:

- The averaged surrounding July 1 U.S. population and armed forces overseas counts from the Census Bureau prior to 1970
- Modified April 1 U.S. populations from the Census Bureau for decennial census years from 1970 through 2000.
- The January 1 U.S. population and armed forces overseas counts from the Census Bureau modified by OCACT mortality rates and Census USAF net immigration for ages over 65 for 2010 and 2019.

For ages 85 and over, $P_{x,s}^z$ for each tab year is set equal to [Built Up Pops Age x, Sex s] * [Total 85+ for Sex s]/[Total Built Up 85+ for Sex s], where the built up estimates are created by taking into account deaths and immigration data from the previous tab year and [Total 85+ for Sex s] is the sum of the calculated U.S. population and armed forces overseas calculated using the same method listed above for each year for ages 0-84.

For years between the tab years, populations are estimated taking into account the components of changes due to births, deaths, legal emigration, adjustments of status, and net LPR immigration (or total net immigration, if known) during that time period. These estimates are then multiplied by the appropriate age-sex-specific ratios so that the error of closure at the tab years is eliminated.

Equation 1.4.2 – Historical Population by age, sex, and marital status ($P_{x,s,m}^z$)

Since eligibility for auxiliary benefits is dependent on marital status, the Social Security area population is disaggregated by marital status. The four marital states are defined as single (having never been married), married, widowed, and divorced.

The distribution of the number of existing marriages are available for decennial census years 1940-2000 from Census public use microdata sample (PUMS) files. These data are aggregated by age group of husband crossed with age group of wife. Additional tabulations from the American Community Survey from 2000-19 are incorporated to adjust these marital prevalence grids for changes since 2000. The grids are transformed from age grouped numbers to single year of age figures from ages 14 to 100+ for husband and wife using the two dimensional H.S. Beers method of interpolation.

Percentages of single, married, widowed, and divorced persons are calculated by using the H.S. Beers method of interpolation on compiled data by age group and sex based on Census and/or ACS PUMS. Data is converted to a December basis for each year by taking a weighted average of surrounding Census and/or ACS data. Note that the ACS is not used until 2006, the first year grouped quarters were included. These percentages are multiplied by the total populations calculated in Equation 1.4.1 for each age, sex, and year to get a preliminary population for each age, sex, and marital status.

To keep the marriage prevalence grids and the marital status percentages smooth and consistent, several algorithms are used. First, the married population is adjusted so that the number of married men equals the number of married women (though, this is not forced to be ultimately true once same-sex marriages were federally recognized as described in the next paragraph). Then, the number of married persons for each age and sex is set equal to the marginal total of the associated year's marital prevalence grid. Finally, the other marital statuses population totals are adjusted to keep the total number of people in all marital statuses the same as calculated before splitting into marital statuses.

The population is modeled to include the following population statuses for December 31, 2013, and later: heterosexual, gay, and lesbian. Gay and lesbian populations are broken out assuming 2.5% of the male population is gay and 4.5% of the female population is lesbian. (Note that this means we assume 2.5% of the male population and 4.5% of the female population are in an existing same-sex marriage, or would same-sex marry, versus opposite-sex marry.) Marriage grids of age of older spouse crossed with age of younger spouse for same-sex couples are needed starting December 31, 2013. The grids and populations were produced using data from the American Community Survey, National Survey of Family Growth, and state-level same-sex marriage data.

Equation 1.4.3 – Historical Other-Than-LPR Population by age and sex ($O_{x,s}^z$)

This subprocess also estimates historical levels of other-than-LPR in the population, by age and sex. For each year, an initial net residual estimate by single year of age and sex is backed out from estimates of beginning and end of year populations, births, deaths, LPR immigrants, adjustments of status, and legal emigrants. This net residual equals the implied initial other in minus other out. These residuals are then modified to ensure reasonableness. Next, using these modified net residuals, along with adjustments of status and other-than-LPR deaths (using the same death rates as for the total population), an initial other-than-LPR stock is built. These stocks are then modified to ensure reasonableness. After 2000, one further adjustment is done to the stocks. From January 2001 through January 2004, the total other-than-LPR populations are set equal to the values that linearly grade from the final OCACT January 2000 total other-than-LPR population to a DHS-based January 2005 total other-than-LPR population. From January 2005 through January 2012, the total other-than-LPR population is forced to match DHS-based total other-than-LPR population estimates. For January 2013 through January 2019, the total other-than-LPR populations are estimated based on the DHS method and emailed data from DHS.

1.5. OTHER-THAN-LPR IMMIGRATION

1.5.a. Overview

The term “other-than-LPR immigration” refers to persons entering the U.S. in a manner other than being lawfully admitted for permanent residence, and who reside in the U.S. for at least 6 months. This includes temporary immigrants (persons lawfully admitted for a limited period of time, such as temporary workers and foreign students), also called nonimmigrants, in addition to undocumented immigrants living in the U.S. These undocumented immigrants can be split into those that were never authorized to enter the U.S. and those that were nonimmigrants but overstayed their visas (visa-overstayers). The term “other-than-LPR emigration” refers to those in the other-than-LPR immigrant population who leave the Social Security area.

For each year z of the projection period, the OTHER-THAN-LPR IMMIGRATION subprocess produces estimates of other-than-LPR immigration ($OI_{x,s,t}^z$), by age (x), sex, and type (t) based on assumptions set by the Trustees. Estimates of projected other-than-LPR emigration ($OE_{x,s,t}^z$), by age, sex, and type are also developed in this subprocess.

The Department of Homeland Security (DHS) estimated the stock of nonimmigrants by age group and sex for April 2008, December 2010, and April 2016. The HISTORICAL POPULATION subprocess already produces historical estimates of other-than-LPR immigrants. These historical data are used to develop recent estimates of the other-than-LPR stock by age, sex, and type, where type is never-authorizeds, nonimmigrants, and visa-overstayers.

The primary equations of OTHER-THAN-LPR IMMIGRATION, by age (x), sex (s), and type (t) for each year (z) of the 75-year projection period are summarized below:

$$OI_{x,s,t}^z = OI_{x,s,t}^z(\cdot) \quad (1.5.1)$$

$$OE_{x,s,t}^z = OE_{x,s,t}^z(\cdot) \quad (1.5.2)$$

$$NO_{x,s,t}^z = OI_{x,s,t}^z - OE_{x,s,t}^z - AOS_{x,s,t}^z \quad (1.5.3)$$

where $NO_{x,s,t}^z$ are the number of net other-than-LPR immigrants, by age (x), sex (s), and type (t) for year z , and $AOS_{x,s,t}^z$ are the number of adjustments to LPR status from the LPR IMMIGRATION subprocess, by age (x), sex (s), and type (t) for year z ;

$$OP_{x,s,t}^z = OP_{x-1,s,t}^{z-1} + OI_{x,s,t}^z - OE_{x,s,t}^z - AOS_{x,s,t}^z - OD_{x,s,t}^z \quad (1.5.4)$$

where, $OP_{x,s,t}^z$ is equal to the other-than-LPR immigrant population, by age (x), sex (s), and type (t) as of December 31st of each year (z), $OD_{x,s,t}^z$ are the number of deaths in the other-than-LPR immigrant population by age (x), sex (s), and type (t) for each year (z), and $AOS_{x,s,t}^z$ are the number of adjustments to LPR status by age (x), sex (s), and type (t) for each year (z).

1.5.b. *Input Data*

Trustees Assumptions -

1. Each year the Board of Trustees of the OASDI Trust Funds specifies the assumed total annual values for other-than-LPR immigration. The ultimate annual level was set at 1,350,000 persons per year for each year beginning in 2026. The level of other-than-LPR immigration was estimated at 1,100,000 for 2019. Due to COVID effects, the level was then estimated to drop to 420,000 and 880,000, for years 2020 and 2021, respectively. Finally, the level was estimated to increase to 1,350,000 for year 2022, and increase again to 1,750,000 for years 2023-25.

Long-Range OASDI Projection Data -

Demography

2. Historical and projected probabilities of death by age last birthday (including a neonatal mortality factor, single year of age for ages 0-99, and age group 100+) and sex, for years 1941-2100. These data are updated each year from the MORTALITY program.
3. Historical net other-than-LPR immigration by single year of age (-1-99+)⁷ and sex for years 1961-2018. These data are updated each year from the HISTORICAL program.
4. Historical December 31 other-than-LPR immigrants by single year of age (0-100+) and sex for years 1963-2018. These data are updated each year from the HISTORICAL program.
5. Historical July 1 other-than-LPR immigrants by single year of age (0-100+) and sex for years 1964-2018. These data are updated each year from the HISTORICAL program.
6. Final historical year (on a December 31 basis). This datum is updated each year from the HISTORICAL program.
7. Historical new arrivals by single year of age (-1-100+) and sex for years 1941-2018. These data are updated each year from the LPR IMMIGRATION program.
8. Historical and projected adjustments of status by single year of age (-1-100+) and sex for years 1941-2100. These data are updated each year from the LPR IMMIGRATION program.

Department of Homeland Security –

9. Components of the unauthorized immigrant population by year for 2005-12, from the Unauthorized Immigrant Population Reports. These data are updated as new data become available.
10. Components of the LPR population by year for 2005-11, from the Unauthorized Immigrant Population Reports. Components of the LPR population by year for 2012-19, from unpublished DHS estimates. These data are updated as new data become available.
11. Nonimmigrant stock in April 2008, December 2010, and April 2016 by age-group and sex. These data are updated as new data become available and internal resources are sufficient to examine and interpret such new data.

⁷ Age -1 represents births that occur during the year.

12. Nonimmigrant admissions by class of admission for various fiscal years 1981-2016. These data are updated as new data become available and internal resources are sufficient to examine and interpret such new data.
13. Total nonimmigrants as of January 1, 2005-11, from the Unauthorized Immigrant Population Reports. Total nonimmigrants as of January 1, 2012-19, from unpublished DHS estimates. These data are updated as new data become available.
14. Total annual approvals for initial grants under the 2012 Deferred Action for Childhood Arrivals (DACA) initiative, for fiscal years 2013-18. These data are updated as new data become available.
15. End-of-year population of people with DACA status, by sex (including unknown) and single year of age (0-50, including unknown), for years 2013-19. These data are updated as new data become available.

U.S. Census Bureau –

16. From the American Community Survey (ACS), foreign-born new persons by ACS year (2000-19), entry year (1900-2019), age (0-100) and sex. These data are updated as new data become available.
17. From the ACS, total foreign-born population and total population for 2000-19, and total population in Puerto Rico for 2005-19, used to calculate undercount factors (Note that the population referred to in each case is the beginning-of-year population). These data are updated as new data become available.
18. From the 2012 ACS, persons, by entry year (1900-2012), age (0-100) and sex, that are:
 - Foreign-born citizens
 - Foreign-born non-citizens that are in school or are high-school graduates
 - Non-citizen parents of citizen children
 - Non-citizen parents of citizen children that are in school or are high school graduates
 These data are not updated.
19. From the 2012 ACS, persons, by entry year (1900-2012), age (0-100) and sex, that are eligible for temporary protected status (TPS) based on originating from various countries by certain dates and are:
 - Foreign-born citizens
 - Foreign-born non-citizens that are in school or are high-school graduates
 - Non-citizen parents of citizen children
 - Non-citizen parents of citizen children that are in school or are high school graduates
 These data are not updated.

Other input data -

20. The number of those potentially eligible under the 2012 DACA initiative by age group and an overall split by sex, from the Migration Policy Institute. These data are updated as new data become available and internal resources are sufficient to examine and interpret such new data.
21. Internally developed numbers of those that were potentially eligible under the 2014 DACA initiative that were not eligible under the 2012 DACA initiative by age and sex. These data will not be updated.
22. Internally developed numbers of those that were potentially eligible under the Deferred Action for Parents of Americans and LPRs (DAPA) initiative, by age and sex. These data will not be updated.
23. Internally developed factors of potential DACA stock attaining DACA status by sex and ages 5-100 for

the first, second, and ultimate DACA years. These data are updated as new data become available and internal resources are sufficient to examine and interpret such new data.

24. Internally developed factors to apply to other-than-LPR immigrants that enter as nonimmigrants. These data will not be updated.
25. Internally developed factors used to create the nonimmigrant other in distribution by age and sex for each year. These factors ensure that there will be enough nonimmigrant stock to transfer to LPR status. These data will not be updated.
26. Internally developed overstay percentages by age. These data are based off a RAND Corporation document using data from the 1980s, and are adjusted based on insights from the DHS. These data are updated as new data become available and internal resources are sufficient to examine and interpret such new data.
27. Internally developed rates of departure for the non-DACA/DAPA potential/actual never-authorizeds for non-recent arrivals. These data are updated as new data become available and internal resources are sufficient to examine and interpret such new data.
28. Internally developed rates of departure for the non-DACA/DAPA potential/actual non-immigrants. These data are set to initial rates in 2015 (when the Executive Actions went into effect including decreased deportation of non-felons) and then gradually increase to the ultimate rates in 2025. These data are updated as new data become available and internal resources are sufficient to examine and interpret such new data.
29. Internally developed rates of departure for the non-DACA/DAPA potential/actual visa overstayers. These data are updated as new data become available and internal resources are sufficient to examine and interpret such new data.
30. Internally developed rates of departure for non-DACA potential/actual never authorizeds for non-recent arrivals to use prior to 2015 (when the Executive Actions went into effect including decreased deportation of non-felons). These data are updated as new data become available and internal resources are sufficient to examine and interpret such new data.
31. Internally developed rates of departure for the non-DACA potential/actual visa overstayers to use prior to 2015 (when the Executive Actions went into effect including decreased deportation of non-felons). These data are updated as new data become available and internal resources are sufficient to examine and interpret such new data.
32. Internally developed number of other-than-LPR immigrants, by age and sex, for years 1999-2010. These data will not be updated.

1.5.c. *Development of Output*

The ACS provides data to help derive the number of foreign-born new arrivals, which is then used to separate the historical net other-than-LPR immigration into those entering and those leaving. There are several other key inputs that go into this calculation, including an estimated undercount factor. This factor accounts for (1) differences between the foreign-born data from the ACS and the component pieces obtained from DHS, (2) differences between the ACS (Public Use Microdata Sample) and Census' total population, and (3) the foreign-born residing in Puerto Rico. The estimated other-than-LPR immigration is calculated by taking the foreign born from the ACS (after smoothing and applying the undercount factors) and subtracting the LPR new arrivals. The estimated historical other-than-LPR emigration is then calculated as the difference between the net other-

than-LPR immigration (calculated in the HISTORICAL subprocess) and the estimated historical other-than-LPR immigration. A series of steps are then taken to smooth the two categories. Based on various assumptions, the historical other-than-LPR immigrants are split into those who arrive or depart the Social Security area as a never-authorized immigrant, nonimmigrant, and visa-overstayer immigrant.

Equation 1.5.1 – Other-Than-LPR Immigration

For each projection year, an age-sex-type distribution is used to distribute the aggregate number of other-than-LPR immigrants by age, sex, and type. This age-sex-type distribution is denoted as $ODIST_{x,s,t}$ and is developed by using average historical estimates of other-than-LPR immigrants entering the country from 2000 through 2007.

The assumed total level of other-than-LPR immigration is denoted by TO^z . Thus, for each year (z) other-than-LPR immigration is defined by the following equation:

$$OI_{x,s,t}^z = TO^z \cdot ODIST_{x,s,t}$$

Equation 1.5.2 – Other-Than-LPR Emigration

$OE_{x,s,t}^z$ denotes the annual number of other-than-LPR immigrants who depart the Social Security area by age (x), sex (s), and type (t). These estimates are based on 2014 TR build-up of stocks from 2008 through 2010 including other-than-LPR immigration discussed above, deaths, adjustments of status (from the LPR IMMIGRATION subprocess), and assumptions about the number of departures from each type. Deaths for the other-than-LPR immigrant population use the same death probabilities as the total population:

$$OD_{x,s,t}^z = q_{x,s}^z \cdot OP_{x,s,t}^z$$

Then, for this 2008-10 period, rates are calculated by dividing $OE_{x,s,t}^z$ by $OP_{x,s,t}^z$ for each age, sex, and type. After smoothing and adjusting for the effects of the recent recession, these rates are used to calculate $OE_{x,s,t}^z$ in projected years by being applied to the other-than-LPR stock populations $OP_{x,s,t}^{z-1}$ for the overstayer and nonimmigrant stocks. For the never authorized stock, these rates are further adjusted and split into two categories so that recent arrivals are exposed to twice the rates as the residual never authorized stock. For the potential DACA population, and for those that are already in the DACA population, the exit rates are lower than for those not eligible for DACA.

This subprocess also splits historical other-than-LPR immigrants into the various categories. It is assumed that all other-than-LPR immigrants were nonimmigrants as of December 31, 1963. Between December 31, 1963, and December 31, 2010, the percentage of total other-than-LPR immigrants by age and sex in each type is linearly interpolated from the percentages at those two points in time. Between December 31, 2010, and December 31, 2015, a similar interpolation is done from the percentages at those two points in time. A final adjustment ensures the total nonimmigrants are appropriate, based on DHS nonimmigrant admissions or, if available, stock estimates.

Finally, this subprocess also projects the DACA population, a subset of the other-than-LPR immigrant population, by age (x) and sex (s). The DACA population consists of other-than-LPR immigrants who meet specific criteria and are granted authorization to work. The eligible DACA population is estimated separately by those that meet the age, residency, and educational requirements. Rates are applied to the eligible population to estimate the net number of individuals who actually apply and obtain DACA status. A final adjustment insures that the DACA population by age and sex is appropriate, based on DHS stock estimates.

Note that the DAPA and the 2014 DACA are no longer being applied to this subprocess. Furthermore, it is assumed that there will be no new 2012 DACA's for years 2019-20. These assumptions may change according to future laws, executive actions, and/or court rulings that may affect the DACA program.

1.6. MARRIAGE

1.6.a Overview

The National Center for Health Statistics (NCHS) collected detailed data on the annual number of new marriages in the Marriage Registration Area (MRA), by age of husband crossed with age of wife, for the period 1978 through 1988 (excluding 1980). In 1988, the MRA consisted of 42 States and D.C. and accounted for 80 percent of all marriages in the U.S. Estimates of the unmarried population in the MRA, by single year of age (or age group if single year of age was not available) and sex, were obtained from the NCHS. Marriage rates for this period are calculated from these data. The age-of-husband crossed with age-of-wife marriage grid rates are transformed from age grouped numbers to single year of age figures from ages 14 to 100+ for husband and wife using the two dimensional H.S. Beers method of interpolation.

Beginning in 1989, the NCHS no longer collected data on the annual number of new marriages in the MRA. However, for years 1989-95, they supplied less detailed data on new marriages from a subset of the MRA. Beginning in 2008, the American Community Survey (ACS) started asking if a person was married in the last 12 months. Using this question, along with ages of spouses, grids of new marriages by age-group-of-husband crossed with age-group-of-wife were developed for years 2007 and later. For the years between 1995 and 2007, the marriage grids were linearly interpolated.

Age-specific marriage rates ($\hat{m}_{x,y}^z$) for a given year (z) are defined as the ratio of (1) the number of marriages for a given age-of-husband (x) crossed with age-of-wife (y) to (2) a theoretical midyear unmarried population at those ages ($P_{x,y}^z$). The theoretical midyear population is defined as the geometric mean⁸ of the midyear unmarried male population and unmarried female population.

An age-adjusted central marriage rate (\widehat{AMR}^z) summarizes the $\hat{m}_{x,y}^z$ for a given year. The standard population chosen for age adjusting is the unmarried male population and unmarried female population in the Social Security area population as of July 1, 2010. The first step in calculating the total age-adjusted central marriage rate for a particular year is to determine an expected number of marriages by applying the age-of-husband-age-of-wife specific central marriage rates for that year to the geometric mean of the corresponding age groups in the standard population.

The \widehat{AMR}^z is then obtained by dividing:

- The expected number of marriages by
- The geometric mean of (1) the number of unmarried men, ages 15 and older, and (2) the unmarried women, ages 15 and older, in the standard population.

The MARRIAGE subprocess projects annual $\hat{m}_{x,y}^z$ by age-of-husband crossed with age-of-wife. The equations for this subprocess are given below:

$$\hat{m}_{x,y}^z = \hat{m}_{x,y}^z(\cdot) \quad (1.6.1)$$

$$\widehat{AMR}^z = \frac{\sum_{x,y} P_{x,y}^S \cdot \hat{m}_{x,y}^z}{\sum_{x,y} P_{x,y}^S} \quad (1.6.2)$$

⁸ The geometric mean, as used in this document, is the square root of the product of two numbers.

where x and y refer to the age of men and women, respectively, and $P_{x,y}^S$ is the theoretical unmarried population in the Social Security area population as of July 1, 2010 (the geometric mean of the corresponding age groups in the standard population).

1.6.b. *Input Data*

Long-Range OASDI Projection Data -

Demography

1. Estimates of the Social Security area population as of December 31, by age, sex, and marital status for years 1977-2018. These data are updated each year based on output of the HISTORICAL POPULATION subprocess.
2. Final historical year (on a December 31 basis). This datum is updated each year from the HISTORICAL POPULATION subprocess.

Assumptions -

3. For each Trustees Report, ultimate values for the \widehat{AMR}^z are assumed. The \widehat{AMR}^z reaches its ultimate value in the 25th year of the 75-year projection period. For the 2020 report, the ultimate \widehat{AMR}^z assumption is 4,000 per 100,000 unmarried couples.

NCHS Data -

4. Number of new marriages in the MRA, by age-of-husband crossed with age-of-wife, for calendar years 1978 through 1988, excluding 1980. These data are not available for years after 1988. The data vary in detail by year. They are broken out by single year age-of-husband crossed with single year age-of-wife for many ages (particularly younger ages).
5. Number of unmarried men and women in the MRA for calendar years 1978 through 1988, excluding 1980. These data are not available for years after 1988. The data are generally broken out by single year age for ages under 40 and by age groups 40-44, 45-49, 50-54, 55-59, 60-64, 65-74, and 75+.
6. Number of new marriages, in a subset of the MRA, by age-group-of-husband crossed with age-group-of-wife (age groups include 15-19, 20-24, 25-29, 30-34, 35-44, 45-54, 55-64, and 65+), for calendar years 1989-95. These data are updated as new data become available and internal resources are sufficient to examine and interpret such new data.
7. The total number of new marriages in the MRA less marriages in those states not included in the MRA unmarried population for the period 1957-88. These data are not updated.
8. The total number of new marriages in the United States for the period 1989-2019. Normally, each year, the NCHS publishes the total number of marriages for one more year.
9. Number of new marriages in the MRA for years 1979 and 1981-88 by age group (age groups include 14-19, 20-24, 25-29, 30-34, 35-44, 45-54, 55-64, and 65+), sex, and prior marital status (single, widowed, and divorced). These data are not available for years after 1988.
10. Number of unmarried people in the MRA (in thousands) for years 1982 - 1988 by age group (age groups include 14-19, 20-24, 25-29, 30-34, 35-44, 45-54, 55-64, and 65+), sex, and prior marital status (single,

widowed, and divorced). These data are not available for years after 1988.

11. Total Marriages and Remarriages for years 1979 and 1981-88.

U.S. Census Bureau Data -

12. Estimates of new marriages by age-group-of-husband crossed with age-group-of-wife from the American Community Survey (ACS) public use microdata sample (PUMS) files occurring, on average, at the end of years 2007-18. An additional year of data is available each year.

13. Data for the 2010 Standard Population. The marriage program uses the unmarried population (single + widow + divorce).

Other Input Data -

14. From the vital statistics offices in various states, number of same-sex marriages from 2004-12. These data are updated as they become available.

1.6.c. Development of Output

Equation 1.6.1 – Age-Specific Marriage Rates

Age-specific marriage rates are determined for a given age-of-husband crossed with age-of-wife, where ages range from 14 through 100+. The historical period includes years of complete NCHS data on the number of marriages and the unmarried population in the MRA for the period 1978 through 1988, excluding 1980. Data for a subset of the MRA, available by age group only, are used for the period 1989 through 1995, and ACS new-married grids by age group are used for the period 2008 through 2016. The marriage grids by age group for the years 1996 through 2007 are linearly interpolated. The total number of marriages from NCHS are also used in the age-specific marriage rate calculations for the period 1989-2018. The projection period of the MARRIAGE subprocess begins in 2019.

The historical age-specific marriage rates are calculated for each year in the historical period based on NCHS data of the number of new marriages by age-of-husband crossed with age-of-wife and the number of unmarried persons by age and sex. The formula used in the calculations is given below:

$$\hat{m}_{x,y}^z = \frac{\hat{M}_{x,y}^z}{P_{x,y}^z}, \text{ where}$$

- x refers to the age of men and y refers to the age of women;
- $\hat{M}_{x,y}^z$ is the number of marriages in year z ; and
- $P_{x,y}^z$ is the geometric mean of the midyear unmarried men and unmarried women in year z .

The rates for the period 1978 through 1988⁹ are then averaged, graduated, and loaded into an 87 by 87 matrix (age-of-husband crossed with age-of-wife for ages 14 through 100+), denoted as MarGrid. This matrix is used in the calculation of the age-specific marriage rates for all later historical years and the years in the projection period.

For the period 1989-2018, the NCHS and ACS provided data on the number of marriages by age-group-of-husband crossed with age-group-of-wife (age groups include 15-19, 20-24, 25-29, 30-34, 35-44, 45-54, 55-64, and 65+). These data are used to change the distribution of MarGrid by these age groups. For each age-group-of-husband crossed with age-group-of-wife, the more detailed marriage rates in MarGrid that are contained

⁹ Data for 1980 is not available and is excluded from the calculations.

within this group are adjusted so that the number of marriages obtained by using the rates in MarGrid match the number implied in the subset.

For each year of the entire 1989-2018 period, an expected total number of marriages is calculated by multiplying the rates in the MarGrid (or the adjusted MarGrid) by the corresponding geometric mean of the unmarried men and unmarried women in the Social Security area population. All rates in MarGrid (or the adjusted MarGrid) are then proportionally adjusted to correspond to the total number of marriages estimated in the year for the Social Security area population. This estimate is obtained by increasing the number of marriages reported in the U.S. to reflect the difference between the Social Security area population and the U.S. population. In addition, we also subtract out same-sex marriages from the NCHS data, as we handle those in a later step. The age-specific rates are then graduated using the Whittaker-Henderson method and are used to calculate the age-adjusted rates for each year.

The age-adjusted marriage rates are expected to reach their ultimate value in the 25th year of the 75-year projection period. Rather than use the last year of data to calculate the starting rate, we calculate the weighted average of the rates for the past five historical data years to derive the starting value. The annual rate of change decreases in absolute value as the ultimate year approaches.

To obtain the age-of-husband-age-of-wife-specific rates for a particular year from the age-adjusted rate projected for that year, the age-of-husband-age-of-wife-specific rates in MarGrid are proportionally scaled so as to produce the age-adjusted rate for the particular year. The MarGrid rates are then adjusted to produce two sets of marriage rates: opposite-sex marriage rates and same-sex marriage rates.

A complete projection of age-of-husband-age-of-wife-specific marriage rates was not done separately for each previous marital status. However, data indicate that the differential in marriage rates by prior marital status is significant. Thus, future relative differences in marriage rates by prior marital status are assumed to be the same as the average of those experienced during 1979 and 1981-88.

1.7. DIVORCE

1.7.a. Overview

For the period 1979 through 1988, the National Center for Health Statistics (NCHS) collected data on the annual number of divorces in the Divorce Registration Area (DRA), by age-group-of-husband crossed with age-group-of-wife. In 1988, the DRA consisted of 31 States and accounted for about 48 percent of all divorces in the U.S. These data are then inflated to represent an estimate of the total number of divorces in the Social Security area. This estimate for the Social Security area is based on the total number of divorces in the 50 States, the District of Columbia, Puerto Rico, and the Virgin Islands. Divorce rates for this period are calculated using this adjusted data on number of divorces and estimates of the married population by age and sex in the Social Security area.

An age-of-husband (x) crossed with age-of-wife (y) specific divorce rate ($\hat{d}_{x,y}^z$) for a given year (z) is defined as the ratio of (1) the number of divorces in the Social Security area for the given age of husband and wife ($\hat{D}_{x,y}^z$) to (2) the corresponding number of married couples in the Social Security area ($P_{x,y}^z$) with the given age of husband and wife. An age-adjusted central divorce rate (\widehat{ADR}^z) summarizes the $\hat{d}_{x,y}^z$ for a given year.

The \widehat{ADR}^z is calculated by determining the expected number of divorces by applying:

- The age-of-husband crossed with age-of-wife specific divorce rates to
- The July 1, 2010, population of married couples in the Social Security area by corresponding age-of-husband and age-of-wife.

The DIVORCE subprocess projects annual $\hat{d}_{x,y}^z$ by age-of-husband crossed with age-of-wife. The primary equations are given below:

$$\hat{d}_{x,y}^z = \hat{d}_{x,y}^z(\cdot) \quad (1.7.1)$$

$$\widehat{ADR}^z = \frac{\sum_{x,y} P_{x,y}^S \cdot \hat{d}_{x,y}^z}{\sum_{x,y} P_{x,y}^S} \quad (1.7.2)$$

where x and y refer to the age of husband and age of wife, respectively, and $P_{x,y}^S$ is the number of married couples in the Social Security area population as of July 1, 2010.

1.7.b. *Input Data*

Long-Range OASDI Projection Data -

Demography

1. Social Security area population of married couples by age-of-husband crossed with age-of-wife as of December 31 for years 1978-2018. These data are updated each year from the HISTORICAL POPULATION subprocess. In addition, the standard population is based on the averaged 2009 and 2010 December 31 marriage grids from the 2015 TR.
2. The total July 1 population in the Social Security area for years 1979-2018. However, only years 1979-88, 1998-2000, and 2008-18 data are used. An additional year of data is added for each additional year of divorce data from the NCHS with a maximum of the final historical year.
3. The total July 1 population in the U.S. resident population plus armed forces overseas for years 1979-2018. However, only years 1979-88, 1998-2000, and 2008-18 data are used. An additional year of data is added for each additional year of divorce data from the American Community Survey (ACS).
4. The total July 1 population in Puerto Rico and the Virgin Islands for years 1979-2018. However, only years 1988, 1998-2000, and 2008-18 are used. An additional year of data is added for each additional year of divorce data from the ACS.
5. Final historical year (on a December 31 basis). This datum is updated each year from the HISTORICAL POPULATION subprocess.

Assumptions -

6. Each year, the assumed ultimate value for the age-adjusted divorce rate is established. The rate reaches its ultimate value in the 25th year of the 75-year projection period. For the 2021 report, the assumed ultimate \overline{ADR}^z is 1,700 per 100,000 married couples.

NCHS Data -

7. The number of divorces in the DRA, by age-of-husband crossed with age-of-wife, for calendar years 1979 through 1988. These data are not available for years after 1988. The data are broken out by single year age-of-husband crossed with single year age-of-wife for many ages (particularly younger ages).
8. The total number of divorces in the United States for the period for 1979-2019. For years 1992+, the number of divorces are derived by multiplying the rate times the population. Data is updated when it becomes available.
9. The total number of divorces in Puerto Rico and the Virgin Islands for years 1988, 1998, 1999, and 2000. New data are incorporated as they become available and resources are sufficient to validate their use.

State Divorce Data -

10. Since NCHS stopped collecting state-specific divorce data by age of husband crossed with age of wife, we directly contacted various state health departments for their most recent data. We were able to get this data from 18 states. The years and age groups available vary by state. In general, the years were from 2009-12. These 18 states that had these data available online, or that sent us the data via email, are

Alabama, Alaska, Idaho, Kansas, Kentucky, Michigan, Missouri, Montana, Nebraska, New Hampshire, Tennessee, Texas, Utah, Vermont, Virginia, Washington, West Virginia, and Wyoming.

Census Bureau Data –

11. The number of divorces for years 2008-18 in Puerto Rico, estimated using the 2008-19 American Community Survey (ACS) public use microdata sample (PUMS) files. A new year of data is generally available each year.

1.7.c. Development of Output

Equation 1.7.1 -

Age-specific divorce rates are calculated for ages 14 through 100+. Detailed NCHS data on the number of divorces by age-group-of-husband crossed with age-group-of-wife are available for the period 1979 through 1988. Data on the total number of divorces in the United States are used for the period 1989 through 2018. With the data from the various states, we developed an age-group-of-husband crossed with age-group-of-wife grid for 2011.

First, the detailed NCHS data on divorces by age group is disaggregated into single year of age of husband (x) and age of wife (y), for ages 14-100+, using the H.S. Beers method of interpolation. Then, the age-specific divorce rates ($\hat{d}_{x,y}^z$), for each year (z) are calculated for the period 1979-88 by taking the number of divorces (inflated to represent the Social Security area, $\hat{D}_{x,y}^z$) and dividing by the married population in the Social Security area at that age-of-husband and age-of-wife ($P_{x,y}^z$). The formula for this calculation is given below:

$$\hat{d}_{x,y}^z = \frac{\hat{D}_{x,y}^z}{P_{x,y}^z} \quad (1.7.3)$$

These rates are then averaged, graduated,¹⁰ and loaded into an 87 by 87 matrix (age-of-husband crossed with age-of-wife for ages 14 through 100+), denoted as DivGrid. DivGrid is then adjusted using the state data grid developed for 2011. DivGrid for years after 1988 is a weighted average of the 1988 DivGrid and the 2011 state data single year grid. This state data single year of age grid is derived by ratioing the 1988 DivGrid cells using the original state age-group data. DivGrid is used in the calculation of the age-specific divorce rates for all later years including the projection period.

For each year in the 1989-2019 period, an expected number of total divorces in the Social Security area is obtained by applying the age-of-husband crossed with age-of-wife rates in DivGrid to the corresponding married population in the Social Security area. The rates in DivGrid are then proportionally adjusted so that they would yield an estimate of the total number of divorces in the Social Security area. The estimate of total divorces is obtained by adjusting the reported number of divorces in the U.S. for (1) the differences between the total divorces in the U.S. and in the combined U.S., Puerto Rico, and Virgin Islands area, and (2) the difference between the population in the combined U.S., Puerto Rico, and Virgin Islands area and in the Social Security area.

The starting age-adjusted divorce rate is set to a weighted average of the past five years of data. This age-adjusted rate is assumed to reach its ultimate value in the 25th year of the 75-year projection period. The annual rate of change decreases in absolute value as the ultimate year approaches.

¹⁰ Using a two-dimensional Whittaker-Henderson method of graduation from <http://www.howardfamily.ca/graduation/index.html>. This graduation is also used by the Society of Actuaries in their mortality improvement scales.

To obtain age-specific rates for use in the projections, the age-of-husband-age-of-wife-specific rates in DivGrid are adjusted proportionally so as to produce the age-adjusted rate assumed for that particular year.

1.8. PROJECTED POPULATION

1.8.a. Overview

For the 2021 Trustees Report, the starting population for the population projections is the December 31, 2018, Social Security area population, by age, sex, and marital status, produced by the HISTORICAL POPULATION subprocess. (For this section, section 1.8, the term “starting year” refers to the year 2018.) The Social Security area population is then projected using a component method. The components of change include births, deaths, net LPR immigration, and net other-than-LPR immigration. The components of change are applied to the starting population by age and sex to prepare estimated populations as of December 31, 2019 and 2020, and to project the population through the 75-year projection period (years 2021-95).

Beginning with December 31, 2013, the historical and projected populations are modeled using the following population statuses: heterosexual, gay, and lesbian. The gay and lesbian populations in the HISTORICAL POPULATION program are broken out assuming 2.5% of the male population and 4.5% of the female population is gay or lesbian, and the same is true for cohorts born in the PROJECTED POPULATION program.

There is a separate equation for each of the components of change as follows:

$$B_{s,p}^z = B_{s,p}^z(\cdot) \quad (1.8.1)$$

where $B_{s,p}^z$ is the number of births of each sex (s) by population status (p) born in year z;

$$D_{x,s,p}^z = D_{x,s,p}^z(\cdot) \quad (1.8.2)$$

where $D_{x,s,p}^z$ is the number of deaths by age (x), sex (s), and population status (p) that occurs in year z; and

$$NI_{x,s}^z = NL_{x,s}^z + NO_{x,s}^z \quad (1.8.3)$$

where $NI_{x,s}^z$ is the total net immigration (both LPR and other-than-LPR) by age (x), sex (s), and population status (p), $NL_{x,s}^z$ is the net LPR immigration (produced by the LPR IMMIGRATION subprocess), and $NO_{x,s}^z$ is the net other-than-LPR immigration (produced by the OTHER-THAN-LPR IMMIGRATION subprocess). The population program further disaggregates the new immigration, $NI_{x,s}^z$, by population status into $NI_{x,s,p}^z$. Once the components of change are calculated, the following equation is used to calculate the Social Security area population by age, sex, and population status:

$$P_{0,s,p}^z = B_{s,p}^z - D_{0,s,p}^z + NI_{0,s,p}^z \quad \text{for age} = 0 \quad (1.8.4)$$

$$P_{x,s,p}^z = P_{x-1,s,p}^{z-1} - D_{x,s,p}^z + NI_{x,s,p}^z \quad \text{for ages} > 0$$

where $P_{x,s,p}^z$ is the population, by age (x), sex (s), and population status (p), as of December 31st of each year (z).

The population is further disaggregated into the following four marital statuses: single (never married), married, widowed, and divorced. The following equation shows the population by age (x), sex (s), population status (p), and marital status (m) for each year (z):

$$P_{x,s,p,m}^z = P_{x,s,p,m}^z(\cdot) \quad (1.8.5)$$

The children (ages 0-18) population is further disaggregated into the following four parent statuses (i.e., fates): both parents are alive, only father is alive, only mother is alive, and both parents deceased. The following

equation shows the children population by age of child (x), sex of parent (s), age group of parent (g), and fate of parent (f) for each year (z):

$$C_{x,s,g,f}^z = C_{x,s,g,f}^z(\cdot) \quad (1.8.6)$$

1.8.b. *Input Data*

Long-Range OASDI Projection Data -

Demography

FERTILITY

1. Historical birth rates by single year of age of mother (14-49) for the years beginning with 1941 and ending with the starting year. These data are updated each year.
2. Projected birth rates by single year of age of mother (14-49) for the years beginning with the year after the starting year and ending with 2100. These data are updated each year.

MORTALITY

3. Historical probabilities of death by age last birthday (including neonatal mortality factor, single year of age for ages 0-99, and age group 100+) and sex for years beginning with 1941 and ending with the starting year. These data are updated each year.
4. Projected probabilities of death by age last birthday (including neonatal mortality factor, single year of age for ages 0-99, and age group 100+) and sex for the years beginning with the year after the starting year and ending with 2100. These data are updated each year.
5. Factors to distribute probabilities of death by marital status. They are dimensioned by sex, single year of age (ages 14-100+), and marital status. These data are updated each year.

LPR IMMIGRATION

6. Projected numbers of LPR immigrants who are new arrivals, by single year of age (-1-100) and sex for years beginning with the year after the starting year and ending with 2100. These data are updated each year. Note that age -1 represents births that occur during the year.
7. Projected numbers of legal emigrants by single year of age (-1-100) and sex for years beginning with the year after the starting year and ending with 2100. These data are updated each year. Note that age -1 represents births that occur during the year.
8. Projected numbers of LPR immigrants who are adjustments of status, by single year of age (-1-100) and sex for years beginning with the year after the starting year and ending with 2100. These data are updated each year. Note that age -1 represents births that occur during the year.

HISTORICAL POPULATION

9. Social Security area population by single year of age (0-99 and 100+), sex, marital status, and population status (eligible for or in an opposite-sex, same-sex male, or same-sex female marriage) for the years beginning with 1940 and ending with the starting year. These data are updated each year.
10. Married couples by single year of age of spouse 1 (ages 14-100+) crossed with single year of age of spouse 2 (ages 14-100+) for the years beginning with 1940 and ending with the starting year by marriage type (opposite-sex, same-sex male, and same-sex female). These data are updated each year.

11. Other-than-LPR population by age and sex for the years beginning with 1963 and ending with the starting year. These data are updated each year.
12. Final historical year (on a December 31 basis). This datum is updated each year from the HISTORICAL POPULATION subprocess.

OTHER-THAN-LPR IMMIGRATION

13. Projected numbers of other-than-LPR immigrants entering the country by age (-1-100) and sex for years beginning with the year after the starting year and ending with 2100. These data are updated each year.
14. Projected numbers of other-than-LPR immigrants leaving the country by age (-1-100) and sex for years beginning with the year after the starting year and ending with 2100. These data are updated each year.
15. Other-than-LPR population by age and sex for the years beginning with the year after the starting year and ending with 2100. These data are updated each year.

MARRIAGE

16. Projected central marriage rates by single year of age of husband (ages 14-100+) crossed with single year of age of wife (ages 14-100+) for each year of the projection period. These data are updated each year.
17. Projected central same-sex marriage rates by single year of age of spouse 1 (ages 14-100+) crossed with single year of age of spouse 2 (ages 14-100+) for each year of the projection period. These data are updated each year.
18. Averaged and graduated marriage rates for the period 1979 and 1981-88 by single year of age (ages 14-100+), sex, and prior marital status (single, divorced, and widowed). These data are updated each year.
19. Total number of marriages for the years beginning with 1989 and ending with 2019. These data are updated each year.

DIVORCE

20. Projected central divorce rates by single year of age of husband (14-100+) crossed with single year of age of wife (14-100+) for each year of the projection period. These data are updated each year.

U.S. Census Bureau Data -

21. CPS data on the average number of children per married couple with children by age group of householder (age groups 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, and 55-64) for 1960-2020. (Note that the program splits the last age group, which is a 10-year age group, into two 5-year age groups.) An additional year of data is added each year.

1.8.c. *Development of Output*

Equation 1.8.1 - Births

The number of births in the Social Security area, B_x^z , is computed for each year, z , of the projection period by applying the age-specific birth rate to the midyear female population aged 14 to 49 as follows:

$$B_x^z = b_x^z \left(\frac{FP_x^z + FP_x^{z+1}}{2} \right)$$

where,

B_x^z = number of births to mothers age x in year z ;
 b_x^z = birth rate of mothers age x in year z ; and
 FP_x^z = female population age x at the beginning of year z .

The total number of births in a given year is the sum of the number of births to mothers at each age. This total number of births is disaggregated by sex by assuming a sex ratio of 1048 male births for every 1000 female births. The total number of births is also disaggregated by population status by assuming 2.5% of boys that are born are gay and 4.5% of girls that are born are lesbian. (Note: These percentages include assuming that one-half of those born bisexual will want to enter into a same-sex marriage.)

Equation 1.8.2 – Deaths

The number of deaths for the Social Security area by age (x), sex (s), and population status (p), $D_{x,s,p}^z$, is computed for each projection year (z) by applying the death probabilities for each age and sex, $q_{x,s}^z$, to the exposed population at the beginning of the year.

$$D_{x,s,p}^z = q_{x,s}^z P_{x,s,p}^z$$

Equation 1.8.5 – Disaggregating the population by marital status

Once the population is projected by single year of age, sex, and population status, it is then disaggregated by population status into the following four marital states; single, married, widowed, and divorced. Estimates of the Social Security area population by single year of age (0-99 and 100+), sex, marital status, and population status as of the starting year of the population projection are obtained from the HISTORICAL POPULATION subprocess. In addition, the HISTORICAL POPULATION subprocess provides the number of married couples by single year of age of husband crossed with single year of age of wife and number of married male/male and female/female marriages, single year of age of spouse 1 crossed with single year of age of spouse 2, as of the starting year.

All births are assigned to the single marital status. For a given age, sex, and population status, deaths are assigned by marital status according to the relative differences in death rates by marital status observed for that age and sex during the calendar years 1995 and 1996, as determined in the MORTALITY subprocess. For a given age, sex, and population status, immigrants are assigned by marital status according to the beginning of year marital distribution of the Social Security area population for that age and sex.

Once the number of marriages, divorces, and widowings during a year are determined, the population by age, sex, population status, and marital status is updated to represent end of year. The unmarried population at the end of the year is estimated from the unmarried population at the beginning of the year by factoring in deaths, marriages, new unmarried immigrants, widowings, and divorces during the year. The married population at the end of the year is estimated from the married population at the beginning of the year by factoring in divorces, widowings, dissolutions of marriages when both husband and wife dies, new married immigrants, and marriages during the year.

Numbers of new marriages are determined for each projection year. The annual number of opposite-sex marriages occurring at each age of husband crossed with each age of wife is obtained by multiplying the age-of-husband-specific and age-of-wife-specific marriage rates with the geometric mean of the midyear unmarried male population and the midyear unmarried female population.

The age-specific midyear unmarried male population¹¹ is estimated from the beginning of the year unmarried population. It is calculated by adjusting the number of unmarried men at the beginning of the year to represent midyear using the relationship between the prior beginning of year and the current beginning of year unmarried male populations. The midyear female unmarried population is approximated similarly.

The numbers of marriages are then distributed by previous marital status (single, widowed, divorced) in the same proportions as would have been produced by applying the previous marital-status-specific marriage rates from the MARRIAGE subprocess to the population by marital status at the beginning of the year.

Numbers of new divorces are determined for each projection year. The number of divorces during a year, occurring at each age of husband crossed with each age of wife, is obtained by multiplying the age-of-husband crossed with age-of-wife divorce rates for that year with the midyear number of married couples in that age crossing.

The number of age-of-husband crossed with age-of-wife midyear married couples is estimated from the beginning of the year married couples. It is calculated by adjusting the number of married couples at the beginning of the year to represent midyear using the relationship between the number of married couples at the beginning of the prior year and the beginning of the current year.

Marriages and divorces for same-sex couples are calculated similarly.

Widowings are computed by applying general population probabilities of death to the marriage prevalence at the beginning of the year. Widowings and deaths by marital status are then reconciled for internal consistency.

Equation 1.8.6 – Disaggregating the children by parent survival status

Once the population is projected by single year of age, sex, population status, and marital status, the number of children are then categorized by age of father, age of mother, and orphan status. The HISTORICAL POPULATION subprocess provides the historical number of children (ages 0-18), number of women (ages 14-49), and the number of married couples by single year of age of husband crossed with single year of age of wife. The projected number of children (ages 0-18), number of women (ages 14-49), and marriage grid age of husband crossed with age of wife is calculated in the projected population.

For women aged 14-49, births are calculated by multiplying the age-specific birth rate, from the FERTILITY subprocess, with the number of women at the corresponding age. The births are then distributed to the age of husband in the same proportions as the age of husband crossed with age of wife married couples grid.

Each year the number of children is then rolled forward a year to the next age of husband, age of wife, and child age. Parent survival is calculated based on the deaths rates from the MORTALITY subprocess. The number of orphans consists of children with at least one parent deceased. The calculated number of children by age of father and age of mother must match the number of children in the historical or projected population. To accomplish this, the calculated number of children is multiplied by the ratio of the number of children in the historical or projected population to the number of children by age of father and age of mother that was calculated using the fertility rates. For any remaining difference, an adjustment of one is made for each age of husband crossed with age of wife until the total number of children match.

Once the population is projected by single year of age, sex, population status, marital status, and children, the mean number of children per married couple with children is determined by year and age of householder. The historical mean number of children by year and age of householder in the population program is calculated from the number of children categorized by age of father, age of mother, and the number of married men by age

¹¹ The midyear population exposed to marriage is the unmarried population (sum of those single, widowed, and divorced).

group from the HISTORICAL POPULATION subprocess. Linear regression is used to model the relationship between the mean number of children in the population program to the mean number of children from the U.S. Census Bureau. The regression model is then used to project the mean number of children by age of householder in the population program.

Process 2:

Economics

2. Economic

The Office of the Chief Actuary uses the Economic process to project OASDI employment and earnings-related variables, such as the average wage for indexing and the effective taxable payroll. The Economic process receives input data from the Demography process and provides output data to the Beneficiaries and the Trust Fund Operations & Actuarial Status processes.

The Economic Process is composed of five subprocesses, U.S. EMPLOYMENT, U.S. EARNINGS, COVERED EMPLOYMENT AND EARNINGS, TAXABLE PAYROLL, and REVENUES. As a rough overview, U.S. EMPLOYMENT and U.S. EARNINGS project U.S. employment and earnings data, respectively, while COVERED EMPLOYMENT AND EARNINGS converts these employment and earnings variables to OASDI covered concepts. TAXABLE PAYROLL, in turn, converts OASDI covered earnings to taxable concepts, which are eventually used to estimate future payroll tax income. REVENUES converts taxable concepts into projected OASDI wage tax liabilities (WTL) and self-employment tax liabilities (SEL) for the short range period.

U.S. EMPLOYMENT and U.S. EARNINGS produce output by quarter, while the output from COVERED EMPLOYMENT AND EARNINGS and TAXABLE PAYROLL are calendar year amounts. REVENUES produces amounts of wages paid to employees during a quarter and tax liabilities owed on those wages, as well as taxes collected on those amounts by quarter and by fiscal and calendar year.

Two appendices are at the end of this documentation. The first appendix, 2-1, provides details for most of the equations given in the following descriptions of the Economic process. The second appendix, 2-2, provides a listing with explanations of acronyms used in this documentation.

2.1. U.S. EMPLOYMENT (USEMP)

2.1.a. Overview

The Bureau of Labor Statistics (BLS) publishes historical monthly estimates for civilian U.S. employment-related concepts from the Current Population Survey (CPS). The principal measures include the civilian labor force (LC) and its two components – employment (E) and unemployment (U), along with the civilian noninstitutional population (N). The BLS also publishes values for the civilian labor force participation rate (LFPR) and the civilian unemployment rate (RU). The LFPR is defined as the ratio of LC to N, while the RU is the ratio of U to LC, expressed to a base of 100. For many of these concepts, the BLS publishes historical data disaggregated by age, gender, marital status, and presence of children.

For various disaggregated groups¹, USEMP projects quarterly and annual values for these principal measures of U.S. employment and population. Equations 2.1.1 through 2.1.6 outline the subprocess' overall structure and solution sequence for the total economy. We project that the military population (M) will remain constant over the projection horizon. We also project that the sum of N and M will grow at the same annual rate projected for the Social Security area population (P) (see Demography Process input).

$$M^t = M^{2020} \quad (2.1.1)$$

$$N^t = [(N^{t-1} + M^{t-1}) * (P^t / P^{t-1})] - M^t \quad (2.1.2)$$

$$RU = RU(\cdot) \quad (2.1.3)$$

$$LFPR = LFPR(\cdot) \quad (2.1.4)$$

$$LC = LFPR * N \quad (2.1.5)$$

$$E = LC * (1 - RU / 100) \quad (2.1.6)$$

Note: the superscript t represents the projection year.

The Demography Process estimates historical values for the total Social Security area population (P) and an important component, the other-than-lawful-permanent-resident, or other-than-LPR, population (OP). OP is further disaggregated into components by visa status: those temporarily authorized to reside or work in the US (OP_A), those who have overstayed their authorization (OP_NA), and those who were never authorized to reside or work in the US (OP_NO). Similarly, USEMP projects annual values for E and employed OP (EO), including its visa-status components (EO_A, EO_NA, EO_NO). USEMP also separates EO to those whose earnings are reported and posted to the Master Earnings File (EO_MEF), those whose earnings are reported posted to the Earnings Suspense File (EO_ESF), those in the underground economy (EO_UND).

¹ Group disaggregation includes age and gender. Some groups are additionally disaggregated by marital status and by presence of children.

A further subgroup of EO_MEF is also calculated: those who are OASDI covered (EO_MEFC). Equations 2.1.7 through 2.1.14 outline the overall structure of the subprocess used to estimate EO and its sub-components.

$$EO_A = EO_A(\cdot) \quad (2.1.7)$$

$$EO_NA = E * OP_NA / N \quad (2.1.8)$$

$$EO_NO = E * OP_NO / N \quad (2.1.9)$$

$$EO = EO_A + EO_NA + EO_NO \quad (2.1.10)$$

$$EO_MEF = EO_MEF(\cdot) \quad (2.1.11)$$

$$EO_MEFC = EO_MEFC(\cdot) \quad (2.1.12)$$

$$EO_ESF = EO_ESF(\cdot) \quad (2.1.13)$$

$$EO_UND = EO - EO_MEF - EO_ESF \quad (2.1.14)$$

Finally, for each age/gender group, USEMP projects total “at-any-time” employed other-than-LPR population (TEO). EO represents the average weekly employment of the other-than-LPR population during a calendar year. TEO represents the total number of individuals in the other-than-LPR population who had any employment during the calendar year. (EO can be roughly viewed as the average number of jobs worked by OP during a calendar year, while TEO represents the total number of individuals who worked those jobs.) Effectively, Equations 2.1.15 through 2.1.19 convert every EO age-gender sub-component to an at-any-time TEO age-gender sub-component counterpart.

$$TEO_MEF = TEO_MEF(\cdot) \quad (2.1.15)$$

$$TEO_MEFC = TEO_MEFC(\cdot) \quad (2.1.16)$$

$$TEO_ESF = TEO_ESF(\cdot) \quad (2.1.17)$$

$$TEO_UND = TEO_UND(\cdot) \quad (2.1.18)$$

$$TEO = TEO_MEF + TEO_ESF + TEO_UND \quad (2.1.19)$$

2.1.b. *Input Data*

Long-Range OASDI Projection Data

These data are updated each year.

Demography

1. Social Security area population as of year-end (1941 – 2100) by age, marital status (single, married, widowed, divorced) and gender (M, F)
2. “Other-than-LPR” population as of year-end (1964 – 2100) by age, gender (M, F), and visa status (OP_A, OP_NA, and OP_NO)
3. Number of children by age of child and age of mother (1960-2100)
4. Life expectancy by age and gender (1950-2100)
5. Exit rates (probability of leaving the “other-than-LPR” population by other than death) by age and gender.
6. Mortality rates by age and gender (1941-2100)

Trust Fund Operations and Actuarial Status – The Trust Fund Operations and Actuarial Status Process provides no direct input to the Economic Process sections. However, the LFPRs generally use input based on the Outgo Process from the prior year’s Trustees Report. For example, the projected LFPRs for the 2021 Trustees Report used input from the 2020 Trustees Report. This is acceptable practice as long as the ultimate disability incidence rate assumption is not changed from the prior year. This input includes:

7. projections for the disability prevalence rates by age and gender (originally from the Beneficiaries subprocess)
8. projections for the disability-insured population (originally from the Beneficiaries subprocess)
9. primary insurance amount (PIA) replacement rates by age and gender.

The disability prevalence rate is defined as the ratio of the number of disabled worker beneficiaries to the disability-insured population. The PIA replacement rate is defined as the ratio of a hypothetical medium-scaled worker’s PIA to his/her career-average indexed earnings level.

Trustees’ Assumptions

Each year the Board of Trustees of the OASDI Trust Funds sets the ultimate average annual growth rate values for key economic variables:

10. Real wage
11. Total economy productivity
12. Average hours worked
13. Ratio of wages to compensation (RWSD)
14. Ratio of compensation to GDP (RWSSY)
15. GDP deflator (PGDP)
16. Consumer Price Index (CPI)

The Board also sets ultimate values for:

17. Annual trust fund real interest rate
18. Unemployment rate

These ultimate values are typically reached during the last half of the short-range (first 10 years) of the projection horizon. Earlier projected values are set to provide a smooth transition from the latest actual historical values to the assumed long-range ultimate ones. As a by-product of this process, values for real GDP and potential GDP are set. The ratio (RTP) of real to potential GDP is an important summary measure of the economic cycle.

The Trustees also agree on the assumed short-range values for the listed variables.

Addfactors

19. Addfactors are adjustments that move an estimate closer to an expected value. They may be used for a variety of reasons associated with data availability, structural changes in the data and/or model, and perceived temporary aberrations in recent historical data. Addfactors were included on male and female LFPRs starting around age 40 to reflect the effects of projected changes in life expectancy. Addfactors are also used to modify LFPRs of several specific age-sex groups to ensure a reasonable shape of the age-LFPR profile. Another addfactor is applied to the labor force in the first few projection years to phase out the differences between the model-predicted values and recent historical data.

Other input data

20. U.S. armed forces (EDMIL) by age and gender, estimated by the Department of Defense and published by the Census Bureau on a monthly basis (1948-2000) by single year of age (17 to 64) and gender. These data are no longer produced by Census.
21. EDMIL by age and gender, estimated by the Economic Process as the difference in monthly resident plus Armed Forces overseas population and the monthly civilian population. These two populations are available from the Census Bureau on a monthly basis (April 2000 to December 2019) by single year of age (16 to 69) and gender. These data are updated once a year.
22. Data for the mobilized military reservist population, by branch of service (September 2001-September 2016) are reported by the US Department of Defense weekly. These data are no longer reported by the Department of Defense.
23. Data from the March Supplement of the Joint BLS/Census Current Population Survey (CPS) by year (1968-2019), for levels of the civilian noninstitutional population, labor force, military, and unemployment. These data are available from the U.S. Census Bureau by single year of age (16 to 85+), gender, marital status (never married, married with a spouse present, and married with no spouse present),

- and presence of children. These data are updated by the U.S. Census Bureau for the BLS annually. This subprocess updates the data every other year (or more often, based on time availability).
24. Data from the March Supplement of the CPS by year (1992-2020), for levels of the civilian noninstitutional population. These data are available from the U.S. Census Bureau by single year of age (16 to 85+), gender, and educational attainment level. These data are updated by the U.S. Census Bureau for the BLS annually. This subprocess updates the data every year, if time availability allows.
 25. Data from the CPS (1948-2021) for levels of civilian employment, civilian labor force, civilian unemployment, and civilian noninstitutional population. These data are available from the BLS by age group and gender. These data are updated by the BLS monthly. This subprocess updates the data several times a year.
 26. Data from the CPS by year (1994-2020), for the civilian noninstitutional population. These data are available from the BLS by single year of age (16 to 85+), gender, marital status, labor force employment status, and (for those not in the labor force) reason for not being in the labor force. These data are updated by the BLS monthly. Monthly data are used to calculate annual averages. This subprocess updates the data every year, if time availability allows.
 27. Data from the Current Employment Statistics survey (CES) (1964 (varies) to September 2020) for establishment employment, average hourly earnings, average weekly earnings, and average weekly hours. These data are available from the BLS by sector. These data are updated by the BLS monthly. This subprocess updates the data several times a year.
 28. Unpublished data from the CPS (1965 - October 2020) for male and female civilian labor force participation rates for older workers. These data are available from the BLS by single year of age (ages 55-79) and by group (75 and over, and 80 and over). These data are updated by the BLS monthly. This subprocess updates the data several times a year.

2.1.c. Development of Output

Equation 2.1.3 - Unemployment Rate (RU)

The RU is disaggregated by age and gender. The age groups include 16-17, 18-19, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, 70-74, and 75 and over. Thus, USEMP contains 28 RU equations, 14 for men and 14 for women. Each disaggregated RU is specified using a first-difference model that depends on the distributed lag in the change in the ratio of real to potential GDP (RTP) and an adjustment to ensure that values converge to its estimated trend level. Coefficients are estimated by regression. Furthermore, projections are constrained to the ultimate age-gender-adjusted RU set by the Trustees. The aggregate age-sex-adjusted RU is dependent on the projected distribution of the labor force by age and

gender. See Appendix 2-1 for details on the equations.

Equation 2.1.4 - Labor Force Participation Rate (LFPR)

The LFPR is disaggregated by age and gender. Age groups include 16 to 17 (i.e., 16-17), 18-19, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55, 56, ... 99, 100 and over. For age groups between 20 and 54, male and female LFPRs are further disaggregated by marital status, categories of which include never married, ever married with spouse present, and ever married with spouse absent (which includes separated, widowed, and divorced). Female LFPRs disaggregated by age (between 20 and 44) and by marital status are further disaggregated by presence of own child. The groups for presence of own child include women with at least one child under the age of six and women without a child under the age of six. Thus, USEMP contains 153 LFPR equations, 69 for men and 84 for women. See Appendix 2-1 for details on the equations.

Given the level of demographic disaggregation, the aggregate LFPR is dependent on the projected distribution of the population by age, gender, marital status, and presence of own child. Each disaggregated LFPR, however, is dependent on the input variables that are most relevant to the demographic group. For example, only the LFPRs for relevant older workers are dependent on changes to the normal retirement age (NRA). Specific examples of the impact of input data on the disaggregated LFPRs are presented below.

- Disability prevalence ratio (RD) is defined as the ratio of disabled worker beneficiaries to the disability-insured population. An increase in RD lowers the LFPR because disabled worker beneficiaries tend to exit the labor force. To reflect this, projected LFPRs for each age-sex group are adjusted for disability by dividing them by $(1+RD)$. This adjustment implicitly assumes that disability can strike any person in a given age-sex group with equal probability. For ages 62 to NRA, RDs are not “pure” RDs in that they are subject to the confounding effect of the availability of retirement benefits. For example, at age 62, a marginally disabled individual may opt to begin receiving retirement benefits rather than go through an uncertain disability application/appeals process. For ages NRA and above, RDs are unavailable because at the NRA all disabled-worker beneficiaries become retired-worker beneficiaries. To avoid these problems, RDs for ages 62-74 are set to their cohort RD at age 61. For example, the RD for men age 62 in year (t) is set to the RD for men age 61 in year (t-1). For those ages 75 and older, the lagged cohort variable provides information on the influence of disability prevalence rates on labor force participation.
- The unemployment rate (RU) is a measure of the economic cycle. An increase in the lagged and current unemployment rate leads to a decrease in the LFPR. The RU affects LFPRs for ages up to 54.
- The normal retirement age (NRA) is assumed to affect the LFPRs for those age 62 through 69 through an earnings test and replacement rate. The replacement rate is defined as the ratio of a hypothetical worker’s PIA to career-average wage level. This value is projected for hypothetical workers with medium-scaled earnings patterns² who retire at ages 62 through 69. The replacement rate is adjusted to include the reduction for early

² More details on the hypothetical scaled workers are provided in Actuarial Note 2020.3, located at: www.ssa.gov/OACT/NOTES/ran3/index.html.

retirement and the delayed retirement credit. An increase in the NRA decreases the adjusted replacement rates, which, in turn, leads to increases in the LFPRs for those between the ages of 62 and 69. The potential earnings test tax rate (POT_ET_TXRT) is used in LFPRs between 62 and 69. It is defined as a tax rate on monthly retirement benefits faced by an individual who opts to collect Social Security benefits before reaching NRA while continuing to work and earn income. An increase in the NRA from 66 to 67 leads to an increase in the potential tax rate for those age 66, which, in turn, leads to a decrease in their LFPR.

- The level of education attainment affects LFPRs for ages 55 and older for men and 50 and older for women. As the average level of educational attainment in any given age-sex group rises, LFPRs generally increase.
- The composition of population by marital status also affects LFPRs. For ages up to 54, LFPRs are disaggregated by marital status. For ages 55 to 74 (both men and women), the effect of marital status is captured by the proportion of people in any given age group who are married with spouse present.
- A LFPR increases with its lagged cohort. Lagged cohort variables affect LFPRs for age 75 and over.
- For those approximately age 40 and over, an increase in life expectancy leads to an increase in LFPRs.

Equation 2.1.7 to 2.1.19 – Employed Other-than-LPR Population (EO) and At-Any-Time Employed Other-than-LPR Population (TEO)

EO is estimated by gender and single-year of age from 16 to 100 based on OP and estimated employment-to-population ratios by visa-status component (OP_A, OP_NA, OP_NO). For this purpose, OP_A is further disaggregated into subgroups by visa type that differ in employment patterns or OASDI coverage status. The other two components are assumed to have equal employment-to-population ratio as the LPR population of the same age and gender. This portion of USEMP contains 4,250 equations, for 85 ages, 2 genders, and 25 components and subgroups. We separate EO_NO into those who worked in 2001 and earlier and those who began working in 2002 and later, since we believe that those who worked in 2001 and earlier are more likely to have OASDI covered wages. Each component is then further separated into EO_MEF, EO_MEFC, EO_ESF, and EO_UND.

Every EO sub-component by age, gender, and visa status is converted to its age-gender TEO sub-component counterpart using an age-gender conversion weight. For example, if the sub-component of EO is for never authorized men age 20 to 24, the conversion weight is defined as the ratio of total economy-wide at-any-time employed men age 20 to 24 (TEM2024) to the sum of military and CPS civilian male employment age 20 to 24. For authorized workers and students on temporary visas, conversion weights take into account their partial presence in the year of arrival and the year of departure.

2.2. U.S. EARNINGS (MODSOL2)

2.2.a. Overview

In the CPS data, E is separated by class of worker. The broad categories include wage and salary workers (EW), the self-employed (ES), and unpaid family workers (EU). For the nonagricultural sector, the self-employed participation rate (SEPR) is defined as the ratio of ES to E, the proportion of employed persons who are self-employed. For the agricultural sector, the SEPR is defined as the ratio of ES to the civilian noninstitutional population.

MODSOL2 projects quarterly values for these principal classes of employment. Equations 2.2.1 through 2.2.4 outline the subprocess' overall structure and solution sequence.

$$SEPR = SEPR(\cdot) \quad (2.2.1)$$

$$ES = SEPR * E \quad (2.2.2)$$

$$EU = EU(\cdot) \quad (2.2.3)$$

$$EW = E - ES - EU \quad (2.2.4)$$

In the National Income and Product Accounts (NIPA), the Bureau of Economic Analysis (BEA) publishes historical quarterly estimates for gross domestic product (GDP), real GDP, and the GDP price deflator (PGDP). Real GDP is equal to the ratio of nominal GDP to PGDP. Potential (or full-employment) GDP is a related concept defined as the level of real GDP that is consistent with a full-employment aggregate RU.

MODSOL2 projects quarterly values for these output measures. Potential GDP is based on the change in full-employment values for: (1) E (including U.S. armed forces), (2) average hours worked per week, and (3) productivity. Full-employment values for E are derived by solving USEMP under full-employment conditions, while the full-employment values for the other variables (average hours worked and productivity) are set by assumption. RTP is the ratio of real GDP to potential GDP and is set by assumption. RTP reaches 1.0 in the short-range period and remains at 1.0 thereafter. Projected real GDP is set equal to the product of potential GDP and RTP. Nominal GDP is the product of real GDP and PGDP. The growth rate in PGDP is set by assumptions.

The BEA also publishes quarterly values for the principal components of U.S. earnings, including total wage worker compensation (WSS), total wage and salary disbursements (WSD), and total proprietor income (Y). These concepts can be aggregated and rearranged. Total compensation (WSSY) is defined as the sum of WSS and Y. The total compensation ratio (RWSSY) is defined as the ratio of WSSY to the GDP. The income ratio (RY) is defined as the ratio of Y to WSSY. The earnings ratio (RWSD) is defined as the ratio of WSD to WSS.

MODSOL2 projects quarterly values for these principal components of U.S. earnings using Equations 2.2.5 through 2.2.11.

$$RWSSY = RWSSY(\cdot) \quad (2.2.5)$$

$$WSSY = RWSSY * GDP \quad (2.2.6)$$

$$RY = RY(\cdot) \quad (2.2.7)$$

$$Y = RY * WSSY \quad (2.2.8)$$

$$WSS = WSSY - Y \quad (2.2.9)$$

$$RWSD = RWSD(\cdot) \quad (2.2.10)$$

$$WSD = RWSD * WSS \quad (2.2.11)$$

2.2.b. *Input Data*

Long-Range OASDI Projection Data

1. Demography- (See Section 2.1.b.)
2. Economics - Data from Section 2.1 include the total employed (E), E by age and gender, LFPRs by age and gender, the aggregate unemployment rate (RU), and the full-employment concepts for LC, RU, and E.
3. *Trustees Assumptions* - (See Section 2.1.b.)

Addfactors

4. Addfactors were included on some employment and output variables to smooth the transition between the latest historical data and the projected values. The need for addfactors is reviewed each year and they are implemented if necessary.

Other input data

5. Data from the NIPA (1929 (varies) to 2020) for GDP, income, wages, compensation, personal consumption expenditures, investment, employer contributions for employee pension and insurance funds, and employer contributions for government social insurance. They are published by the BEA quarterly and/or annually. This subprocess updates the data several times a year.
6. OASDI employee, employer, and self-employed tax rates from 1937 to 2100.

These contribution rates are set according to the Social Security Act of 1935 as amended through 2015. The rates are updated when legislation mandates a change.

7. The historical Consumer Price Index (CPI) is published monthly by the BLS. This subprocess updates the data several times a year.
8. The historical CPI for medical services is published monthly by the BLS. Quarterly values are projected based on the projected growth in the aggregate CPI and an additional amount defined as the growth rate differential in the two price measures that was assumed in the latest President's Fiscal Year Budget. The series is updated annually.
9. U.S. armed forces (EDMIL) by age and gender were estimated by the Department of Defense and published by the Census Bureau on a monthly basis (1948-2000) by single year of age (17 to 64) and gender. These data are no longer produced by Census.
10. EDMIL by age and gender are estimated by the Economic process as the difference in the monthly resident plus Armed Forces overseas population and the monthly civilian population. These two populations are available from the Census Bureau on a monthly basis (April 2000 to December 2019) by single year of age (16 to 69) and gender. These data are updated once a year.
11. Wages for railroad workers are wages covered by the Railroad Retirement Act. The annual data are for the period 1971 to 2017. An additional year of data from the Railroad Retirement Board is usually available for including in preparation of the next annual Trustees Report.
12. Unpublished data from the CPS (1988-2019) on employment by class of worker (i.e., agricultural, nonagricultural, unpaid family, private industry, government, wage and salary, self-employed). These data are available from the BLS by age group and gender. These data are updated by the BLS annually. This subprocess updates the data annually.
13. Data from the NIPA (1947-2020) for wages and compensation of households and institutions are published by the BEA quarterly. This subprocess updates the data several times a year.
14. Other program-related parameters, including the average indexing wage, the benefit increase, the taxable maximum, and the annual retirement earnings test exempt amounts, are obtained annually from the Short-Range section of OCACT. This subprocess updates the data annually.
15. Unpublished data from the CES & CPS for total hours worked in the economy. These data are available from the BLS. These data are updated by the BLS quarterly (1948-2020) and annually (1948-2019). This subprocess

updates the data several times a year.

16. The Federal minimum hourly wage is based on the Fair Labor Standards Act from the Department of Labor for 1938 to 2020. The wage is updated when there is legislation mandating a change.
17. Time trends (set by Economic process) are used in the agriculture sector for employment, real output, and compensation in the short-range period. These short-range trends are extended for each year's Trustees Report, reflecting a new short-range period.

2.2.c. Development of Output

Equation 2.2.1 - Self-Employed Participation Rate (SEPR)

The SEPR is disaggregated by age, gender, and industry. The age groups include 16-17, 18-19, 20-24, 25-34, 35-44, 45-54, 55-64, and 65 and over. The industry groups include agriculture and nonagriculture.

For the nonagriculture sector, the SEPRs by age and gender are defined as the ratio of the nonagriculture self-employment to total employment. Thus, the aggregate nonagriculture SEPR is dependent on the projected distribution of employment by age and gender. All nonagriculture SEPRs by age and gender are dependent on the RTP. Increases in the RTP lead to decreases in the SEPRs.

For the agriculture sector, the male SEPRs by age (as well as the female SEPR for ages 16-17) are defined as the ratio of agriculture self-employment to the civilian noninstitutional population. Thus, the aggregate agriculture SEPR for men is dependent on the projected distribution of the population by age. The agriculture SEPRs for men by age are dependent on the ratio of total agriculture employment (EA) to the total civilian population aged 16 and over. (EA is projected in a farm sub-program. Real farm output is projected to increase with the population, while farm productivity, defined as output per worker, is projected to continue to follow its historical trend. EA is projected as the ratio of farm output to farm productivity.) An increase in the ratio of EA to the total civilian population aged 16 and over leads to an increase in the agriculture SEPRs for men.

The female SEPRs by age (for ages 18 and higher) for the agriculture sector are defined as the ratio of the female to male agriculture self-employment. Thus, the aggregate agriculture SEPR for women is dependent on the projected distribution of male agriculture employment by age. For female age groups age 18 and over, the SEPRs are dependent on the RTP and the corresponding ratio of total female to male employment. Generally, an increase in the RTP leads to increases in the SEPRs. An increase in the total employment ratio also leads to an increase in the SEPR.

Equation 2.2.2 – Self-Employed Workers (ES)

ES is disaggregated by age, gender, and industry. The age groups include 16-17, 18-19, 20-24, 25-34, 35-44, 45-54, 55-64, and 65 and over. The industry groups include agriculture and nonagriculture. For the nonagricultural sector, ES is derived from SEPR by scaling it to the total nonagricultural self-employed workers (ENAS), which is projected as a constant share of nonagricultural employment over the long range. For the agricultural sector, it is similarly scaled to the total agricultural self-employed workers (EAS), which is projected as the residual after subtracting projected wage workers and unpaid family workers from total agricultural employment.

Equation 2.2.3 - Unpaid Family Workers (EU)

EU is disaggregated by age, gender, and industry. The age groups include 16-17, 18-19, 20-24, 25-34, 35-44, 45-54, 55-64, and 65 and over. The industry groups include agriculture and nonagriculture.

From 1970 to 2014, the level of EU fell from about 0.5 to 0.02 million in the agriculture sector and from about 0.5 to 0.06 million in the nonagricultural sector. For projections, the levels of EU by age and gender in the agriculture sector are assumed constant at about five thousand or less. The EUs by age and gender in the nonagricultural sector are projected as a constant ratio to ES.

Equation 2.2.4 - Wage Workers (EW)

For the nonagricultural sector, the number of wage workers is the residual after subtracting self-employed workers and unpaid family workers from total workers. For the agricultural sector, we first project wage workers, and the number of self-employed workers is the residual after subtracting wage workers and unpaid family workers from total agricultural workers. Agricultural wage workers in each age/gender group are projected as a function of the business cycle and the age-gender-group's share of total US workers. The age groups include 16-17, 18-19, 20-24, 25-34, 35-44, 45-54, 55-64, and 65 and over. The nonagriculture sector is further disaggregated: private household workers are projected by age and sex, while Federal Government (Civilian and Military, separately) and State & Local Government workers are projected in total.

Equation 2.2.5 - Total Compensation Ratio (RWSSY)

The Trustees set the ultimate annual growth rate for RWSSY. For the short-range period, total WSS, WSD, and Y are aggregated from sector components. Total GDP, WSS, and WSD are divided into the farm and nonfarm sectors. The nonfarm sector is further separated into the government and government enterprises, households, nonprofit institutions, and residual (private nonfarm business excluding government enterprises (PBNFXGE)) sectors. Total Y is divided into the farm and residual (i.e., PBNFXGE) sectors.

The methodology used to estimate GDP, WSS, WSD, and Y differs by sector.

Farm - Nominal GDP is the product of real GDP and the farm price deflator. Real farm GDP is projected from estimates of real farm per capita output. EA is projected from estimates of farm productivity. EAW is projected to continue its historical increase relative to EA. Farm compensation (WSSPF) is the product of estimates for average farm compensation (AWSSPF) and EAW, while farm proprietor income (YF) is the product of estimates of average farm proprietor income (AYF) and EAS. AYF is projected based, in part, on the growth in AWSSPF.

Government and Government Enterprises - This sector is further disaggregated to Federal Civilian, Federal Military, and State and Local. In each sector, WSD is the product of estimates for average wages and employment. WSS is the sum of WSD and estimates for non-wage components of compensation. GDP is the sum of WSS and estimates of consumption of fixed capital.

Household - WSS is the product of estimates for average compensation and employment. WSD is WSS less employer contributions for the OASDHI tax. GDP is the sum of WSS and the gross value added of owner-occupied housing.

Nonprofit Institutions - The Nonprofit Institutions sector is further disaggregated to Health, Education, and Social Services sectors. In each sector, WSS is the product of estimates for average compensation and employment. WSD is WSS less the estimates for non-wage components of compensation. GDP is WSS plus a residual component of output.

Private Nonfarm Business Excluding Government Enterprises (PBNFXGE) - GDP in the PBNFXGE sector is total economy-wide GDP less the sum of the other sector GDPs. WSS is projected as a ratio to GDP less Y. The ratio is projected to be mostly stable, varying only temporarily with changes in RTP. Y is projected to grow with GDP and the ratio of EAS to total employment in the sector.

Thus, total labor compensation (WSSY) is summed from sector components, while the total compensation ratio (RWSSY) is the ratio of total WSSY to total GDP. It is important to note that the pure program-generated estimate for the total RWSSY is adjusted to ensure a smooth transition between the latest historical data and the Trustees' ultimate assumptions.

Equation 2.2.7 - Income Ratio (RY)

Y is disaggregated to the farm and PBNFXGE sectors. (see description for Equation 2.2.5)

Equation 2.2.10 - Earnings Ratio (RWSD)

In the NIPA, the difference between WSS and WSD is defined as employer contributions for employee pension and insurance funds (OLI) and employer contributions for government social insurance (SOC). OLI is mostly health and life insurance, and pension and profit sharing. SOC is composed of employer contributions to Federal and State & Local government social insurance funds. Federal government funds include OASDI, HI, UI, and

other small groups. State and Local government funds mostly include workers' compensation.

RWSD is defined as the ratio of WSD to WSS. RWSD is projected to mostly decline on a year-by-year basis over the entire 75-year projection horizon due to projected increases in employer contributions to employee group health insurance premiums (ECEGHIP) and pensions. ECEGHIP is projected by the Center for Medicare and Medicaid Services (CMS). Employer contributions to employee pension funds are assumed to increase as life expectancy increases.

2.3. OASDI COVERED EMPLOYMENT AND EARNINGS (COV)

2.3.a. Overview

Total at-any-time employment (TE) is defined as the sum of total OASDI covered employment (TCE) and total noncovered employment (NCE). TCE can be decomposed to workers who only report OASDI covered self-employed earnings (SEO) and to wage and salary workers who report some OASDI covered wages (WSW). Combination workers (CMB_TOT) are those who have both OASDI covered wages and self-employed income. Workers with some self-employment income (CSW) are the sum of SEO and CMB_TOT.

COV projects annual values for TE and the principle measures of OASDI covered employment. Equations 2.3.1 through 2.3.9 outline the overall structure and solution sequence used to project these concepts. The combination employment ratio (RCMB) is defined as the ratio of CMB_TOT to WSW.

$$\text{(Equation 2.3.1 not used in this version.)} \quad (2.3.1)$$

$$TE = TE(\cdot) \quad (2.3.2)$$

$$NCE = NCE(\cdot) \quad (2.3.3)$$

$$TCE = TE - NCE \quad (2.3.4)$$

$$SEO = SEO(\cdot) \quad (2.3.5)$$

$$WSW = TCE - SEO \quad (2.3.6)$$

$$RCMB = RCMB(\cdot) \quad (2.3.7)$$

$$CMB_TOT = RCMB * WSW \quad (2.3.8)$$

$$CSW = SEO + CMB_TOT \quad (2.3.9)$$

Total OASDI covered earnings is defined as the sum of OASDI covered wages (WSC) and total covered self-employed income (CSE_TOT). Both components can be expressed as ratios to their U.S. earnings counterparts. The covered wage ratio (RWSC) is defined as the ratio of WSC to WSD, while the covered self-employed ratio (RCSE) is the ratio of CSE_TOT to Y.

COV projects annual values for the principal measures of OASDI covered earnings using Equations 2.3.10 through 2.3.13.

$$RWSC = RWSC(\cdot) \quad (2.3.10)$$

$$WSC = RWSC * WSD \quad (2.3.11)$$

$$RCSE = RCSE(\cdot) \quad (2.3.12)$$

$$CSE_TOT = RCSE * Y \quad (2.3.13)$$

COV projects various annual measures of average OASDI covered earnings, including the average covered wage (ACW), average covered self-employed income (ACSE), and average covered earnings (ACE).

$$ACW = WSC / WSW \quad (2.3.14)$$

$$ACSE = CSE_TOT / CSW \quad (2.3.15)$$

$$ACE = (WSC + CSE_TOT) / TCE \quad (2.3.16)$$

The average wage index (AWI) is based on the average wage of all workers with wages reported on Forms W-2 and posted to the Master Earnings File (MEF). By law, it is used to set the OASDI contribution and benefit base (TAXMAX).

COV projects annual values for the AWI and TAXMAX.

$$AWI = AWI(\cdot) \quad (2.3.17)$$

$$TAXMAX = TAXMAX(\cdot) \quad (2.3.18)$$

2.3.b. Input Data

Long-Range OASDI Projection Data

1. Demography - (See Section 2.1.b.)
2. Economics- Employment and earnings-related data from Sections 2.1 and 2.2.
3. *Trustees Assumptions* - (See Section 2.1.b.)

Addfactors

4. Addfactors were included on some employment variables to smooth the transition from the latest historical data to program estimates. The need for addfactors is reviewed each year and they are implemented if necessary.

Other input data

5. Ratios of OASDI covered to NIPA wages by sector. NIPA wages by sector are available quarterly and annually from 1947 to 2020. They are published by the BEA and updated several times during the year. OASDI covered wages (1971 to 2018) are updated annually by the Economic process. Covered data

for the latest historical year are estimated from tabulations of Form 941 and W-2 data.

6. U.S. armed forces (EDMIL) by age and gender were estimated by the Department of Defense and published by the Census Bureau on a monthly basis (1948-2000) by single year of age (17 to 64) and gender. These data are no longer produced by Census.
7. EDMIL by age and gender are estimated by the Economic process as the difference in the monthly resident plus Armed Forces overseas population and the monthly civilian population. These two populations are available from the Census Bureau on a monthly basis (April 2000 to December 2019) by single year of age (16 to 69) and gender. These data are updated once a year.
8. Railroad employment is covered by the Railroad Retirement Act. The annual historical data are for the period 1971 to 2018. An additional new year of historical data from the Railroad Retirement Board is usually available for inclusion in preparation of the next annual Trustees Report.
9. Data obtained from Office of Research, Evaluation, and Statistics (ORES) are tabulations of quarterly Form 941 data. Data currently used are the OASDI, HI, and income taxable wages by sector for the most recent five years. The data represent changes in reported wages since the prior quarterly report. The most recent data are appended to previously reported data. Annual totals are computed and used to derive estimates of OASDI covered wages by sector for the latest historical years.
10. Data obtained from the most recently available 1.0% CWHS active file, maintained on Social Security's mainframe and made available by ORES. The years of data are 1951 to the third year prior to the current Trustees Report year. The data are used for comparison of OASDI covered earnings from other sources.
11. Data obtained from extracting information from the 1.0% Employee-Employer Files, maintained on Social Security's mainframe and made available by ORES. Each year two files are created: a Version 1 file for the third year prior to the current Trustees Report and a Version 3 file for the fifth year prior to the current Trustees Report. Data currently being used are government and farm sector OASDI, HI, and total wages and employment. Data from the latest files are used to estimate OASDI covered wages for the years available on each file.
12. Data obtained from quarterly IRS Form 941 files, provided by Office of Systems (OS). Data currently used are the OASDI and HI taxable wages for 1978 to the most recent year available. The data represent changes in reported wages since the prior quarterly report. The most recent data are appended to

previously reported data. Annual totals are computed and used to derive estimates of HI taxable wages, which are then used to develop OASDI covered wages for the most recent historical years.

13. Data from the Quarterly EPOXY Report, received in hard copy and, more recently, electronic formats obtained from OS. The data currently used are the number of workers with OASDI taxable earnings, number of workers with HI taxable earnings, distribution of number of HI workers by wage intervals, distribution of number of OASDI workers by wage intervals, number of persons with OASDI taxable wages, number of persons with HI taxable wages, number of persons with OASDI taxable self-employment earnings, and number of persons with HI taxable self-employment earnings. Data are also available on an age-sex specific basis for number of persons with OASDI taxable wages, number of persons with HI taxable wages, number of persons with OASDI taxable self-employment earnings, and number of persons with HI taxable self-employment earnings.
14. Data obtained from the Quarterly Trust Fund Letter, received from Office of Financial Policy and Operations (OFPO). Data currently used are OASDI and HI taxable wages accumulated from all Forms 941 and W-2 to date, and changes in self-employment earnings and self-reported tips since the prior Letter. These data are for years 1978 to the most recent year available.
15. Data obtained from OS on amounts of OASDI taxable wages on the Earnings Suspense File for 1937 through the second year prior to the current Trustees Report year. The data are used in estimating total OASDI covered employment.
16. Ratio of OASDI covered to NIPA wages, and ratio of OASDI taxable to covered wages. NIPA wages by sector are available quarterly from 1947 to 2020 and annually from 1947 to 2020. They are published by the BEA and updated several times during the year. OASDI covered and taxable wages (1971 to 2018) are updated annually by the Economic process. Covered and taxable data for more recent historical years are estimated from preliminary tabulations of Form 941 and W-2 data. Projected values for covered ratios are set to the latest historical year for the military, state and local, farm, and private household sectors. The projected value for the federal civilian sector covered ratio is projected to grow to 1.0 by 2030. The projected values for the private nonfarm business and the private sector vary with the relative size of the other-than-LPR population.

2.3.c. *Development of Output*

Equation 2.3.2 - Total Employment (TE)

Based on the CPS, BLS estimates the total number of persons with any work experience

(WE) during a calendar year. Average weeks worked (AWW) during a calendar year is defined as $AWW = E * 52 / WE$. Based on a 100 percent count of earnings reports (i.e., Form W-2 and Schedule SE) tabulated by SSA, OCACT estimates the total number of persons employed at any time during a calendar year (TE). Compared to WE, TE is a broader measure of employment. WE is an estimate of the number of workers in the civilian noninstitutional US population age 16 and over. TE is an estimate of employment in the broader Social Security area population, which includes U.S. territories, the military, and institutions. TE also includes employment of workers who age 15 and younger.

AWW is disaggregated by gender and age and is projected as a function of a time trend and unemployment rate. WE is projected as $(E * 52) / AWW$. TE is projected as the product of its lagged value and the growth rate for WE. TE is adjusted by two multiplicative factors due to differences between E and TE over the recent historical period. The first factor accounts for the difference in growth between N and P between the last historical value for TE and the last historical value for E. The second factor adjusts the model estimate to the value of our most recent historical year, and our latest estimate based on partial quarterly data for that year.

Equation 2.3.3 - Noncovered Employment (NCE)

NCE is disaggregated by age and gender. Age groups include 14-15, 16-17, 18-19, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, and 70 and over.

Employment may not be OASDI covered for a variety of reasons mostly related to the type of work. Consequently, NCE is further disaggregated to the type-of-work components listed below.

Federal Civilian Government - All Federal civilian employees are HI (i.e., Medicare) covered. All Federal Civilian employees hired in January 1984 and later are covered under the Federal Employees Retirement System (FERS) and are OASDI covered. Employees hired before January 1984 are covered under the Civil Service Retirement System (CSRS) and are not OASDI covered. This “closed group” of relatively older CSRS employees is projected to fall to near zero by 2030.

State and Local Government - In 1983, about 70 percent of State and Local Government (S&L) employment and wages were covered under OASDI and HI. Beginning April 1986, all newly hired S&L employees were covered under HI. Beginning January 1990, all S&L employees not under an S&L retirement system were covered under OASDHI.

By 2013, about 28 and 4 percent of S&L employment (and wages) are still not covered under OASDI and HI respectively. The closed group of relatively older S&L employees not covered under HI is projected to fall to near zero by 2030. S&L employment not covered under OASDI is projected to grow at about the same rate as the labor force.

Students at Public Schools - Prior to 2000, students working at S&L public schools were covered under OASDI and HI if the other school employees were covered. In 2000, legislation offered an “open season” allowing schools to remove their students from

coverage. Virtually all major schools opted for removal. Hence, almost no students working at their public schools are covered under OASDI or HI. Students at public schools are projected to grow at about the same rate as the population aged 18 to 24.

Election Workers - Most S&L election workers are subject to an earnings test and are not covered under OASDHI. The earnings test was raised from \$100 to \$1,000 beginning January 1995 and indexed beginning in 2000. Election workers are projected to grow at about the same rate as LC.

Private Household - The threshold for coverage of domestic employees' earnings was raised from \$50 per calendar quarter to \$1,000 per calendar year (CY) per employee. Domestic workers are no longer covered if under age 18. Private household employment is projected to grow at about the same rate as E and vary with RTP.

Students at Private Schools - All students working in private schools are not covered under OASDHI. Students at private schools are projected to grow at about the same rate as the population aged 18 to 24.

Railroad - Employers do not submit payments for payroll taxes to the IRS for railroad employees. Railroad employees are projected by the Railroad Retirement Board.

Underground Economy Workers - Set to the at-any-time employed in the other-than-LPR population who have no reported earnings and therefore are part of the underground economy (i.e., TEO_UND).

Foreign Students and Exchange Visitors

Equation 2.3.5 - Self-Employed Only (SEO)

SEO is projected to grow at the same rate as ES.

Equation 2.3.7 - Ratio of Combination Workers (RCMB)

Total CMB_TOT can be separated into two groups depending on whether they have OASDI covered wages under or over the TAXMAX. CMB_TOT with covered wages under the TAXMAX have taxable wages and self-employed income. CMB_TOT with covered wages over the TAXMAX have taxable wages only. CMB_TOT with covered wages over the TAXMAX would have paid taxes on their self-employed income if the TAXMAX had been eliminated.

Total CMB_TOT is projected as a ratio to WSW. This ratio is dependent on the RTP. If RTP rises, then the CMB_TOT increases.

Equation 2.3.10 - Ratio of Covered Wages (RWSC)

RWSC is disaggregated by the following sectors: Federal Civilian government, Federal

Military, S&L government, and Private.

Federal Civilian government - Total Federal civilian employment and wages are split by retirement system. Those under FERS are OASDI covered, while those under CSRS are not. Hence, the RWSC for the Federal civilian employment is defined as the ratio of wages for employment under FERS to total Federal civilian wages. Employment and wages are projected for workers under each retirement system. Employment under CSRS is a closed group that is expected to fall to zero by about 2030. Employment under FERS is defined as total Federal employment less employment under CSRS. Total Federal civilian employment is projected to be constant over the short-range period, and about equal to the growth in the LC thereafter. The growth rates in the average wage for those under CSRS and FERS are projected based on, for the first five years, pay raises assumed under the most recent OMB FY Budget and on the growth rate in the CPI.

Federal Military - The RWSC for the Federal military sector is projected to remain constant at its latest actual historical level.

S&L government - The RWSC for the S&L government sector is projected to remain constant at its latest actual historical level.

Private - The private sector is separated into sub-sectors including private households, farm, railroad, tips, and a residual private “base”. The RWSCs for the private household and farm sub-sectors are projected to remain constant at their latest actual historical levels. By definition, the RWSCs for the railroad and tips sub-sectors are projected to remain constant at 0.0 and 1.0, respectively. The projected RWSC for the private base sub-sector is dependent on the ratio of EO wage workers in the private base sub-sector who are covered under the OASDI program to all EO wage workers in the private base sub-sector. We assume that all of EO will be wage workers employed in the private residual base sub-sector of the economy and that the proportion of EO that is covered under the OASDI program will decrease. Therefore, we assume that the RWSC for the private residual base sector will also decrease.

Equation 2.3.12 - Ratio of Covered Self-Employed Earnings (RCSE)

The RCSE is projected to remain constant at its latest actual historical level.

Equation 2.3.17 - Average Wage Index (AWI)

The growth in the AWI is projected to be equal to the growth in the average wage for employees with any wages (covered and noncovered) posted to the MEF (AWS_MEF). Total wages posted to the MEF (WS_MEF) is equal to WSC less wages posted to the ESF plus any non-OASDI covered wages posted to the MEF. Similarly, the total number of employees with any wages posted to the MEF (WSW_MEF) is equal to WSW less employees posted only to the ESF plus any employees with no OASDI covered wages posted to the MEF.

Equation 2.3.18 - OASDI Taxable Maximum (TAXMAX)

By law, the growth in the AWI is used to increase the TAXMAX.

2.4. Effective TAXABLE PAYROLL (TAXPAY)

2.4.a. Overview

TAXPAY estimates annual OASDI taxable earnings values including total employee taxable wages (WTEE), total employer taxable wages (WTER), and taxable self-employment income (SET). By law, each individual is required to pay the employee share of OASDI tax on wages from all covered jobs and the self-employment tax from self-employment income up to the TAXMAX. Each employer is required to withhold the employee share of the OASDI tax on the wages of each worker up to the TAXMAX, as well as paying the identical employer share. If an employee works more than one covered wage job and the sum of all covered wages exceeds the TAXMAX, the employee is due a refund. The employers involved are not due the refund. Hence, WTER is greater than WTEE. The difference (i.e., WTER less WTEE) is defined as multi-employer refund wages (MER). Individuals with covered wage employment who are also self-employed only owe taxes on their self-employment income to the extent that it does not exceed the TAXMAX after being added to the individual's covered wages.

TAXPAY estimates the annual OASDI effective taxable payroll (ETP) using the components discussed above. ETP is the amount of earnings in a year which, when multiplied by the combined employee-employer tax rate, yields the total amount of taxes due from wages and self-employment income in the year. ETP is used in estimating OASDI income and in determining income and cost rates and the actuarial balance. ETP is defined as WTER plus SET less one-half of MER.

The components of ETP are estimated by a collection of ratios. The employee taxable ratio (RWTEE) is defined as the ratio of WTEE to WSC. The multi-employer refund wage ratio (RMER) is defined as the ratio of MER to WSC. The self-employment income taxable ratio (RSET) is defined as the ratio of SET to CSE_TOT. Equations 2.4.1 through 2.4.8 outline the projection methodology.

$$RWTEE = RWTEE(\cdot) \quad (2.4.1)$$

$$WTEE = RWTEE * WSC \quad (2.4.2)$$

$$RMER = RMER(\cdot) \quad (2.4.3)$$

$$MER = RMER * WSC \quad (2.4.4)$$

$$WTER = WTEE + MER \quad (2.4.5)$$

$$RSET = RSET(\cdot) \quad (2.4.6)$$

$$SET = RSET * CSE_TOT \quad (2.4.7)$$

$$ETP = WTER + SET - 0.5 * MER \quad (2.4.8)$$

In order to conform to the Trustees' assumption that the ratio of ETP to the sum of WSC and CSE_TOT is 0.825 in the final short range year (see below), TAXPAY solves equations 2.4.1 through 2.4.8 iteratively, altering the trend adjustment on RWTEE until the assumed ratio is reached.

2.4.b. *Input Data*

1. Trustees Assumptions

The Board of Trustees of the OASDI Trust Funds assumes that the ratio of effective OASDI taxable payroll to covered earnings for the final calendar year of the short-range period (2030) is 0.825.

Data used to obtain values input directly to model

2. Data obtained from ORES by email for the amounts of single and multi-employer refunds for the latest 5 years. Each year, data are updated.
3. Data obtained from the most recently available 1.0% CWHS active file, maintained on Social Security's mainframe and made available by ORES. The years of data are 1951 to the third year prior to the current Trustees Report year. The data are used in estimating OASDI taxable wages for 1951 through 1977.
4. Data for taxable wages and self-employment income for 1978 on and total OASDI taxable earnings for 1951 on from the quarterly EPOXY Report, provided by OS. The data currently used in subprocess 2.4 are the amounts of OASDI taxable earnings, wages, and self-employment income and amounts of multi-employer excess wages.
5. Data obtained from the Quarterly Trust Fund Letter (QTFL), received from OFPO. Data currently used are OASDI and HI taxable wages accumulated from all Forms 941 and W-2 to date, and changes in self-employment income and self-reported tips since the prior QTFL. The wage data are for years 1978 to the most recent year available and the self-employment income data for years 1951 to the most recent year available.

Long-Range OASDI Projection Data

Historical and projected data from Section 2.3 are used as inputs.

2.4.c. *Development of Output*

Equation 2.4.1 - Employee Taxable Ratio (RWTEE)

Over the short-range projection horizon, the projected value for RWTEE is the sum of the model's "raw" estimate and an addfactor consisting of four components. The raw estimate for RWTEE is dependent on the distribution of workers by wage interval, the RELMAX,

RTP, the age-gender distribution of wage workers, a time trend adjustment, and a base-year error adjustment. The projected distribution of workers by wage interval is an average (or amalgam) distribution over the 2000 through 2018 period. Holding other factors constant, a distribution with relatively more workers with wages over the TAXMAX leads to a lower RWTEE. The RELMAX is defined as the ratio of the TAXMAX to the ACW. A higher RELMAX leads to a higher RWTEE.

An increase in the RTP leads to a lower RWTEE. The change in the projected RWTEE due to the change in the age-gender distribution of wage workers is calculated by allowing employment by age and gender to change while holding taxable ratios (and average covered wages) by age and gender constant to levels in 1996. The time trend adjustment reduces the level of RWTEE by about 0.6 percentage point over the short-range projection horizon. The base-year error adjustment starts with the value obtained by subtracting the estimated value of RWTEE for the latest historical (or base) year from the actual value and phases this amount out linearly over the ten years of the short-range projection period.

Equation 2.4.2 - Employee Taxable Wages (WTEE)

WTEE is computed by multiplying the ratio of taxable employee wages to covered wages by the level of covered wages.

Equation 2.4.3 - Multi-Employer Refund Wage Ratio (RMER)

The RMER is functionally related to the RWTEE. As RWTEE approaches one, RMER approaches zero. In between the limit values, RMER is positive. Given the present position of RWTEE and RMER on the function, a projected decline in RWTEE leads to an increase in RMER.

The projected RMER is also dependent on RU. An increase in RU leads to a decrease in RMER.

RMER is assumed to remain constant in years after the short-range projection period.

Equation 2.4.4 - Multi-Employer Refund Wages (MER)

MER is computed by multiplying the ratio of multi-employer refund wages to covered wages by the level of covered wages.

Equation 2.4.5 - Employer Taxable Wages (WTER)

WTER is computed by adding employer taxable wages to multi-employer refund wages.

Equation 2.4.6 - Self-Employed Net Income Taxable Ratio (RSET)

The RSET is disaggregated by type of self-employed worker, SEO and CMB_TOT.

SEO - The RSET is dependent on the distribution of self-employed workers by income interval and a RELMAX. The projected distribution of self-employed workers by income interval is set to the 2015 distribution. The RELMAX is defined here as the ratio of the TAXMAX to the average income for SEO. A higher RELMAX leads to a higher RSET.

CMB_TOT - Taxable self-employed net income for CMB_TOT is projected in two steps. First, a taxable earnings (wages and self-employment income) ratio for CMB_TOT is projected based on the 2015 distribution and a RELMAX defined as the ratio of the TAXMAX to the average covered earnings. The projected level of taxable earnings for CMB_TOT is the product of the estimated taxable earnings ratio for CMB_TOT and their covered earnings. Second, a taxable wage ratio for CMB_TOT is projected based on a RELMAX defined as the ratio of the TAXMAX to the average covered wage for CMB_TOT. The projected level of taxable wages for CMB_TOT is the product of the estimated taxable wage ratio for CMB_TOT and their covered wages.

Taxable self-employed net income for CMB_TOT is obtained by subtracting taxable wages from taxable earnings for CMB_TOT.

A “combined” RSET is calculated as the ratio of the sum of taxable self-employment income for SEO and CMB_TOT to CSE_TOT. As with the RWTEE, the combined RSET is adjusted over the short-range period due to other factors (i.e., RTP, the age-gender distribution of workers, and a trend). The effect of the other factors are taken from RWTEE and “scaled.” That is, RSET is adjusted by a percent effect (as opposed to percentage point) that is equal to the percent change in RWTEE due to changes in these other factors.

It is important to note that while the RWTEE is held constant after the short-range period, the RSETs for self-employed workers are not. After the short-range period, the projected RSETs for SEO and CMB_TOT continue to be dependent on their respective RELMAXs. Since by law the TAXMAX grows at the rate of the AWI and since ACSE is assumed to grow faster than the ACW (since only ACW declines with the growth in fringe benefits), the RELMAXs for self-employed workers decline over the long-range period while the RELMAX for wage workers is approximately constant. Hence, the RSETs for SEO and CMB_TOT are projected to decline over the long-range period while the RWTEE is held constant.

Equation 2.4.7 - Taxable Self-Employment Income (SET)

SET is computed by multiplying the self-employment income taxable ratio by the level of covered self-employment income.

Equation 2.4.8 – Effective Taxable Payroll (ETP)

ETP is computed by adding employer taxable wages and taxable self-employment income and subtracting from that total one-half of multi-employer refund wages. (Only employees can obtain refunds of excess taxes withheld in multi-employer refund wage cases.)

As noted above, in order to meet the Trustees’ assumption that the ratio of ETP to total

covered earnings is 0.825 in the last year of the short-range period, equations 2.4.1 through 2.4.8 are solved repeatedly with changes to the time-trend adjustment in equation 2.4.1 until the ratio is obtained.

2.5. REVENUES

2.5.a. Overview

Over the short-range projection horizon (i.e., first 10 years), REVENUES computes tax liabilities from wages and self-employment income, as well as the amount of taxes to be transferred from the trust funds to the general fund of the Treasury due to multi-employer refund wages. In Equation 2.5.1, WTL is the product of taxable wages and the combined OASDI employee-employer tax rate (TRW). In Equation 2.5.2, SEL is the product of SET and the OASDI self-employed tax rate (TRSE). In Equation 2.5.3, MERL is the product of MER and the OASDI employee tax rate (TRWEE).

$$WTL = WTER * TRW \quad (2.5.1)$$

$$SEL = SET * TRSE \quad (2.5.2)$$

$$MERL = MER * TRWEE \quad (2.5.3)$$

Also over the short-range horizon, REVENUES decomposes WTL into quarterly wage tax liabilities (WTLQ), then to quarterly wage tax collections (WTLQC). REVENUES also decomposes SEL into quarterly self-employment income tax collections (SELQC).

$$WTLQ = WTLQ(\cdot) \quad (2.5.4)$$

$$WTLQC = WTLQC(\cdot) \quad (2.5.5)$$

$$SELQC = SELQC(\cdot) \quad (2.5.6)$$

When determining the amount of payroll tax collections for the year, REVENUES subtracts the amount MERL from the sum of the WTLQC amounts.

Finally, over the first two projected quarters, REVENUES estimates of WTLQC and SELQC are replaced with ones from the most recent OMB FY Budget.

2.5.b. Input Data

Data used to obtain values input directly to model

1. Data obtained from ORES as tabulations of quarterly Form 941 data. Data currently used are the OASDI, HI, and income taxable wages by sector (Federal Civilian, military, farm, and State and Local) for the most recent five years. The data represent changes in reported wages since the prior quarterly report. The most recent data are appended to previously reported data. Annual totals are computed and used to derive estimates of OASDI taxable wages by sector for the latest historical years.
2. Data obtained from quarterly IRS Form 941 files, provided by OS. Data currently used are the OASDI and HI taxable wages for 1978 to the most recent year available. The

OASDI data include separate amounts for tips and for agricultural wages. The data represent changes in reported wages since the prior quarterly report. The most recent data are appended to previously reported data. Annual totals are computed and used to derive estimates of OASDI total and farm taxable wages for the most recent historical years.

3. Data from the DB2 database named MCWHS, maintained by Office of Systems. This database contains a table with MEF detail records for SSNs in the 1% CWHS sample. OCACT uses the table, in conjunction with sector information from the employee-employer and IRS Form 941 files described in items 11 and 12 of section 2.3.b, to produce estimates of OASDI and HI taxable wages for the Federal Civilian, military, and State and Local government sectors.

Values input directly to model

Historical and projected data from Sections 2.1, 2.2, and 2.3 are used as input. Data for the following variables have final year of 2100. Each variable is shown with the starting year of the data.

- | | |
|---------------------|--|
| 4. ADDSETREEOD | Total add factor to OASDI taxable to covered self-employment income ratio, 2015 |
| 5. ADDWSTREEOD | Total add factor to OASDI taxable to covered wage ratio, 2015 |
| 6. ADDWSTREEODTREND | Component of total add factor to OASDI taxable to covered wage ratio due to trend in ratio, 2015 |
| 7. AIW | Average wage for indexing (\$), 1971 |
| 8. AWSCFM | Average covered wage for farm workers (\$), 1971 |
| 9. AWSCML | Average covered wage for military (\$), 1971 |
| 10. DMWCHI | Deemed military wage credits for HI (\$ millions), 1983 |
| 11. DMWCOD | Deemed military wage credits for OASDI (\$ millions), 1983 |
| 12. ECFCHO | Number of HI-only covered Federal Civilian workers (millions), 1983 |
| 13. ECFCOD | Number of OASDI covered Federal Civilian workers (millions), 1983 |
| 14. ECHITOT | Number of HI covered workers (millions), 1987 |
| 15. ECSEHI | Number of HI covered self-employed workers (millions), 1988 |
| 16. ECSENO MAX | Number of covered self-employed workers if no taxable maximum (millions), 1988 |
| 17. ECSEO | Number of OASDI covered self-employed only workers (millions), 1981 |
| 18. ECSEOD | Number of OASDI covered self-employed workers (millions), 1981 |
| 19. EC SLNOIS | Number of non-OASDI covered State and Local |

20. ECSLP91	workers including students (millions), 1983 Number of State and Local workers covered under OASDI under pre-1991 law (millions), 1983
21. ECSLNRP	Number of OASDI covered State and Local workers with no retirement plan (millions), 1983
22. ECSLOD	Number of OASDI covered State and Local workers (millions), 1983
23. ECWSHI	Number of HI covered wage workers (millions), 1981
24. ECWSOD	Number of OASDI covered wage workers (millions), 1981
25. ECWSOD_MEF	Number of OASDI covered wage workers on the Master Earnings File (MEF) in millions, 1981
26. ECWSOD(sex, age)_MEF	Number of OASDI covered wage workers on the Master Earnings File (MEF) in millions by sex (M/F) and age group (Under 16, 16-19, 20-24, 25-29, ..., 60-64, 65-69, 70 and over (millions), 1981
27. ESLCG	Number of State and Local workers not covered under HI (millions), 1983
28. ESLSTUD	Number of noncovered students at public schools employed by their school (millions), 1983
29. GAPLAG	Ratio of real to potential GDP (units), 1971
30. RTP	Ratio of real to potential GDP (units), 1971
31. RU	Civilian unemployment rate (percent), 1971
32. SEECMB	Self-employed earnings of all SE workers who also earned wages in same year (\$ millions), 1991
33. SEECHI	HI covered self-employed earnings (\$ millions), 1991
34. SEECNOMAX	Covered self-employed earnings if no taxable maximum (\$ millions), 1991
35. SEECOD	OASDI covered self-employed earnings (\$ millions), 1991
36. SEECOD_OLD	OASDI covered self-employed earnings excluding self-employed earnings of workers with covered wages greater than or equal to the OASDI taxable maximum (\$ millions), 1971
37. SEETODCMB	OASDI taxable self-employment earnings of combination workers (\$ billions), 1995
38. SEETODEXOG	Total OASDI taxable self-employment earnings (\$ millions), 1995
39. SEETODSEO	OASDI taxable self-employment earnings of self-employed only workers (\$ billions), 1995
40. TAXMAXHI	HI taxable maximum (\$) – 0 indicates no maximum, 1971
41. TAXMAXOD	OASDI taxable maximum (\$), 1971

42. TCFCD	Proportion of annual Federal Civilian wages earned in each quarter (units), 1971
43. TCMD	Proportion of annual military wages earned in each quarter (units), 1971
44. TCPD	Proportion of annual private sector wages earned in each quarter (units), 1971
45. TCSLD	Proportion of annual State and Local wages earned in each quarter (units), 1971
46. TETODCMB	Total OASDI taxable earnings of combination workers (\$ millions), 1995
47. WSCCMB	Wages earned in same year by all SE workers with both types of earnings (\$ millions), 1991
48. WSCFCHO	HI Covered wages of Federal Civilian HI-only workers (\$ millions), 1983
49. WSCFCOD	OASDI Covered wages of Federal Civilian workers (\$ millions), 1971
50. WSCFM	Covered wages of farm workers (\$ millions), 1971
51. WSCHI	HI covered wages (\$ millions), 1971
52. WSCML	Covered wages of members of the Armed Forces (\$ millions), 1971
53. WSCOD	OASDI covered wages (\$ millions), 1971
54. WSCOD_SF	OASDI covered wages on the Suspense File (\$ millions), 1971
55. WSCPHH	Covered wages of private household workers (\$ millions), 1971
56. WSCPNF	Covered wages of private nonfarm workers (\$ millions), 1971
57. WSCSLHI	HI covered State and Local wages (\$ millions), 1971
58. WSCSLNRP	Covered wages of State and Local workers with no retirement plan (\$ millions), 1991
59. WSCSLOD	OASDI covered State and Local wages (\$ millions), 1971
60. WSCSLP91	Wages of State and Local workers covered under OASDI under pre-1991 law (\$ millions), 1971
61. WSD	Total NIPA wages (\$ millions), 1971
62. WSP	Total NIPA private sector wages (\$ millions), 1971
63. WSS	Total NIPA compensation (\$ millions), 1971
64. WSSLCG	Wages of State and Local workers not covered under HI (\$ millions), 1983
65. WSSLNOIS	Wages of non-OASDI covered State and Local workers including students (\$ millions), 1983
66. WSSLSTUD	Wages of noncovered students at public schools employed by their school (\$ millions), 1983
67. WSMEREFODEXOG	OASDI multi-employer refund wages

	(\$ millions), 2014
68. WSTEEODEXOG	Total OASDI taxable wages (\$ millions), 2015
69. WSTRRTPHI	Wages of railroad workers taxable under HI (\$ millions), 1971
70. WSTTIPSSR	Taxable tips reported by tip earner instead of employer (\$ millions), 1978
71. WTWPO	Proportion of annual Postal Service wages earned in each quarter (units), 1971

Other direct input data

72. FICA, SECA, and Federal Employer tax transfers by month from the Department of the Treasury for years 1984 to 2020.
73. FICA, SECA, and Federal Employer tax transfers by month from the Department of the Treasury for January 2021 through May 2021.
74. FICA and SECA tax transfers by month split by liability period from the Department of the Treasury for January 1984 to June 2021.
75. Historical annual HI taxable self-employment earnings for 1983 to 1993. (Values from 1994 on are equal to HI covered earnings and are obtained from subprocess COV.) **Does not affect SOSI**
76. Historical annual OASDI taxable self-employment earnings for 1971 to 2019.
77. Historical annual HI multi-employer refund wages for 1983 to 1993 (Values for 1994 on are zero because of the elimination of the HI taxable maximum.) **Does not affect SOSI**
78. Historical annual OASDI multi-employer refund wages for 1971 to 2019.
79. Historical annual HI single-employer refund wages for 1991 to 1993 (No values prior to 1991 because HI taxable maximum equals OASDI taxable maximum for those years. Values for 1994 on are zero because of the elimination of the HI taxable maximum.) **Does not affect SOSI**
80. Historical annual OASDI single-employer refund wages for 1971 to 2019.
81. Historical annual HI taxable wages for 1983 to 1993. (Values from 1994 on are equal to HI covered wages and are obtained from subprocess COV.) **Does not affect SOSI**
82. Historical annual OASDI taxable wages for 1971 to 2019.
83. Historical annual HI-only taxable Federal Civilian wages for 1983 to 1993. (Values from 1994 on are equal to HI-only covered wages and are obtained from subprocess COV.) **Does not affect SOSI**

84. Historical annual OASDI taxable Federal Civilian wages for 1983 to 2019.
85. Historical annual HI taxable Federal Civilian wages for 1983 to 1993. (Values from 1994 on are equal to HI covered wages and are obtained from subprocess COV.) **Does not affect SOSI**
86. Historical annual OASDI taxable farm sector wages for 1971 to 2019.
87. Historical annual HI taxable farm sector wages for 1991 to 1993. (Values from 1994 on are equal to HI covered wages and are obtained from subprocess COV.) **Does not affect SOSI**
88. Historical annual OASDI taxable military sector wages for 1971 to 2019.
89. Historical annual HI taxable military sector wages for 1991 to 1993. (Values from 1994 on are equal to HI covered wages and are obtained from subprocess COV.) **Does not affect SOSI**
90. Historical annual OASDI taxable State and Local government sector wages for 1971 to 2019.
91. Historical annual HI taxable State and Local government sector wages for 1983 to 1993. (Values from 1994 on are equal to HI covered wages and are obtained from subprocess COV.) **Does not affect SOSI**
92. Historical annual OASDI taxable tips for employees as reported by employers for 1971 to 2019.
93. Historical and projected annual OASDI taxable tips for employers as reported by employers for 1980 to 1987. (No tips were taxable for employers prior to 1980. Employer taxable tips equal employee in 1988 and after.)
94. Quarterly OASDI and **HI** taxable wages for calendar year 2020. (These values do not add up to the annual values as described above. These quarterly distributions are used to split the annual 2020 values.) **HI values do not affect SOSI**
95. Historical FICA and SECA appropriation adjustments for OASI, DI, and **HI** by month for 1968 to 2020. **HI values do not affect SOSI**
96. Actual FICA and SECA appropriation adjustments for OASI, DI, and **HI** for March 2021 and preliminary values for June 2021. **HI values do not affect SOSI**
97. Historical FICA revenues for OASI, DI, and **HI** by quarter for 1984 to 2020. **HI values do not affect SOSI**

98. Historical SECA revenues for OASI, DI, and HI by quarter for 1984 to 2020. HI values do not affect SOSI
99. Historical Federal Employer revenues for OASI, DI, and HI by quarter for 1984 to 2020. HI values do not affect SOSI
100. Historical Deposits by States for OASI, DI, and HI by quarter for 1984 to 2020. (This is an obsolete type of revenue which has had no valid non-zero amount since 2002.) HI values do not affect SOSI
101. Historical single-employer refunds of excess taxes for OASI, DI, and HI by quarter for 1984 to 2020. HI values do not affect SOSI
102. Historical FICA credits for OASI and DI by quarter for 1984 to 2020.
103. Historical SECA credits for OASI, DI, and HI by quarter for 1984 to 2020. HI values do not affect SOSI
104. Historical multi-employer refunds of excess taxes for OASI, DI, and HI by month for 1968 to 2020. HI values do not affect SOSI

Miscellaneous historical covered employment and earnings data:

105. HI Covered self-employed workers for 1986 to 1987 – variable ECSEHI. Does not affect SOSI
106. Number of OASDI covered wage workers for 1971 to 1980 – variable ECWSOD.
107. HI covered self-employment earnings for 1971 to 1990 – variable SEECHI. Does not affect SOSI
108. Covered self-employment earnings if there were no taxable maximum for 1971 to 1990 – variable SEECNOMAX.
109. OASDI covered self-employment earnings for 1971 to 1990 – variable SEECOD.

Miscellaneous historical and fixed projected data:

110. Quarterly distribution of annual OASDI taxable farm wages for 1971 to 2100 – variable TTFMD.
111. Quarterly OASDI covered private nonfarm sector wages for 1971 to 1975 – variable QWSCPWF.
112. Quarterly OASDI covered State and Local government sector wages for 1971 to 1977 – variable QWSCSLOD.

113. Quarterly OASDI covered military sector wages for 1971 to 1977 – variable QWSCML.
114. Quarterly OASDI covered Federal Civilian sector wages for 1971 to 1977 – variable QWSCFCOD.
115. Quarterly OASDI taxable private nonfarm sector wages for 1971 to 1977 – variable QWSTPNFEEOD.
116. Quarterly OASDI taxable State and Local government sector wages for 1971 to 1980Q1 – variable QWSTSLEEOD.
117. Quarterly OASDI taxable military sector wages for 1971 to 1977 – variable QWSTMLEEOD.
118. Quarterly OASDI taxable Federal Civilian sector wages for 1971 to 1977 – variable QWSTFCEEOD.
119. Quarterly OASDI taxable farm sector wages for 1971 to 1980Q2 – variable QWSTFMEEOD.
120. OASDI employee, employer, and self-employment tax rates from 1971 to 2100. These contribution rates are set according to the Social Security Act of 1935 and subsequent changes to the Act and to the Internal Revenue Code. The rates are updated when legislation mandates a change. The rates were unchanged from 2000 to 2015. The Bipartisan Budget Act of 2015 reallocated the OASI and DI employee-employer and self-employment tax rates for years 2016 through 2018. The OASDI rates remain the same and the rates revert to the ones in effect for 2000 to 2015 in 2019 and thereafter – variables RATEEO, RATEED, RATEEH, RATERO, RATERD, RATERH, RATSEO, RATSED, RATSEH.
121. Annual OASDI employee credit tax rate for 1984 – variable CRATEEOD.
122. Annual OASDI self-employment credit tax rates for 1984 to 1989 – variable CRATSEODH.
123. Annual reductions in OASDI employee and self-employment tax rates due to the payroll tax holiday for 2011 and 2012.
124. Annual trend variable for taxable to covered wage ratio calculation for 1971 to 2100 – variable TREND. (No longer used)
125. Annual trend variable for taxable to covered self-employment earnings ratio calculation for 1971 to 2100 – variable SETRND. (No longer used)
126. Average OASDI covered wages by age groups and gender for 1996.

127. Ratio of OASDI taxable to covered wages by age groups and gender for 1996.
128. Corrections to prior FICA appropriation adjustments made in March 2000.
129. Projected single-employer refund wages by calendar year for 2020 through 2031.
130. Projected ratio of OASDI taxable tips for the current year to the prior year for 2020 through 2031.
131. FICA and SECA appropriation adjustments for OASI and DI related to HIRE Act of 2010.
132. Estimated quarterly transfers provided to OTA for calendar years 2000 through 2021, 2nd quarter.
133. Data needed to compute estimates of the ACA's additional HI tax effective starting 2013. **Does not affect SOSI**
134. Estimated tax transfers by liability period for the ACA's additional HI tax for 2013Q1 through 2021Q2 used in computing adjustments. **Does not affect SOSI**
135. Estimated tax transfers for the ACA's additional HI tax for 2013Q1 through 2021Q2 used in computing adjustments. **Does not affect SOSI**
136. Estimated SECA appropriation adjustments for OASI, DI, and HI in December 2022. **HI values do not affect SOSI**
137. Estimated FICA and SECA appropriation adjustments for OASI, DI, and HI for differences between actual liabilities for 2021 transferred in January through June 2021 and estimated liabilities in Trustees Report. **HI values do not affect SOSI**

2.5.c. Development of Output

Equation 2.5.1 – Annual Covered Wage Tax Liabilities (WTL)

WTL is computed by multiplying the combined OASDI employee-employer tax rate by the OASDI taxable wages input from the PAYROLL subprocess. REVENUES estimates annual taxable wages for the Federal Civilian, Federal Military, S&L, Private Household, Farm, Self-reported Tips, and residual Private Nonfarm sectors. Liabilities by sector are computed by multiplying the combined OASDI employee-employer tax rate by OASDI taxable wages for each sector.

Equation 2.5.2 – Annual Self-Employed Income Tax Collections (SEL)

SEL is computed by multiplying the OASDI self-employment tax rate by the OASDI self-

employment taxable income input from the PAYROLL subprocess.

Equation 2.5.3 – Annual Multi-Employer Refund Wage Liabilities (MERL)

MERL is computed by multiplying the OASDI employee tax rate by the OASDI multi-employer refund wages input from the PAYROLL subprocess.

Equation 2.5.4 – Quarterly Covered Wage Tax Liabilities (WTLQ)

Total WTLQ is summed from sector components that include Federal Civilian, Federal Military, S&L, Private Household, Farm, Self-reported Tips, and residual Private Nonfarm. Sector WTLQs are determined by computing ratios of quarterly to annual liabilities for each quarter. These are calculated for the Private Nonfarm, S&L, Federal Civilian and Military sectors. Each is dependent on the quarterly distribution of WSD and the RWTEE for the relevant sector, and on a payday adjustment that takes into account the actual number of paydays that fall into a particular calendar quarter. WTLQ ratios for the other sectors are assumed to be constants over the projection horizon.

Equation 2.5.5 - Quarterly Wage Tax Collections (WTLQC)

Employers incur tax liabilities when they pay wages to their employees. These liabilities are required to be deposited in the general fund of the U.S. Treasury by employers based on the amount of total payroll tax liability (income taxes plus Social Security and Medicare taxes withheld) accumulated. Some very large employers must deposit their tax liabilities the next banking day after paying their employees. Other levels of accumulated tax liabilities require depositing within three days and by the middle of the following month. If employers follow these deposit requirements, the result is that all tax liability for a particular quarter is deposited by the last day of the month following the end of the quarter. Thus, the WTLQC for any particular quarter are the sum of the tax liabilities deposited for wages paid in the same quarter and the liabilities deposited for wages paid in the prior quarter.

WTLQC are summed from sector components that include the Federal Civilian, Federal Military, Farm, S&L, and residual Private Nonfarm (including Private Household and Self-Reported Tips). For the Federal Civilian and Military sectors, the WTLQC are set equal to their respective WTLQ since tax liabilities for the two sectors are considered collected immediately. The WTLQC for Farm is also set equal to its WTLQ, due in part to the fact that farms report tax liabilities annually. For the S&L and Private Nonfarm sectors, the WTLQC amount is computed by adding the product of WTLQ and the proportion of WTLQ that should be deposited in the same quarter in which the wages were paid to the WTLQ from the previous quarter which was not deposited in that quarter. Each quarter's proportion is based on the deposit requirements and estimates of accumulated tax liabilities, which in turn are based on firm size (or total wages paid). Separate proportions are estimated for the S&L and the Private Nonfarm sector because of the large difference in wage distributions between them.

Equation 2.5.6 - Quarterly Self-Employed Income Tax Collections (SELQC)

For wage workers, annual liabilities (WTL) are distributed to quarterly liabilities (WTLQ), which in turn are distributed to quarterly collections (WTLQC). However, for self-employed workers, annual liabilities (SEL) are distributed directly to SELQC, since self-employment income is only reported on an annual basis (on IRS Form 1040 Schedule SE).

SEL for a particular calendar year are distributed as collections to the four quarters of that year and to the first three quarters of the next year. This distribution uses quarterly proportions that are based on an historical pattern of the amount of SEL collected in each month, as estimated by the OTA. The OTA estimates reflect IRS regulations that require self-employed workers to deposit estimated tax liabilities four times a year (January, April, June, and September) and to make up any shortfall when filing Federal income tax returns in the following year. The program computes the collection distribution ratios based on the OTA estimates, which are input to the program. The ratios for projected years are determined by averaging the ratios for all years 1997 through the most recent year for which the OTA estimates are complete.

Appendix 2-1 *Equations*

2.1 *U.S. Employment (USEMP)*

UNEMPLOYMENT RATES, PRELIMINARY

[Full estimation details for these equations are available upon request.]

MALE

$$\text{RM1617_P} = \text{RM1617_P}(-1) + [-31.52172 * \text{D}(\text{RTP}) - 24.81520 * \text{D}(\text{RTP}(-1)) - 14.72058 * \text{D}(\text{RTP}(-2)) - 31.33966 * \text{D}(\text{RTP}(-3))]$$

$$\text{RM1819_P} = \text{RM1819_P}(-1) + [-56.74492 * \text{D}(\text{RTP}) - 24.87344 * \text{D}(\text{RTP}(-1)) - 35.84160 * \text{D}(\text{RTP}(-2)) - 0.70217 * \text{D}(\text{RTP}(-3))]$$

$$\text{RM2024_P} = \text{RM2024_P}(-1) + [-52.29093 * \text{D}(\text{RTP}) - 19.73984 * \text{D}(\text{RTP}(-1)) - 15.03600 * \text{D}(\text{RTP}(-2)) - 13.47522 * \text{D}(\text{RTP}(-3))]$$

$$\text{RM2529_P} = \text{RM2529_P}(-1) + [-39.13841 * \text{D}(\text{RTP}) - 24.46060 * \text{D}(\text{RTP}(-1)) - 17.51574 * \text{D}(\text{RTP}(-2)) - 0.81036 * \text{D}(\text{RTP}(-3))]$$

$$\text{RM3034_P} = \text{RM3034_P}(-1) + [-23.76350 * \text{D}(\text{RTP}) - 22.76416 * \text{D}(\text{RTP}(-1)) - 8.88851 * \text{D}(\text{RTP}(-2)) - 11.24022 * \text{D}(\text{RTP}(-3))]$$

$$\text{RM3539_P} = \text{RM3539_P}(-1) + [-28.26931 * \text{D}(\text{RTP}) - 9.02630 * \text{D}(\text{RTP}(-1)) - 14.93420 * \text{D}(\text{RTP}(-2)) - 9.03795 * \text{D}(\text{RTP}(-3))]$$

$$\text{RM4044_P} = \text{RM4044_P}(-1) + [-18.16004 * \text{D}(\text{RTP}) - 23.06573 * \text{D}(\text{RTP}(-1)) - 9.94244 * \text{D}(\text{RTP}(-2)) - 4.34840 * \text{D}(\text{RTP}(-3))]$$

$$\text{RM4549_P} = \text{RM4549_P}(-1) + [-22.41979 * \text{D}(\text{RTP}) - 16.69081 * \text{D}(\text{RTP}(-1)) - 11.82097 * \text{D}(\text{RTP}(-2)) - 1.04890 * \text{D}(\text{RTP}(-3))]$$

$$\text{RM5054_P} = \text{RM5054_P}(-1) + [-23.03219 * \text{D}(\text{RTP}) - 11.75062 * \text{D}(\text{RTP}(-1)) - 8.80293 * \text{D}(\text{RTP}(-2)) - 10.36992 * \text{D}(\text{RTP}(-3))]$$

$$\text{RM5559_P} = \text{RM5559_P}(-1) + [-25.47548 * \text{D}(\text{RTP}) - 14.31543 * \text{D}(\text{RTP}(-1)) - 6.15894 * \text{D}(\text{RTP}(-2)) - 1.00513 * \text{D}(\text{RTP}(-3))]$$

$$\text{RM6064_P} = \text{RM6064_P}(-1) + [-2.10594 * \text{D}(\text{RTP}) - 17.27891 * \text{D}(\text{RTP}(-1)) - 4.87379 * \text{D}(\text{RTP}(-2)) - 15.36088 * \text{D}(\text{RTP}(-3))]$$

$$\text{RM6569_P} = \text{RM6569_P}(-1) + [-13.51567 * \text{D}(\text{RTP}) - 2.41017 * \text{D}(\text{RTP}(-1)) - 17.92290 * \text{D}(\text{RTP}(-2)) + 1.48482 * \text{D}(\text{RTP}(-3))]$$

$$\text{RM7074_P} = \text{RM7074_P}(-1) + [-9.76394 * \text{D}(\text{RTP}) - 7.44785 * \text{D}(\text{RTP}(-1)) - 15.23719 * \text{D}(\text{RTP}(-2)) + 7.17596 * \text{D}(\text{RTP}(-3))]$$

$$\text{RM75O_P} = \text{RM75O_P}(-1) + [2.47935 * \text{D}(\text{RTP}) - 27.63188 * \text{D}(\text{RTP}(-1)) + 8.35078 * \text{D}(\text{RTP}(-2)) - 3.11058 * \text{D}(\text{RTP}(-3))]$$

FEMALE

$$\text{RF1617_P} = \text{RF1617_P}(-1) + [-13.33857 * \text{D}(\text{RTP}) - 1.17140 * \text{D}(\text{RTP}(-1)) - 69.40326 * \text{D}(\text{RTP}(-2)) + 9.53209 * \text{D}(\text{RTP}(-3))]$$

$$\text{RF1819_P} = \text{RF1819_P}(-1) + [-42.47852 * \text{D}(\text{RTP}) - 11.03359 * \text{D}(\text{RTP}(-1)) - 0.89011 * \text{D}(\text{RTP}(-2)) - 33.04213 * \text{D}(\text{RTP}(-3))]$$

$$\text{RF2024_P} = \text{RF2024_P}(-1) + [-23.03635 * \text{D}(\text{RTP}) - 17.33653 * \text{D}(\text{RTP}(-1)) - 10.59182 * \text{D}(\text{RTP}(-2)) - 7.73281 * \text{D}(\text{RTP}(-3))]$$

$$\text{RF2529_P} = \text{RF2529_P}(-1) + [-15.61692 * \text{D}(\text{RTP}) - 12.87215 * \text{D}(\text{RTP}(-1)) - 15.16923 * \text{D}(\text{RTP}(-2)) - 3.55380 * \text{D}(\text{RTP}(-3))]$$

$$\text{RF3034_P} = \text{RF3034_P}(-1) + [-14.71130 * \text{D}(\text{RTP}) - 6.25917 * \text{D}(\text{RTP}(-1)) - 18.72755 * \text{D}(\text{RTP}(-2)) - 2.42329 * \text{D}(\text{RTP}(-3))]$$

$$\text{RF3539_P} = \text{RF3539_P}(-1) + [-19.78502 * \text{D}(\text{RTP}) - 7.31538 * \text{D}(\text{RTP}(-1)) - 8.69549 * \text{D}(\text{RTP}(-2)) - 5.45418 * \text{D}(\text{RTP}(-3))]$$

$$\begin{aligned}
RF4044_P &= RF4044_P(-1) + [-8.62310*D(RTP) - 10.87758*D(RTP(-1)) - 6.86064*D(RTP(-2)) \\
&\quad - 4.08914*D(RTP(-3))] \\
RF4549_P &= RF4549_P(-1) + [-9.41764*D(RTP) - 14.45854*D(RTP(-1)) - 0.04687*D(RTP(-2)) \\
&\quad - 4.15977*D(RTP(-3))] \\
RF5054_P &= RF5054_P(-1) + [-11.37880*D(RTP) - 4.16526*D(RTP(-1)) - 12.32611*D(RTP(-2)) \\
&\quad - 5.79074*D(RTP(-3))] \\
RF5559_P &= RF5559_P(-1) + [-9.11833*D(RTP) - 2.89447*D(RTP(-1)) - 17.40333*D(RTP(-2)) \\
&\quad + 0.37368*D(RTP(-3))] \\
RF6064_P &= RF6064_P(-1) + [-21.93195*D(RTP) + 2.80763*D(RTP(-1)) - 11.61244*D(RTP(-2)) \\
&\quad - 1.60187*D(RTP(-3))] \\
RF6569_P &= RF6569_P(-1) + [6.59281*D(RTP) + 5.16431*D(RTP(-1)) - 35.27026*D(RTP(-2)) \\
&\quad + 8.58449*D(RTP(-3))] \\
RF7074_P &= RF7074_P(-1) + [-4.25967*D(RTP) + 10.26851*D(RTP(-1)) - 19.18862*D(RTP(-2)) \\
&\quad + 6.93390*D(RTP(-3))] \\
RF75O_P &= RF75O_P(-1) + [-40.60841*D(RTP) + 42.55417*D(RTP(-1)) - 29.60376*D(RTP(-2)) \\
&\quad - 14.09088*D(RTP(-3))]
\end{aligned}$$

UNEMPLOYMENT RATES, AGE-GENDER ADJUSTED, PRELIMINARY

$$RUM_ASA_P = (RM1617_P * LM1617_BY + RM1819_P * LM1819_BY + RM2024_P * LM2024_BY + RM2529_P * LM2529_BY + RM3034_P * LM3034_BY + RM3539_P * LM3539_BY + RM4044_P * LM4044_BY + RM4549_P * LM4549_BY + RM5054_P * LM5054_BY + RM5559_P * LM5559_BY + RM6064_P * LM6064_BY + RM6569_P * LM6569_BY + RM7074_P * LM7074_BY + RM75O_P * LM75O_BY) / LCM_BY$$

$$RUF_ASA_P = (RF1617_P * LF1617_BY + RF1819_P * LF1819_BY + RF2024_P * LF2024_BY + RF2529_P * LF2529_BY + RF3034_P * LF3034_BY + RF3539_P * LF3539_BY + RF4044_P * LF4044_BY + RF4549_P * LF4549_BY + RF5054_P * LF5054_BY + RF5559_P * LF5559_BY + RF6064_P * LF6064_BY + RF6569_P * LF6569_BY + RF7074_P * LF7074_BY + RF75O_P * LF75O_BY) / LCF_BY$$

$$RU_ASA_P = (RUM_ASA_P * LCM_BY + RUF_ASA_P * LCF_BY) / LC_BY$$

UNEMPLOYMENT RATES

MALE

$$\begin{aligned}
RM1617 &= RM1617_P * (1 + RU_ASA_ADJ / RU_ASA_P) \\
RM1819 &= RM1819_P * (1 + RU_ASA_ADJ / RU_ASA_P) \\
RM2024 &= RM2024_P * (1 + RU_ASA_ADJ / RU_ASA_P) \\
RM2529 &= RM2529_P * (1 + RU_ASA_ADJ / RU_ASA_P) \\
RM3034 &= RM3034_P * (1 + RU_ASA_ADJ / RU_ASA_P) \\
RM3539 &= RM3539_P * (1 + RU_ASA_ADJ / RU_ASA_P) \\
RM4044 &= RM4044_P * (1 + RU_ASA_ADJ / RU_ASA_P) \\
RM4549 &= RM4549_P * (1 + RU_ASA_ADJ / RU_ASA_P) \\
RM5054 &= RM5054_P * (1 + RU_ASA_ADJ / RU_ASA_P) \\
RM5559 &= RM5559_P * (1 + RU_ASA_ADJ / RU_ASA_P) \\
RM6064 &= RM6064_P * (1 + RU_ASA_ADJ / RU_ASA_P) \\
RM6569 &= RM6569_P * (1 + RU_ASA_ADJ / RU_ASA_P) \\
RM7074 &= RM7074_P * (1 + RU_ASA_ADJ / RU_ASA_P) \\
RM75O &= RM75O_P * (1 + RU_ASA_ADJ / RU_ASA_P)
\end{aligned}$$

FEMALE

$$\begin{aligned}
RF1617 &= RF1617_P * (1 + RU_ASA_ADJ / RU_ASA_P) \\
RF1819 &= RF1819_P * (1 + RU_ASA_ADJ / RU_ASA_P) \\
RF2024 &= RF2024_P * (1 + RU_ASA_ADJ / RU_ASA_P)
\end{aligned}$$

RF2529= RF2529_P * (1 + RU_ASA_ADJ / RU_ASA_P)
 RF3034= RF3034_P * (1 + RU_ASA_ADJ / RU_ASA_P)
 RF3539= RF3539_P * (1 + RU_ASA_ADJ / RU_ASA_P)
 RF4044= RF4044_P * (1 + RU_ASA_ADJ / RU_ASA_P)
 RF4549= RF4549_P * (1 + RU_ASA_ADJ / RU_ASA_P)
 RF5054= RF5054_P * (1 + RU_ASA_ADJ / RU_ASA_P)
 RF5559= RF5559_P * (1 + RU_ASA_ADJ / RU_ASA_P)
 RF6064= RF6064_P * (1 + RU_ASA_ADJ / RU_ASA_P)
 RF6569= RF6569_P * (1 + RU_ASA_ADJ / RU_ASA_P)
 RF7074= RF7074_P * (1 + RU_ASA_ADJ / RU_ASA_P)
 RF750 = RF750_P * (1 + RU_ASA_ADJ / RU_ASA_P)

UNEMPLOYMENT RATES, AGE-GENDER ADJUSTED

RUM_ASA = (RM1617 * LM1617_BY+ RM1819 * LM1819_BY + RM2024 * LM2024_BY + RM2529 * LM2529_BY
 + RM3034 * LM3034_BY + RM3539 * LM3539_BY + RM4044 * LM4044_BY + RM4549 * LM4549_BY
 + RM5054 * LM5054_BY + RM5559 * LM5559_BY + RM6064 * LM6064_BY + RM6569 * LM6569_BY
 + RM7074 * LM7074_BY + RM750 * LM750_BY) / LCM_BY

RUF_ASA = (RF1617 * LF1617_BY + RF1819 * LF1819_BY + RF2024 * LF2024_BY + RF2529 * LF2529_BY
 + RF3034 * LF3034_BY + RF3539 * LF3539_BY + RF4044 * LF4044_BY + RF4549 * LF4549_BY
 + RF5054 * LF5054_BY + RF5559 * LF5559_BY + RF6064 * LF6064_BY + RF6569 * LF6569_BY
 + RF7074 * LF7074_BY + RF750 * LF750_BY) / LCF_BY

RU_ASA = (RUM_ASA * LCM_BY + RUF_ASA * LCF_BY) / LC_BY

UNEMPLOYMENT RATES, FULL EMPLOYMENT DIFFERENTIALS

MALE

DRM1617_FE = -31.52172*(1 - RTP) - 24.81520*(1 - RTP(-1)) - 14.72058*(1 - RTP(-2)) - 31.33966*(1 - RTP(-3))
 DRM1819_FE = -56.74492*(1 - RTP) - 24.87344*(1 - RTP(-1)) - 35.84160*(1 - RTP(-2)) - 0.70217*(1 - RTP(-3))
 DRM2024_FE = -52.29093*(1 - RTP) - 19.73984*(1 - RTP(-1)) - 15.03600*(1 - RTP(-2)) - 13.47522*(1 - RTP(-3))
 DRM2529_FE = -39.13841*(1 - RTP) - 24.46060*(1 - RTP(-1)) - 17.51574*(1 - RTP(-2)) - 0.81036*(1 - RTP(-3))
 DRM3034_FE = -23.76350*(1 - RTP) - 22.76416*(1 - RTP(-1)) - 8.88851*(1 - RTP(-2)) - 11.24022*(1 - RTP(-3))
 DRM3539_FE = -28.26931*(1 - RTP) - 9.02630*(1 - RTP(-1)) - 14.93420*(1 - RTP(-2)) - 9.03795*(1 - RTP(-3))
 DRM4044_FE = -18.16004*(1 - RTP) - 23.06573*(1 - RTP(-1)) - 9.94244*(1 - RTP(-2)) - 4.34840*(1 - RTP(-3))
 DRM4549_FE = -22.41979*(1 - RTP) - 16.69081*(1 - RTP(-1)) - 11.82097*(1 - RTP(-2)) - 1.04890*(1 - RTP(-3))
 DRM5054_FE = -23.03219*(1 - RTP) - 11.75062*(1 - RTP(-1)) - 8.80293*(1 - RTP(-2)) - 10.36992*(1 - RTP(-3))
 DRM5559_FE = -25.47548*(1 - RTP) - 14.31543*(1 - RTP(-1)) - 6.15894*(1 - RTP(-2)) - 1.00513*(1 - RTP(-3))
 DRM6064_FE = -2.10594*(1 - RTP) - 17.27891*(1 - RTP(-1)) - 4.87379*(1 - RTP(-2)) - 15.36088*(1 - RTP(-3))
 DRM6569_FE = -13.51567*(1 - RTP) - 2.41017*(1 - RTP(-1)) - 17.92290*(1 - RTP(-2)) + 1.48482*(1 - RTP(-3))
 DRM7074_FE = -9.76394*(1 - RTP) - 7.44785*(1 - RTP(-1)) - 15.23719*(1 - RTP(-2)) + 7.17596*(1 - RTP(-3))
 DRM750_FE = 2.47935*(1 - RTP) - 27.63188*(1 - RTP(-1)) + 8.35078*(1 - RTP(-2)) - 3.11058*(1 - RTP(-3))

FEMALE

DRF1617_FE = -13.33857*(1 - RTP) - 1.17140*(1 - RTP(-1)) - 69.40326*(1 - RTP(-2)) + 9.53209*(1 - RTP(-3))
 DRF1819_FE = -42.47852*(1 - RTP) - 11.03359*(1 - RTP(-1)) - 0.89011*(1 - RTP(-2)) - 33.04213*(1 - RTP(-3))
 DRF2024_FE = -23.03635*(1 - RTP) - 17.33653*(1 - RTP(-1)) - 10.59182*(1 - RTP(-2)) - 7.73281*(1 - RTP(-3))
 DRF2529_FE = -15.61692*(1 - RTP) - 12.87215*(1 - RTP(-1)) - 15.16923*(1 - RTP(-2)) - 3.55380*(1 - RTP(-3))
 DRF3034_FE = -14.71130*(1 - RTP) - 6.25917*(1 - RTP(-1)) - 18.72755*(1 - RTP(-2)) - 2.42329*(1 - RTP(-3))
 DRF3539_FE = -19.78502*(1 - RTP) - 7.31538*(1 - RTP(-1)) - 8.69549*(1 - RTP(-2)) - 5.45418*(1 - RTP(-3))
 DRF4044_FE = -8.62310*(1 - RTP) - 10.87758*(1 - RTP(-1)) - 6.86064*(1 - RTP(-2)) - 4.08914*(1 - RTP(-3))
 DRF4549_FE = -9.41764*(1 - RTP) - 14.45854*(1 - RTP(-1)) - 0.04687*(1 - RTP(-2)) - 4.15977*(1 - RTP(-3))
 DRF5054_FE = -11.37880*(1 - RTP) - 4.16526*(1 - RTP(-1)) - 12.32611*(1 - RTP(-2)) - 5.79074*(1 - RTP(-3))
 DRF5559_FE = -9.11833*(1 - RTP) - 2.89447*(1 - RTP(-1)) - 17.40333*(1 - RTP(-2)) + 0.37368*(1 - RTP(-3))
 DRF6064_FE = -21.93195*(1 - RTP) + 2.80763*(1 - RTP(-1)) - 11.61244*(1 - RTP(-2)) - 1.60187*(1 - RTP(-3))

$$\begin{aligned} \text{DRF6569_FE} &= 6.59281*(1 - \text{RTP}) + 5.16431*(1 - \text{RTP}(-1)) - 35.27026*(1 - \text{RTP}(-2)) + 8.58449*(1 - \text{RTP}(-3)) \\ \text{DRF7074_FE} &= - 4.25967*(1 - \text{RTP}) + 10.26851*(1 - \text{RTP}(-1)) - 19.18862*(1 - \text{RTP}(-2)) + 6.93390*(1 - \text{RTP}(-3)) \\ \text{DRF750_FE} &= - 40.60841*(1 - \text{RTP}) + 42.55417*(1 - \text{RTP}(-1)) - 29.60376*(1 - \text{RTP}(-2)) - 14.09088*(1 - \text{RTP}(-3)) \end{aligned}$$

**UNEMPLOYMENT RATES, FULL EMPLOYMENT DIFFERENTIALS
TOTALS**

MALE

$$\begin{aligned} \text{RM1617_FE} &= \text{RM1617} + \text{DRM1617_FE} \\ \text{RM1819_FE} &= \text{RM1819} + \text{DRM1819_FE} \\ \text{RM2024_FE} &= \text{RM2024} + \text{DRM2024_FE} \\ \text{RM2529_FE} &= \text{RM2529} + \text{DRM2529_FE} \\ \text{RM3034_FE} &= \text{RM3034} + \text{DRM3034_FE} \\ \text{RM3539_FE} &= \text{RM3539} + \text{DRM3539_FE} \\ \text{RM4044_FE} &= \text{RM4044} + \text{DRM4044_FE} \\ \text{RM4549_FE} &= \text{RM4549} + \text{DRM4549_FE} \\ \text{RM5054_FE} &= \text{RM5054} + \text{DRM5054_FE} \\ \text{RM5559_FE} &= \text{RM5559} + \text{DRM5559_FE} \\ \text{RM6064_FE} &= \text{RM6064} + \text{DRM6064_FE} \\ \text{RM6569_FE} &= \text{RM6569} + \text{DRM6569_FE} \\ \text{RM7074_FE} &= \text{RM7074} + \text{DRM7074_FE} \\ \text{RM750_FE} &= \text{RM750} + \text{DRM750_FE} \end{aligned}$$

FEMALE

$$\begin{aligned} \text{RF1617_FE} &= \text{RF1617} + \text{DRF1617_FE} \\ \text{RF1819_FE} &= \text{RF1819} + \text{DRF1819_FE} \\ \text{RF2024_FE} &= \text{RF2024} + \text{DRF2024_FE} \\ \text{RF2529_FE} &= \text{RF2529} + \text{DRF2529_FE} \\ \text{RF3034_FE} &= \text{RF3034} + \text{DRF3034_FE} \\ \text{RF3539_FE} &= \text{RF3539} + \text{DRF3539_FE} \\ \text{RF4044_FE} &= \text{RF4044} + \text{DRF4044_FE} \\ \text{RF4549_FE} &= \text{RF4549} + \text{DRF4549_FE} \\ \text{RF5054_FE} &= \text{RF5054} + \text{DRF5054_FE} \\ \text{RF5559_FE} &= \text{RF5559} + \text{DRF5559_FE} \\ \text{RF6064_FE} &= \text{RF6064} + \text{DRF6064_FE} \\ \text{RF6569_FE} &= \text{RF6569} + \text{DRF6569_FE} \\ \text{RF7074_FE} &= \text{RF7074} + \text{DRF7074_FE} \\ \text{RF750_FE} &= \text{RF750} + \text{DRF750_FE} \end{aligned}$$

LABOR FORCE PARTICIPATION RATES (LFPR)

[Full estimation details for these equations are available upon request.]

MALE LFPR EQUATIONS

MALE 1617 AND 1819

$$\begin{aligned} \text{PM1617_P} &= [(- 0.00134*\text{RM1617} - 0.00167*\text{RM1617}(-1) - 0.00132*\text{RM1617}(-2) - 0.00065*\text{RM1617}(-3) \\ &\quad + 0.00003*\text{RM1617}(-4) + 0.00035*\text{RM1617}(-5)) \\ &\quad - 0.01009 * \text{TR_M1617} + 0.49638 \\ &\quad + 0.98199] / (1 + \text{RM1617DI}) \end{aligned}$$

$$\begin{aligned} \text{PM1819_P} &= [(- 0.00068*\text{RM1819} - 0.00092*\text{RM1819}(-1) - 0.00083*\text{RM1819}(-2) - 0.00056*\text{RM1819}(-3) \\ &\quad - 0.00025*\text{RM1819}(-4) - 0.00002*\text{RM1819}(-5)) \\ &\quad - 0.00780 * \text{TR_M1819} + 0.47128 \\ &\quad + 0.97741] / (1 + \text{RM1819DI}) \end{aligned}$$

$$\text{PM1617} = \text{PM1617_P}$$

PM1819 = PM1819_P

MALE 20 TO 54, NEVER MARRIED

PM2024NM_P = [(- 0.00053*RM2024 - 0.00074*RM2024(-1) - 0.00071*RM2024(-2) - 0.00054*RM2024(-3)
- 0.00030*RM2024(-4) - 0.00010*RM2024(-5))
- 0.00310 * TR_M2024
+ 1.12658] / (1 + RM2024DI)

PM2529NM_P = [(- 0.00031*RM2529 - 0.00053*RM2529(-1) - 0.00066*RM2529(-2) - 0.00068*RM2529(-3)
- 0.00058*RM2529(-4) - 0.00036*RM2529(-5))
- 0.00091 * TR_M2529
+ 0.99893] / (1 + RM2529DI)

PM3034NM_P = [(- 0.00001*RM3034 - 0.00018*RM3034(-1) - 0.00040*RM3034(-2) - 0.00058*RM3034(-3)
- 0.00064*RM3034(-4) - 0.00047*RM3034(-5))
+ 0.89877] / (1 + RM3034DI)

PM3539NM_P = [(0.00048*RM3539 + 0.00037*RM3539(-1) - 0.00007*RM3539(-2) - 0.00058*RM3539(-3)
- 0.00091*RM3539(-4) - 0.00080*RM3539(-5))
+ 0.87015] / (1 + RM3539DI)

PM4044NM_P = [(- 0.00018*RM4044 - 0.00026*RM4044(-1) - 0.00027*RM4044(-2) - 0.00022*RM4044(-3)
- 0.00014*RM4044(-4) - 0.00006*RM4044(-5))
+ 0.83848] / (1 + RM4044DI)

PM4549NM_P = [(0.00004*RM4549 - 0.00011*RM4549(-1) - 0.00036*RM4549(-2) - 0.00059*RM4549(-3)
- 0.00068*RM4549(-4) - 0.00052*RM4549(-5))
+ 0.80657] / (1 + RM4549DI)

PM5054NM_P = [(0.00115*RM5054 + 0.00089*RM5054(-1) - 0.00017*RM5054(-2) - 0.00141*RM5054(-3)
- 0.00220*RM5054(-4) - 0.00194*RM5054(-5))
+ 0.77113] / (1 + RM5054DI)

MALE 20 TO 54, MARRIED, SPOUSE PRESENT

PM2024MS_P = [(- 0.00053*RM2024 - 0.00074*RM2024(-1) - 0.00071*RM2024(-2) - 0.00054*RM2024(-3)
- 0.00030*RM2024(-4) - 0.00010*RM2024(-5))
- 0.00141 * TR_M2024
+ 1.10245] / (1 + RM2024DI)

PM2529MS_P = [(- 0.00031*RM2529 - 0.00053*RM2529(-1) - 0.00066*RM2529(-2) - 0.00068*RM2529(-3)
- 0.00058*RM2529(-4) - 0.00036*RM2529(-5))
- 0.00048 * TR_M2529
+ 1.02497] / (1 + RM2529DI)

PM3034MS_P = [(- 0.00001*RM3034 - 0.00018*RM3034(-1) - 0.00040*RM3034(-2) - 0.00058*RM3034(-3)
- 0.00064*RM3034(-4) - 0.00047*RM3034(-5))
+ 0.97980] / (1 + RM3034DI)

PM3539MS_P = [(0.00048*RM3539 + 0.00037*RM3539(-1) - 0.00007*RM3539(-2) - 0.00058*RM3539(-3)
- 0.00091*RM3539(-4) - 0.00080*RM3539(-5))
+ 0.98104] / (1 + RM3539DI)

PM4044MS_P = [(- 0.00018*RM4044 - 0.00026*RM4044(-1) - 0.00027*RM4044(-2) - 0.00022*RM4044(-3)
- 0.00014*RM4044(-4) - 0.00006*RM4044(-5))
+ 0.98178] / (1 + RM4044DI)

PM4549MS_P = [(0.00004*RM4549 - 0.00011*RM4549(-1) - 0.00036*RM4549(-2) - 0.00059*RM4549(-3)
- 0.00068*RM4549(-4) - 0.00052*RM4549(-5))
+ 0.98395] / (1 + RM4549DI)

PM5054MS_P = [(0.00115*RM5054 + 0.00089*RM5054(-1) - 0.00017*RM5054(-2) - 0.00141*RM5054(-3)

$$- 0.00220*RM5054(-4) - 0.00194*RM5054(-5)) \\ + 0.97763] / (1 + RM5054DI)$$

MALE 20 TO 54, MARRIED, SPOUSE ABSENT

$$PM2024MA_P = [(- 0.00053*RM2024 - 0.00074*RM2024(-1) - 0.00071*RM2024(-2) - 0.00054*RM2024(-3) \\ - 0.00030*RM2024(-4) - 0.00010*RM2024(-5)) \\ - 0.00441 * TR_M2024 \\ + 1.33014] / (1 + RM2024DI)$$

$$PM2529MA_P = [(- 0.00031*RM2529 - 0.00053*RM2529(-1) - 0.00066*RM2529(-2) - 0.00068*RM2529(-3) \\ - 0.00058*RM2529(-4) - 0.00036*RM2529(-5)) \\ - 0.00061 * TR_M2529 \\ + 0.98620] / (1 + RM2529DI)$$

$$PM3034MA_P = [(- 0.00001*RM3034 - 0.00018*RM3034(-1) - 0.00040*RM3034(-2) - 0.00058*RM3034(-3) \\ - 0.00064*RM3034(-4) - 0.00047*RM3034(-5)) \\ + 0.93031] / (1 + RM3034DI)$$

$$PM3539MA_P = [(0.00048*RM3539 + 0.00037*RM3539(-1) - 0.00007*RM3539(-2) - 0.00058*RM3539(-3) \\ - 0.00091*RM3539(-4) - 0.00080*RM3539(-5)) \\ + 0.92144] / (1 + RM3539DI)$$

$$PM4044MA_P = [(- 0.00018*RM4044 - 0.00026*RM4044(-1) - 0.00027*RM4044(-2) - 0.00022*RM4044(-3) \\ - 0.00014*RM4044(-4) - 0.00006*RM4044(-5)) \\ + 0.90958] / (1 + RM4044DI)$$

$$PM4549MA_P = [(0.00004*RM4549 - 0.00011*RM4549(-1) - 0.00036*RM4549(-2) - 0.00059*RM4549(-3) \\ - 0.00068*RM4549(-4) - 0.00052*RM4549(-5)) \\ + 0.88963] / (1 + RM4549DI)$$

$$PM5054MA_P = [(0.00115*RM5054 + 0.00089*RM5054(-1) - 0.00017*RM5054(-2) - 0.00141*RM5054(-3) \\ - 0.00220*RM5054(-4) - 0.00194*RM5054(-5)) \\ + 0.85714] / (1 + RM5054DI)$$

MALE AGE 20 TO 54

$$PM2024_P = (PM2024NM_P * NM2024NM + PM2024MS_P * NM2024MS + PM2024MA_P * NM2024MA) / NM2024 \\ PM2529_P = (PM2529NM_P * NM2529NM + PM2529MS_P * NM2529MS + PM2529MA_P * NM2529MA) / NM2529 \\ PM3034_P = (PM3034NM_P * NM3034NM + PM3034MS_P * NM3034MS + PM3034MA_P * NM3034MA) / NM3034 \\ PM3539_P = (PM3539NM_P * NM3539NM + PM3539MS_P * NM3539MS + PM3539MA_P * NM3539MA) / NM3539 \\ PM4044_P = (PM4044NM_P * NM4044NM + PM4044MS_P * NM4044MS + PM4044MA_P * NM4044MA) / NM4044 \\ PM4549_P = (PM4549NM_P * NM4549NM + PM4549MS_P * NM4549MS + PM4549MA_P * NM4549MA) / NM4549 \\ PM5054_P = (PM5054NM_P * NM5054NM + PM5054MS_P * NM5054MS + PM5054MA_P * NM5054MA) / NM5054$$

$$PM2024 = PM2024_P \\ PM2529 = PM2529_P \\ PM3034 = PM3034_P \\ PM3539 = PM3539_P \\ PM4044 = PM4044_P \\ PM4549 = PM4549_P \\ PM5054 = PM5054_P$$

$$PM2024NM = PM2024NM_P * PM2024 / PM2024_P \\ PM2529NM = PM2529NM_P * PM2529 / PM2529_P \\ PM3034NM = PM3034NM_P * PM3034 / PM3034_P \\ PM3539NM = PM3539NM_P * PM3539 / PM3539_P \\ PM4044NM = PM4044NM_P * PM4044 / PM4044_P \\ PM4549NM = PM4549NM_P * PM4549 / PM4549_P \\ PM5054NM = PM5054NM_P * PM5054 / PM5054_P$$

$$PM2024MS = PM2024MS_P * PM2024 / PM2024_P \\ PM2529MS = PM2529MS_P * PM2529 / PM2529_P$$

PM3034MS = PM3034MS_P * PM3034 / PM3034_P
PM3539MS = PM3539MS_P * PM3539 / PM3539_P
PM4044MS = PM4044MS_P * PM4044 / PM4044_P
PM4549MS = PM4549MS_P * PM4549 / PM4549_P
PM5054MS = PM5054MS_P * PM5054 / PM5054_P

PM2024MA = PM2024MA_P * PM2024 / PM2024_P
PM2529MA = PM2529MA_P * PM2529 / PM2529_P
PM3034MA = PM3034MA_P * PM3034 / PM3034_P
PM3539MA = PM3539MA_P * PM3539 / PM3539_P
PM4044MA = PM4044MA_P * PM4044 / PM4044_P
PM4549MA = PM4549MA_P * PM4549 / PM4549_P
PM5054MA = PM5054MA_P * PM5054 / PM5054_P

MALE AGE 55 TO 74

PM55_P = [0.00197*EDSCOREM55 - 0.09058*MSSHARE_M55 + 0.95174] / (1 + RM55DI)

PM56_P = [-0.01821*EDSCOREM56 - 0.10617*MSSHARE_M56 + 0.98672] / (1 + RM56DI)

PM57_P = [-0.01861*EDSCOREM57 - 0.21015*MSSHARE_M57 + 1.05211] / (1 + RM57DI)

PM58_P = [0.01509*EDSCOREM58 - 0.22982*MSSHARE_M58 + 0.99265] / (1 + RM58DI)

PM59_P = [0.00952*EDSCOREM59 - 0.30748*MSSHARE_M59 + 1.04218] / (1 + RM59DI)

PM60_P = [0.03704*EDSCOREM60 - 0.32468*MSSHARE_M60 + 0.96171] / (1 + RM60DI)

PM61_P = [0.00718*EDSCOREM61 - 0.44317*MSSHARE_M61 + 1.07617] / (1 + RM61DI)

PM62_P = [-0.60*RRADJ_M62 - 0.02*POT_ET_TXRT_62
+ 0.10092*EDSCOREM62 - 0.55927*MSSHARE_M62 + 1.09868] / (1 + RM62DI)

PM63_P = [-0.55*RRADJ_M63 - 0.02*POT_ET_TXRT_63
+ 0.06427*EDSCOREM63 - 0.95029*MSSHARE_M63 + 1.39001] / (1 + RM63DI)

PM64_P = [-0.50*RRADJ_M64 - 0.02*POT_ET_TXRT_64
+ 0.05462*EDSCOREM64 - 1.01565*MSSHARE_M64 + 1.40427] / (1 + RM64DI)

PM65_P = [-0.45*RRADJ_M65 - 0.02*POT_ET_TXRT_65
+ 0.14092*EDSCOREM65 - 0.56959*MSSHARE_M65 + 0.83637] / (1 + RM65DI)

PM66_P = [-0.40*RRADJ_M66 - 0.02*POT_ET_TXRT_66
+ 0.08401*EDSCOREM66 - 0.71602*MSSHARE_M66 + 0.99009] / (1 + RM66DI)

PM67_P = [-0.35*RRADJ_M67 - 0.02*POT_ET_TXRT_67
+ 0.09458*EDSCOREM67 - 0.80679*MSSHARE_M67 + 0.98465] / (1 + RM67DI)

PM68_P = [-0.30*RRADJ_M68 - 0.02*POT_ET_TXRT_68
+ 0.11230*EDSCOREM68 - 0.91881*MSSHARE_M68 + 0.99375] / (1 + RM68DI)

PM69_P = [-0.30*RRADJ_M69 - 0.02*POT_ET_TXRT_69
+ 0.09610*EDSCOREM69 - 0.83697*MSSHARE_M69 + 0.93415] / (1 + RM69DI)

PM70_P = [0.09997*EDSCOREM70 - 0.22828*MSSHARE_M70 + 0.28131] / (1 + RM70DI)

PM71_P = [0.12351*EDSCOREM71 + 0.13152*MSSHARE_M71 - 0.04761] / (1 + RM71DI)

PM72_P = [0.13620*EDSCOREM72 - 0.19020*MSSHARE_M72 + 0.15703] / (1 + RM72DI)

PM73_P = [0.13710*EDSCOREM73 + 0.26157*MSSHARE_M73 - 0.20437] / (1 + RM73DI)

PM74_P = [0.13464*EDSCOREM74 + 0.42963*MSSHARE_M74 - 0.34404] / (1 + RM74DI)

MALE AGE 75 TO 79

$$PM75_P = PM74(-4) * 0.92$$

$$PM76_P = PM75(-4) * 0.92$$

$$PM77_P = PM76(-4) * 0.92$$

$$PM78_P = PM77(-4) * 0.92$$

$$PM79_P = PM78(-4) * 0.92$$

MALE AGE 55 TO 79

$$PM55 = PM55_P$$

$$PM56 = PM56_P$$

$$PM57 = PM57_P$$

$$PM58 = PM58_P$$

$$PM59 = PM59_P$$

$$PM60 = PM60_P$$

$$PM61 = PM61_P$$

$$PM62 = PM62_P$$

$$PM63 = PM63_P$$

$$PM64 = PM64_P$$

$$PM65 = PM65_P$$

$$PM66 = PM66_P$$

$$PM67 = PM67_P$$

$$PM68 = PM68_P$$

$$PM69 = PM69_P$$

$$PM70 = PM70_P$$

$$PM71 = PM71_P$$

$$PM72 = PM72_P$$

$$PM73 = PM73_P$$

$$PM74 = PM74_P$$

$$PM75 = PM75_P$$

$$PM76 = PM76_P$$

$$PM77 = PM77_P$$

$$PM78 = PM78_P$$

$$PM79 = PM79_P$$

MALE AGE 80 AND OVER

$$PM80_P = PM79(-4) * 0.965 ^ 1$$

$$PM81_P = PM79(-8) * 0.965 ^ 2$$

$$PM82_P = PM79(-12) * 0.965 ^ 3$$

$$PM83_P = PM79(-16) * 0.965 ^ 4$$

$$PM84_P = PM79(-20) * 0.965 ^ 5$$

$PM85_P = MOVAVG(8, PM79(-24)) * 0.965^6$
 $PM86_P = MOVAVG(8, PM79(-28)) * 0.965^7$
 $PM87_P = MOVAVG(8, PM79(-32)) * 0.965^8$
 $PM88_P = MOVAVG(8, PM79(-36)) * 0.965^9$
 $PM89_P = MOVAVG(8, PM79(-40)) * 0.965^{10}$
 $PM90_P = MOVAVG(8, PM79(-44)) * 0.965^{11}$
 $PM91_P = MOVAVG(8, PM79(-48)) * 0.965^{12}$
 $PM92_P = MOVAVG(8, PM79(-52)) * 0.965^{13}$
 $PM93_P = MOVAVG(8, PM79(-56)) * 0.965^{14}$
 $PM94_P = MOVAVG(8, PM79(-60)) * 0.965^{15}$
 $PM95_P = PM94_P * 0.965$
 $PM96_P = PM95_P * 0.965$
 $PM97_P = PM96_P * 0.965$
 $PM98_P = PM97_P * 0.965$
 $PM99_P = PM98_P * 0.965$
 $PM100_P = PM99_P * 0.965$

$PM80O_P = (PM80_P * NM80 + PM81_P * NM81 + PM82_P * NM82 + PM83_P * NM83 + PM84_P * NM84$
 $+ PM85_P * NM85 + PM86_P * NM86 + PM87_P * NM87 + PM88_P * NM88 + PM89_P * NM89$
 $+ PM90_P * NM90 + PM91_P * NM91 + PM92_P * NM92 + PM93_P * NM93 + PM94_P * NM94$
 $+ PM95_P * NM95 + PM96_P * NM96 + PM97_P * NM97 + PM98_P * NM98 + PM99_P * NM99$
 $+ PM100_P * NM100) / NM80O$

$PM80O = PM80O_P$
 $PM80 = PM80_P * PM80O / PM80O_P$
 $PM81 = PM81_P * PM80O / PM80O_P$
 $PM82 = PM82_P * PM80O / PM80O_P$
 $PM83 = PM83_P * PM80O / PM80O_P$
 $PM84 = PM84_P * PM80O / PM80O_P$
 $PM85 = PM85_P * PM80O / PM80O_P$
 $PM86 = PM86_P * PM80O / PM80O_P$
 $PM87 = PM87_P * PM80O / PM80O_P$
 $PM88 = PM88_P * PM80O / PM80O_P$
 $PM89 = PM89_P * PM80O / PM80O_P$
 $PM90 = PM90_P * PM80O / PM80O_P$
 $PM91 = PM91_P * PM80O / PM80O_P$
 $PM92 = PM92_P * PM80O / PM80O_P$
 $PM93 = PM93_P * PM80O / PM80O_P$
 $PM94 = PM94_P * PM80O / PM80O_P$
 $PM95 = PM95_P * PM80O / PM80O_P$
 $PM96 = PM96_P * PM80O / PM80O_P$
 $PM97 = PM97_P * PM80O / PM80O_P$

PM98 = PM98_P * PM80O / PM80O_P
 PM99 = PM99_P * PM80O / PM80O_P
 PM100 = PM100_P * PM80O / PM80O_P

FEMALE LFPR EQUATIONS

FEMALE 1617 AND 1819

PF1617_P = [- 0.00214*RF1617 - 0.00256*RF1617(-1) - 0.00184*RF1617(-2) - 0.00062*RF1617(-3)
 + 0.00050*RF1617(-4) + 0.00091*RF1617(-5)
 - 0.33769 * RF1617CU6 + 0.01223
 - 0.00981 * TR_F1617 + 0.50143
 + 0.98698] / (1 + RF1617DI)

PF1819_P = [- 0.00118*RF1819 - 0.00145*RF1819(-1) - 0.00112*RF1819(-2) - 0.00050*RF1819(-3)
 + 0.00010*RF1819(-4) + 0.00037*RF1819(-5)
 - 0.42102 * RF1819CU6 - 0.01160
 - 0.00829 * TR_F1819 + 0.55840
 + 0.98042] / (1 + RF1819DI)

PF1617 = PF1617_p
 PF1819 = PF1819_p

FEMALE 20 TO 44, NEVER MARRIED WITH AT LEAST ONE CHILD UNDER 6:

PF2024NMC6U_P = [- 0.00006*RF2024 - 0.00026*RF2024(-1) - 0.00050*RF2024(-2) - 0.00070*RF2024(-3)
 - 0.00075*RF2024(-4) - 0.00055*RF2024(-5)
 + 0.69322] / (1 + RF2024DI)

PF2529NMC6U_P = [0.00056*RF2529 + 0.00034*RF2529(-1) - 0.00030*RF2529(-2) - 0.00101*RF2529(-3)
 - 0.00143*RF2529(-4) - 0.00121*RF2529(-5)
 + 0.72672] / (1 + RF2529DI)

PF3034NMC6U_P = [0.00078*RF3034 + 0.00075*RF3034(-1) + 0.00024*RF3034(-2) - 0.00042*RF3034(-3)
 - 0.00090*RF3034(-4) - 0.00087*RF3034(-5)
 + 0.72034] / (1 + RF3034DI)

PF3539NMC6U_P = [0.00040*RF3539 + 0.00032*RF3539(-1) - 0.00004*RF3539(-2) - 0.00046*RF3539(-3)
 - 0.00074*RF3539(-4) - 0.00066*RF3539(-5)
 + 0.72644] / (1 + RF3539DI)

PF4044NMC6U_P = [0.00113*RF4044 + 0.00097*RF4044(-1) + 0.00007*RF4044(-2) - 0.00102*RF4044(-3)
 - 0.00176*RF4044(-4) - 0.00160*RF4044(-5)
 + 0.71893] / (1 + RF4044DI)

FEMALE 20 TO 44, NEVER MARRIED WITH NO CHILDREN CHILD UNDER 6:

PF2024NMNC6_P = [- 0.00006*RF2024 - 0.00026*RF2024(-1) - 0.00050*RF2024(-2) - 0.00070*RF2024(-3)
 - 0.00075*RF2024(-4) - 0.00055*RF2024(-5)
 - 0.00297 * TR_F2024
 + 1.07188] / (1 + RF2024DI)

PF2529NMNC6_P = [0.00056*RF2529 + 0.00034*RF2529(-1) - 0.00030*RF2529(-2) - 0.00101*RF2529(-3)
 - 0.00143*RF2529(-4) - 0.00121*RF2529(-5)
 - 0.00077 * TR_F2529
 + 0.94654] / (1 + RF2529DI)

PF3034NMNC6_P = [0.00078*RF3034 + 0.00075*RF3034(-1) + 0.00024*RF3034(-2) - 0.00042*RF3034(-3)
 - 0.00090*RF3034(-4) - 0.00087*RF3034(-5)
 + 0.84866] / (1 + RF3034DI)

$$\text{PF3539NMNC6_P} = [0.00040*\text{RF3539} + 0.00032*\text{RF3539}(-1) - 0.00004*\text{RF3539}(-2) - 0.00046*\text{RF3539}(-3) \\ - 0.00074*\text{RF3539}(-4) - 0.00066*\text{RF3539}(-5) \\ + 0.83195] / (1 + \text{RF3539DI})$$

$$\text{PF4044NMNC6_P} = [0.00113*\text{RF4044} + 0.00097*\text{RF4044}(-1) + 0.00007*\text{RF4044}(-2) - 0.00102*\text{RF4044}(-3) \\ - 0.00176*\text{RF4044}(-4) - 0.00160*\text{RF4044}(-5) \\ + 0.82513] / (1 + \text{RF4044DI})$$

FEMALE 20 TO 44, MARRIED, SPOUSE PRESENT, WITH AT LEAST ONE CHILD UNDER 6:

$$\text{PF2024MSC6U_P} = [- 0.00006*\text{RF2024} - 0.00026*\text{RF2024}(-1) - 0.00050*\text{RF2024}(-2) - 0.00070*\text{RF2024}(-3) \\ - 0.00075*\text{RF2024}(-4) - 0.00055*\text{RF2024}(-5) \\ + 0.53561] / (1 + \text{RF2024DI})$$

$$\text{PF2529MSC6U_P} = [0.00056*\text{RF2529} + 0.00034*\text{RF2529}(-1) - 0.00030*\text{RF2529}(-2) - 0.00101*\text{RF2529}(-3) \\ - 0.00143*\text{RF2529}(-4) - 0.00121*\text{RF2529}(-5) \\ + 0.61916] / (1 + \text{RF2529DI})$$

$$\text{PF3034MSC6U_P} = [0.00078*\text{RF3034} + 0.00075*\text{RF3034}(-1) + 0.00024*\text{RF3034}(-2) - 0.00042*\text{RF3034}(-3) \\ - 0.00090*\text{RF3034}(-4) - 0.00087*\text{RF3034}(-5) \\ + 0.65485] / (1 + \text{RF3034DI})$$

$$\text{PF3539MSC6U_P} = [0.00040*\text{RF3539} + 0.00032*\text{RF3539}(-1) - 0.00004*\text{RF3539}(-2) - 0.00046*\text{RF3539}(-3) \\ - 0.00074*\text{RF3539}(-4) - 0.00066*\text{RF3539}(-5) \\ + 0.66673] / (1 + \text{RF3539DI})$$

$$\text{PF4044MSC6U_P} = [0.00113*\text{RF4044} + 0.00097*\text{RF4044}(-1) + 0.00007*\text{RF4044}(-2) - 0.00102*\text{RF4044}(-3) \\ - 0.00176*\text{RF4044}(-4) - 0.00160*\text{RF4044}(-5) \\ + 0.67612] / (1 + \text{RF4044DI})$$

FEMALE 20 TO 44, MARRIED, SPOUSE PRESENT WITH NO CHILDREN CHILD UNDER 6:

$$\text{PF2024MSNC6_P} = [- 0.00006*\text{RF2024} - 0.00026*\text{RF2024}(-1) - 0.00050*\text{RF2024}(-2) - 0.00070*\text{RF2024}(-3) \\ - 0.00075*\text{RF2024}(-4) - 0.00055*\text{RF2024}(-5) \\ - 0.00175 * \text{TR_F2024} \\ + 0.98277] / (1 + \text{RF2024DI})$$

$$\text{PF2529MSNC6_P} = [0.00056*\text{RF2529} + 0.00034*\text{RF2529}(-1) - 0.00030*\text{RF2529}(-2) - 0.00101*\text{RF2529}(-3) \\ - 0.00143*\text{RF2529}(-4) - 0.00121*\text{RF2529}(-5) \\ - 0.00050 * \text{TR_F2529} \\ + 0.88519] / (1 + \text{RF2529DI})$$

$$\text{PF3034MSNC6_P} = [0.00078*\text{RF3034} + 0.00075*\text{RF3034}(-1) + 0.00024*\text{RF3034}(-2) - 0.00042*\text{RF3034}(-3) \\ - 0.00090*\text{RF3034}(-4) - 0.00087*\text{RF3034}(-5) \\ + 0.80007] / (1 + \text{RF3034DI})$$

$$\text{PF3539MSNC6_P} = [0.00040*\text{RF3539} + 0.00032*\text{RF3539}(-1) - 0.00004*\text{RF3539}(-2) - 0.00046*\text{RF3539}(-3) \\ - 0.00074*\text{RF3539}(-4) - 0.00066*\text{RF3539}(-5) \\ + 0.79025] / (1 + \text{RF3539DI})$$

$$\text{PF4044MSNC6_P} = [0.00113*\text{RF4044} + 0.00097*\text{RF4044}(-1) + 0.00007*\text{RF4044}(-2) - 0.00102*\text{RF4044}(-3) \\ - 0.00176*\text{RF4044}(-4) - 0.00160*\text{RF4044}(-5) \\ + 0.81064] / (1 + \text{RF4044DI})$$

FEMALE 20 TO 44, MARRIED, SPOUSE ABSENT, WITH AT LEAST ONE CHILD UNDER 6:

$$\text{PF2024MAC6U_P} = [- 0.00006*\text{RF2024} - 0.00026*\text{RF2024}(-1) - 0.00050*\text{RF2024}(-2) - 0.00070*\text{RF2024}(-3) \\ - 0.00075*\text{RF2024}(-4) - 0.00055*\text{RF2024}(-5)$$

$$+ 0.73283] / (1 + RF2024DI)$$

$$PF2529MAC6U_P = [0.00056*RF2529 + 0.00034*RF2529(-1) - 0.00030*RF2529(-2) - 0.00101*RF2529(-3) \\ - 0.00143*RF2529(-4) - 0.00121*RF2529(-5) \\ + 0.76433] / (1 + RF2529DI)$$

$$PF3034MAC6U_P = [0.00078*RF3034 + 0.00075*RF3034(-1) + 0.00024*RF3034(-2) - 0.00042*RF3034(-3) \\ - 0.00090*RF3034(-4) - 0.00087*RF3034(-5) \\ + 0.77183] / (1 + RF3034DI)$$

$$PF3539MAC6U_P = [0.00040*RF3539 + 0.00032*RF3539(-1) - 0.00004*RF3539(-2) - 0.00046*RF3539(-3) \\ - 0.00074*RF3539(-4) - 0.00066*RF3539(-5) \\ + 0.77156] / (1 + RF3539DI)$$

$$PF4044MAC6U_P = [0.00113*RF4044 + 0.00097*RF4044(-1) + 0.00007*RF4044(-2) - 0.00102*RF4044(-3) \\ - 0.00176*RF4044(-4) - 0.00160*RF4044(-5) \\ + 0.76058] / (1 + RF4044DI)$$

FEMALE 20 TO 44, MARRIED, SPOUSE ABSENT WITH NO CHILDREN CHILD UNDER 6:

$$PF2024MANC6_P = [- 0.00006*RF2024 - 0.00026*RF2024(-1) - 0.00050*RF2024(-2) - 0.00070*RF2024(-3) \\ - 0.00075*RF2024(-4) - 0.00055*RF2024(-5) \\ - 0.00223 * TR_F2024 \\ + 0.97444] / (1 + RF2024DI)$$

$$PF2529MANC6_P = [0.00056*RF2529 + 0.00034*RF2529(-1) - 0.00030*RF2529(-2) - 0.00101*RF2529(-3) \\ - 0.00143*RF2529(-4) - 0.00121*RF2529(-5) \\ + 0.00025 * TR_F2529 \\ + 0.79370] / (1 + RF2529DI)$$

$$PF3034MANC6_P = [0.00078*RF3034 + 0.00075*RF3034(-1) + 0.00024*RF3034(-2) - 0.00042*RF3034(-3) \\ - 0.00090*RF3034(-4) - 0.00087*RF3034(-5) \\ + 0.83788] / (1 + RF3034DI)$$

$$PF3539MANC6_P = [0.00040*RF3539 + 0.00032*RF3539(-1) - 0.00004*RF3539(-2) - 0.00046*RF3539(-3) \\ - 0.00074*RF3539(-4) - 0.00066*RF3539(-5) \\ + 0.84820] / (1 + RF3539DI)$$

$$PF4044MANC6_P = [0.00113*RF4044 + 0.00097*RF4044(-1) + 0.00007*RF4044(-2) - 0.00102*RF4044(-3) \\ - 0.00176*RF4044(-4) - 0.00160*RF4044(-5) \\ + 0.85568] / (1 + RF4044DI)$$

$$PF2024NM_P = (PF2024NMC6U_P * NF2024NMC6U + PF2024NMNC6_P * NF2024NMNC6) / NF2024NM$$

$$PF2024MS_P = (PF2024MSC6U_P * NF2024MSC6U + PF2024MSNC6_P * NF2024MSNC6) / NF2024MS$$

$$PF2024MA_P = (PF2024MAC6U_P * NF2024MAC6U + PF2024MANC6_P * NF2024MANC6) / NF2024MA$$

$$PF2529NM_P = (PF2529NMC6U_P * NF2529NMC6U + PF2529NMNC6_P * NF2529NMNC6) / NF2529NM$$

$$PF2529MS_P = (PF2529MSC6U_P * NF2529MSC6U + PF2529MSNC6_P * NF2529MSNC6) / NF2529MS$$

$$PF2529MA_P = (PF2529MAC6U_P * NF2529MAC6U + PF2529MANC6_P * NF2529MANC6) / NF2529MA$$

$$PF3034NM_P = (PF3034NMC6U_P * NF3034NMC6U + PF3034NMNC6_P * NF3034NMNC6) / NF3034NM$$

$$PF3034MS_P = (PF3034MSC6U_P * NF3034MSC6U + PF3034MSNC6_P * NF3034MSNC6) / NF3034MS$$

$$PF3034MA_P = (PF3034MAC6U_P * NF3034MAC6U + PF3034MANC6_P * NF3034MANC6) / NF3034MA$$

$$PF3539NM_P = (PF3539NMC6U_P * NF3539NMC6U + PF3539NMNC6_P * NF3539NMNC6) / NF3539NM$$

$$PF3539MS_P = (PF3539MSC6U_P * NF3539MSC6U + PF3539MSNC6_P * NF3539MSNC6) / NF3539MS$$

$PF3539MA_P = (PF3539MAC6U_P * NF3539MAC6U + PF3539MANC6_P * NF3539MANC6) / NF3539MA$
 $PF4044NM_P = (PF4044NMC6U_P * NF4044NMC6U + PF4044NMNC6_P * NF4044NMNC6) / NF4044NM$
 $PF4044MS_P = (PF4044MSC6U_P * NF4044MSC6U + PF4044MSNC6_P * NF4044MSNC6) / NF4044MS$
 $PF4044MA_P = (PF4044MAC6U_P * NF4044MAC6U + PF4044MANC6_P * NF4044MANC6) / NF4044MA$

FEMALE AGE 45 TO 54

FEMALE 45 TO 54, NEVER MARRIED:

$PF4549NM_P = [0.00155*RF4549 + 0.00133*RF4549(-1) + 0.00007*RF4549(-2) - 0.00145*RF4549(-3)$
 $- 0.00247*RF4549(-4) - 0.00224*RF4549(-5)$
 $+ 0.80950] / (1 + RF4549DI)$

$PF5054NM_P = [0.00093*RF5054 + 0.00074*RF5054(-1) - 0.00008*RF5054(-2) - 0.00104*RF5054(-3)$
 $- 0.00168*RF5054(-4) - 0.00149*RF5054(-5)$
 $+ 0.78085] / (1 + RF5054DI)$

FEMALE 45 TO 54, MARRIED, SPOUSE PRESENT:

$PF4549MS_P = [0.00155*RF4549 + 0.00133*RF4549(-1) + 0.00007*RF4549(-2) - 0.00145*RF4549(-3)$
 $- 0.00247*RF4549(-4) - 0.00224*RF4549(-5)$
 $+ 0.81276] / (1 + RF4549DI)$

$PF5054MS_P = [0.00093*RF5054 + 0.00074*RF5054(-1) - 0.00008*RF5054(-2) - 0.00104*RF5054(-3)$
 $- 0.00168*RF5054(-4) - 0.00149*RF5054(-5)$
 $+ 0.48809$
 $+ 0.16476*EDSCOREF5054] / (1 + RF5054DI)$

FEMALE 45 TO 54, MARRIED, SPOUSE ABSENT:

$PF4549MA_P = [0.00155*RF4549 + 0.00133*RF4549(-1) + 0.00007*RF4549(-2) - 0.00145*RF4549(-3)$
 $- 0.00247*RF4549(-4) - 0.00224*RF4549(-5)$
 $+ 0.84789] / (1 + RF4549DI)$

$PF5054MA_P = [0.00093*RF5054 + 0.00074*RF5054(-1) - 0.00008*RF5054(-2) - 0.00104*RF5054(-3)$
 $- 0.00168*RF5054(-4) - 0.00149*RF5054(-5)$
 $+ 0.81452] / (1 + RF5054DI)$

FEMALE AGE 20 TO 54

$PF2024_P = (PF2024NM_P * NF2024NM + PF2024MS_P * NF2024MS + PF2024MA_P * NF2024MA) / NF2024$
 $PF2529_P = (PF2529NM_P * NF2529NM + PF2529MS_P * NF2529MS + PF2529MA_P * NF2529MA) / NF2529$
 $PF3034_P = (PF3034NM_P * NF3034NM + PF3034MS_P * NF3034MS + PF3034MA_P * NF3034MA) / NF3034$
 $PF3539_P = (PF3539NM_P * NF3539NM + PF3539MS_P * NF3539MS + PF3539MA_P * NF3539MA) / NF3539$
 $PF4044_P = (PF4044NM_P * NF4044NM + PF4044MS_P * NF4044MS + PF4044MA_P * NF4044MA) / NF4044$
 $PF4549_P = (PF4549NM_P * NF4549NM + PF4549MS_P * NF4549MS + PF4549MA_P * NF4549MA) / NF4549$
 $PF5054_P = (PF5054NM_P * NF5054NM + PF5054MS_P * NF5054MS + PF5054MA_P * NF5054MA) / NF5054$

$PF2024 = PF2024_P$
 $PF2529 = PF2529_P$
 $PF3034 = PF3034_P$
 $PF3539 = PF3539_P$
 $PF4044 = PF4044_P$
 $PF4549 = PF4549_P$
 $PF5054 = PF5054_P$

$PF2024NM = PF2024NM_P * PF2024 / PF2024_P$
 $PF2529NM = PF2529NM_P * PF2529 / PF2529_P$
 $PF3034NM = PF3034NM_P * PF3034 / PF3034_P$
 $PF3539NM = PF3539NM_P * PF3539 / PF3539_P$

PF4044NM = PF4044NM_P * PF4044 / PF4044_P
PF4549NM = PF4549NM_P * PF4549 / PF4549_P
PF5054NM = PF5054NM_P * PF5054 / PF5054_P

PF2024MS = PF2024MS_P * PF2024 / PF2024_P
PF2529MS = PF2529MS_P * PF2529 / PF2529_P
PF3034MS = PF3034MS_P * PF3034 / PF3034_P
PF3539MS = PF3539MS_P * PF3539 / PF3539_P
PF4044MS = PF4044MS_P * PF4044 / PF4044_P
PF4549MS = PF4549MS_P * PF4549 / PF4549_P
PF5054MS = PF5054MS_P * PF5054 / PF5054_P

PF2024MA = PF2024MA_P * PF2024 / PF2024_P
PF2529MA = PF2529MA_P * PF2529 / PF2529_P
PF3034MA = PF3034MA_P * PF3034 / PF3034_P
PF3539MA = PF3539MA_P * PF3539 / PF3539_P
PF4044MA = PF4044MA_P * PF4044 / PF4044_P
PF4549MA = PF4549MA_P * PF4549 / PF4549_P
PF5054MA = PF5054MA_P * PF5054 / PF5054_P

PF2024NMC6U = PF2024NMC6U_P * PF2024 / PF2024_P
PF2529NMC6U = PF2529NMC6U_P * PF2529 / PF2529_P
PF3034NMC6U = PF3034NMC6U_P * PF3034 / PF3034_P
PF3539NMC6U = PF3539NMC6U_P * PF3539 / PF3539_P
PF4044NMC6U = PF4044NMC6U_P * PF4044 / PF4044_P

PF2024NMNC6 = PF2024NMNC6_P * PF2024 / PF2024_P
PF2529NMNC6 = PF2529NMNC6_P * PF2529 / PF2529_P
PF3034NMNC6 = PF3034NMNC6_P * PF3034 / PF3034_P
PF3539NMNC6 = PF3539NMNC6_P * PF3539 / PF3539_P
PF4044NMNC6 = PF4044NMNC6_P * PF4044 / PF4044_P

PF2024MSC6U = PF2024MSC6U_P * PF2024 / PF2024_P
PF2529MSC6U = PF2529MSC6U_P * PF2529 / PF2529_P
PF3034MSC6U = PF3034MSC6U_P * PF3034 / PF3034_P
PF3539MSC6U = PF3539MSC6U_P * PF3539 / PF3539_P
PF4044MSC6U = PF4044MSC6U_P * PF4044 / PF4044_P

PF2024MSNC6 = PF2024MSNC6_P * PF2024 / PF2024_P
PF2529MSNC6 = PF2529MSNC6_P * PF2529 / PF2529_P
PF3034MSNC6 = PF3034MSNC6_P * PF3034 / PF3034_P
PF3539MSNC6 = PF3539MSNC6_P * PF3539 / PF3539_P
PF4044MSNC6 = PF4044MSNC6_P * PF4044 / PF4044_P

PF2024MAC6U = PF2024MAC6U_P * PF2024 / PF2024_P
PF2529MAC6U = PF2529MAC6U_P * PF2529 / PF2529_P
PF3034MAC6U = PF3034MAC6U_P * PF3034 / PF3034_P
PF3539MAC6U = PF3539MAC6U_P * PF3539 / PF3539_P
PF4044MAC6U = PF4044MAC6U_P * PF4044 / PF4044_P

PF2024MANC6 = PF2024MANC6_P * PF2024 / PF2024_P
PF2529MANC6 = PF2529MANC6_P * PF2529 / PF2529_P
PF3034MANC6 = PF3034MANC6_P * PF3034 / PF3034_P
PF3539MANC6 = PF3539MANC6_P * PF3539 / PF3539_P
PF4044MANC6 = PF4044MANC6_P * PF4044 / PF4044_P

FEMALE 55 TO 74

PF55_P = [0.18398*EDSCOREF55 + 0.24605*MSSHARE_F55 + 0.26185] / (1 + RF55DI)

PF56_P = [0.20704*EDSCOREF56 - 0.12398*MSSHARE_F56 + 0.4493] / (1 + RF56DI)

PF57_P = [0.18422*EDSCOREF57 - 0.08766*MSSHARE_F57 + 0.45094] / (1 + RF57DI)

$$PF58_P = [0.19406 * EDSCOREF58 - 0.06963 * MSSHARE_F58 + 0.40382] / (1 + RF58DI)$$

$$PF59_P = [0.21112 * EDSCOREF59 + 0.10742 * MSSHARE_F59 + 0.24105] / (1 + RF59DI)$$

$$PF60_P = [0.20633 * EDSCOREF60 - 0.41539 * MSSHARE_F60 + 0.54178] / (1 + RF60DI)$$

$$PF61_P = [0.19349 * EDSCOREF61 - 0.66676 * MSSHARE_F61 + 0.68556] / (1 + RF61DI)$$

$$PF62_P = [-0.5100 * RRADJ_F62 - 0.02 * POT_ET_TXRT_62 + 0.21249 * EDSCOREF62 + 0.01966 * MSSHARE_F62 + 0.32876] / (1 + RF62DI)$$

$$PF63_P = [-0.4675 * RRADJ_F63 - 0.02 * POT_ET_TXRT_63 + 0.21116 * EDSCOREF63 + 0.02896 * MSSHARE_F63 + 0.26792] / (1 + RF63DI)$$

$$PF64_P = [-0.4250 * RRADJ_F64 - 0.02 * POT_ET_TXRT_64 + 0.20467 * EDSCOREF64 - 0.32725 * MSSHARE_F64 + 0.44786] / (1 + RF64DI)$$

$$PF65_P = [-0.3825 * RRADJ_F65 - 0.02 * POT_ET_TXRT_65 + 0.20002 * EDSCOREF65 - 0.05830 * MSSHARE_F65 + 0.22187] / (1 + RF65DI)$$

$$PF66_P = [-0.3400 * RRADJ_F66 - 0.02 * POT_ET_TXRT_66 + 0.17479 * EDSCOREF66 + 0.22954 * MSSHARE_F66 + 0.04361] / (1 + RF66DI)$$

$$PF67_P = [-0.2975 * RRADJ_F67 - 0.02 * POT_ET_TXRT_67 + 0.17673 * EDSCOREF67 - 0.43942 * MSSHARE_F67 + 0.38490] / (1 + RF67DI)$$

$$PF68_P = [-0.2550 * RRADJ_F68 - 0.02 * POT_ET_TXRT_68 + 0.15513 * EDSCOREF68 + 0.35232 * MSSHARE_F68 - 0.07497] / (1 + RF68DI)$$

$$PF69_P = [-0.2550 * RRADJ_F69 - 0.02 * POT_ET_TXRT_69 + 0.11603 * EDSCOREF69 + 0.87528 * MSSHARE_F69 - 0.32696] / (1 + RF69DI)$$

$$PF70_P = [0.09629 * EDSCOREF70 + 0.54302 * MSSHARE_F70 - 0.25141] / (1 + RF70DI)$$

$$PF71_P = [0.11053 * EDSCOREF71 + 0.16137 * MSSHARE_F71 - 0.08070] / (1 + RF71DI)$$

$$PF72_P = [0.09151 * EDSCOREF72 + 0.25854 * MSSHARE_F72 - 0.11711] / (1 + RF72DI)$$

$$PF73_P = [0.08611 * EDSCOREF73 + 0.28099 * MSSHARE_F73 - 0.12804] / (1 + RF73DI)$$

$$PF74_P = [0.11821 * EDSCOREF74 - 0.01277 * MSSHARE_F74 - 0.04121] / (1 + RF74DI)$$

FEMALE, AGES 75 TO 79

$$PF75_P = PF74(-4) * 0.9$$

$$PF76_P = PF75(-4) * 0.9$$

$$PF77_P = PF76(-4) * 0.9$$

$$PF78_P = PF77(-4) * 0.9$$

$$PF79_P = PF78(-4) * 0.9$$

FEMALE AGE 55 TO 79

$$PF55 = PF55_P$$

$$PF56 = PF56_P$$

$$PF57 = PF57_P$$

$$PF58 = PF58_P$$

PF59 = PF59_P
 PF60 = PF60_P
 PF61 = PF61_P
 PF62 = PF62_P
 PF63 = PF63_P
 PF64 = PF64_P
 PF65 = PF65_P
 PF66 = PF66_P
 PF67 = PF67_P
 PF68 = PF68_P
 PF69 = PF69_P
 PF70 = PF70_P
 PF71 = PF71_P
 PF72 = PF72_P
 PF73 = PF73_P
 PF74 = PF74_P
 PF75 = PF75_P
 PF76 = PF76_P
 PF77 = PF77_P
 PF78 = PF78_P
 PF79 = PF79_P

FEMALE, AGES 80 AND OVER

PF80_P = PF79(-4) * 0.965 ^ 1
 PF81_P = PF79(-8) * 0.965 ^ 2
 PF82_P = PF79(-12) * 0.965 ^ 3
 PF83_P = PF79(-16) * 0.965 ^ 4
 PF84_P = PF79(-20) * 0.965 ^ 5
 PF85_P = MOVAVG(8,PF79(-24)) * 0.965 ^ 6
 PF86_P = MOVAVG(8,PF79(-28)) * 0.965 ^ 7
 PF87_P = MOVAVG(8,PF79(-32)) * 0.965 ^ 8
 PF88_P = MOVAVG(8,PF79(-36)) * 0.965 ^ 9
 PF89_P = MOVAVG(8,PF79(-40)) * 0.965 ^ 10
 PF90_P = MOVAVG(8,PF79(-44)) * 0.965 ^ 11
 PF91_P = MOVAVG(8,PF79(-48)) * 0.965 ^ 12
 PF92_P = MOVAVG(8,PF79(-52)) * 0.965 ^ 13
 PF93_P = MOVAVG(8,PF79(-56)) * 0.965 ^ 14
 PF94_P = MOVAVG(8,PF79(-60)) * 0.965 ^ 15
 PF95_P = PF94_P * 0.965
 PF96_P = PF95_P * 0.965

$$PF97_P = PF96_P * 0.965$$

$$PF98_P = PF97_P * 0.965$$

$$PF99_P = PF98_P * 0.965$$

$$PF100_P = PF99_P * 0.965$$

$$PF80O_P = (PF80_P * NF80 + PF81_P * NF81 + PF82_P * NF82 + PF83_P * NF83 + PF84_P * NF84 + PF85_P * NF85 + PF86_P * NF86 + PF87_P * NF87 + PF88_P * NF88 + PF89_P * NF89 + PF90_P * NF90 + PF91_P * NF91 + PF92_P * NF92 + PF93_P * NF93 + PF94_P * NF94 + PF95_P * NF95 + PF96_P * NF96 + PF97_P * NF97 + PF98_P * NF98 + PF99_P * NF99 + PF100_P * NF100) / NF80O$$

$$PF80O = PF80O_P$$

$$PF80 = PF80_P * PF80O / PF80O_P$$

$$PF81 = PF81_P * PF80O / PF80O_P$$

$$PF82 = PF82_P * PF80O / PF80O_P$$

$$PF83 = PF83_P * PF80O / PF80O_P$$

$$PF84 = PF84_P * PF80O / PF80O_P$$

$$PF85 = PF85_P * PF80O / PF80O_P$$

$$PF86 = PF86_P * PF80O / PF80O_P$$

$$PF87 = PF87_P * PF80O / PF80O_P$$

$$PF88 = PF88_P * PF80O / PF80O_P$$

$$PF89 = PF89_P * PF80O / PF80O_P$$

$$PF90 = PF90_P * PF80O / PF80O_P$$

$$PF91 = PF91_P * PF80O / PF80O_P$$

$$PF92 = PF92_P * PF80O / PF80O_P$$

$$PF93 = PF93_P * PF80O / PF80O_P$$

$$PF94 = PF94_P * PF80O / PF80O_P$$

$$PF95 = PF95_P * PF80O / PF80O_P$$

$$PF96 = PF96_P * PF80O / PF80O_P$$

$$PF97 = PF97_P * PF80O / PF80O_P$$

$$PF98 = PF98_P * PF80O / PF80O_P$$

$$PF99 = PF99_P * PF80O / PF80O_P$$

$$PF100 = PF100_P * PF80O / PF80O_P$$

LFPR EQUATIONS, AGE 16 AND OVER

$$PM16O_P = (PM1617_P * NM1617 + PM1819_P * NM1819 + PM2024_P * NM2024 + PM2529_P * NM2529 + PM3034_P * NM3034 + PM3539_P * NM3539 + PM4044_P * NM4044 + PM4549_P * NM4549 + PM5054_P * NM5054 +$$

PM55_P * NM55 +
PM56_P * NM56 +
PM57_P * NM57 +
PM58_P * NM58 +
PM59_P * NM59 +
PM60_P * NM60 +
PM61_P * NM61 +
PM62_P * NM62 +
PM63_P * NM63 +
PM64_P * NM64 +
PM65_P * NM65 +
PM66_P * NM66 +
PM67_P * NM67 +
PM68_P * NM68 +
PM69_P * NM69 +
PM70_P * NM70 +
PM71_P * NM71 +
PM72_P * NM72 +
PM73_P * NM73 +
PM74_P * NM74 +
PM75_P * NM75 +
PM76_P * NM76 +
PM77_P * NM77 +
PM78_P * NM78 +
PM79_P * NM79 +
PM80_O_P * NM80O) / NM16O

PF16O_P = (PF1617_P * NF1617 +
PF1819_P * NF1819 +
PF2024_P * NF2024 +
PF2529_P * NF2529 +
PF3034_P * NF3034 +
PF3539_P * NF3539 +
PF4044_P * NF4044 +
PF4549_P * NF4549 +
PF5054_P * NF5054 +
PF55_P * NF55 +
PF56_P * NF56 +
PF57_P * NF57 +
PF58_P * NF58 +
PF59_P * NF59 +
PF60_P * NF60 +
PF61_P * NF61 +
PF62_P * NF62 +
PF63_P * NF63 +
PF64_P * NF64 +
PF65_P * NF65 +
PF66_P * NF66 +
PF67_P * NF67 +
PF68_P * NF68 +
PF69_P * NF69 +
PF70_P * NF70 +
PF71_P * NF71 +
PF72_P * NF72 +
PF73_P * NF73 +
PF74_P * NF74 +
PF75_P * NF75 +
PF76_P * NF76 +
PF77_P * NF77 +
PF78_P * NF78 +
PF79_P * NF79 +
PF80O_P * NF80O) / NF16O

$$P160_P = (PM160_P * NM160 + PF160_P * NF160) / (NM160 + NF160)$$

LABOR FORCE, AGE 16 AND OVER

$$LCM_P = PM160_P * NM160$$

$$LCF_P = PF160_P * NF160$$

$$LC_P = LCM_P + LCF_P$$

LABOR FORCE PARTICIPATION RATES, FULL EMPLOYMENT

MALE

$$DPM1617_FE = (-0.00134*RM1617_FE - 0.00167*RM1617_FE(-1) - 0.00132*RM1617_FE(-2) - 0.00065*RM1617_FE(-3) + 0.00003*RM1617_FE(-4) + 0.00035*RM1617_FE(-5)) - (-0.00134*RM1617 - 0.00167*RM1617(-1) - 0.00132*RM1617(-2) - 0.00065*RM1617(-3) + 0.00003*RM1617(-4) + 0.00035*RM1617(-5))$$

$$DPM1819_FE = (-0.00068*RM1819_FE - 0.00092*RM1819_FE(-1) - 0.00083*RM1819_FE(-2) - 0.00056*RM1819_FE(-3) - 0.00025*RM1819_FE(-4) - 0.00002*RM1819_FE(-5)) - (-0.00068*RM1819 - 0.00092*RM1819(-1) - 0.00083*RM1819(-2) - 0.00056*RM1819(-3) - 0.00025*RM1819(-4) - 0.00002*RM1819(-5))$$

$$DPM2024_FE = (-0.00053*RM2024_FE - 0.00074*RM2024_FE(-1) - 0.00071*RM2024_FE(-2) - 0.00054*RM2024_FE(-3) - 0.00030*RM2024_FE(-4) - 0.00010*RM2024_FE(-5)) - (-0.00053*RM2024 - 0.00074*RM2024(-1) - 0.00071*RM2024(-2) - 0.00054*RM2024(-3) - 0.00030*RM2024(-4) - 0.00010*RM2024(-5))$$

$$DPM2529_FE = (-0.00031*RM2529_FE - 0.00053*RM2529_FE(-1) - 0.00066*RM2529_FE(-2) - 0.00068*RM2529_FE(-3) - 0.00058*RM2529_FE(-4) - 0.00036*RM2529_FE(-5)) - (-0.00031*RM2529 - 0.00053*RM2529(-1) - 0.00066*RM2529(-2) - 0.00068*RM2529(-3) - 0.00058*RM2529(-4) - 0.00036*RM2529(-5))$$

$$DPM3034_FE = (-0.00001*RM3034_FE - 0.00018*RM3034_FE(-1) - 0.00040*RM3034_FE(-2) - 0.00058*RM3034_FE(-3) - 0.00064*RM3034_FE(-4) - 0.00047*RM3034_FE(-5)) - (-0.00001*RM3034 - 0.00018*RM3034(-1) - 0.00040*RM3034(-2) - 0.00058*RM3034(-3) - 0.00064*RM3034(-4) - 0.00047*RM3034(-5))$$

$$DPM3539_FE = (0.00048*RM3539_FE + 0.00037*RM3539_FE(-1) - 0.00007*RM3539_FE(-2) - 0.00058*RM3539_FE(-3) - 0.00091*RM3539_FE(-4) - 0.00080*RM3539_FE(-5)) - (0.00048*RM3539 + 0.00037*RM3539(-1) - 0.00007*RM3539(-2) - 0.00058*RM3539(-3) - 0.00091*RM3539(-4) - 0.00080*RM3539(-5))$$

$$DPM4044_FE = (-0.00018*RM4044_FE - 0.00026*RM4044_FE(-1) - 0.00027*RM4044_FE(-2) - 0.00022*RM4044_FE(-3) - 0.00014*RM4044_FE(-4) - 0.00006*RM4044_FE(-5)) - (-0.00018*RM4044 - 0.00026*RM4044(-1) - 0.00027*RM4044(-2) - 0.00022*RM4044(-3) - 0.00014*RM4044(-4) - 0.00006*RM4044(-5))$$

$$DPM4549_FE = (0.00004*RM4549_FE - 0.00011*RM4549_FE(-1) - 0.00036*RM4549_FE(-2) - 0.00059*RM4549_FE(-3) - 0.00068*RM4549_FE(-4) - 0.00052*RM4549_FE(-5)) - (0.00004*RM4549 - 0.00011*RM4549(-1) - 0.00036*RM4549(-2) - 0.00059*RM4549(-3) - 0.00068*RM4549(-4) - 0.00052*RM4549(-5))$$

$$DPM5054_FE = (0.00115*RM5054_FE + 0.00089*RM5054_FE(-1) - 0.00017*RM5054_FE(-2) - 0.00141*RM5054_FE(-3) - 0.00220*RM5054_FE(-4) - 0.00194*RM5054_FE(-5)) - (0.00115*RM5054 + 0.00089*RM5054(-1) - 0.00017*RM5054(-2) - 0.00141*RM5054(-3) - 0.00220*RM5054(-4) - 0.00194*RM5054(-5))$$

FEMALE

$$DPF1617_FE = (-0.00214*RF1617_FE - 0.00256*RF1617_FE(-1) - 0.00184*RF1617_FE(-2) - 0.00062*RF1617_FE(-3) + 0.00050*RF1617_FE(-4) + 0.00091*RF1617_FE(-5)) -$$

$$(-0.00214*RF1617 - 0.00256*RF1617(-1) - 0.00184*RF1617(-2) - 0.00062*RF1617(-3) + 0.00050*RF1617(-4) + 0.00091*RF1617(-5))$$

$$\begin{aligned} DPF1819_FE = & (-0.00118*RF1819_FE - 0.00145*RF1819_FE(-1) - 0.00112*RF1819_FE(-2) \\ & - 0.00050*RF1819_FE(-3) + 0.00010*RF1819_FE(-4) + 0.00037*RF1819_FE(-5)) - \\ & (-0.00118*RF1819 - 0.00145*RF1819(-1) - 0.00112*RF1819(-2) - 0.00050*RF1819(-3) \\ & + 0.00010*RF1819(-4) + 0.00037*RF1819(-5)) \end{aligned}$$

$$\begin{aligned} DPF2024_FE = & (-0.00006*RF2024_FE - 0.00026*RF2024_FE(-1) - 0.00050*RF2024_FE(-2) \\ & - 0.00070*RF2024_FE(-3) - 0.00075*RF2024_FE(-4) - 0.00055*RF2024_FE(-5)) - \\ & (-0.00006*RF2024 - 0.00026*RF2024(-1) - 0.00050*RF2024(-2) - 0.00070*RF2024(-3) \\ & - 0.00075*RF2024(-4) - 0.00055*RF2024(-5)) \end{aligned}$$

$$\begin{aligned} DPF2529_FE = & (0.00056*RF2529_FE + 0.00034*RF2529_FE(-1) - 0.00030*RF2529_FE(-2) \\ & - 0.00101*RF2529_FE(-3) - 0.00143*RF2529_FE(-4) - 0.00121*RF2529_FE(-5)) - \\ & (0.00056*RF2529 + 0.00034*RF2529(-1) - 0.00030*RF2529(-2) - 0.00101*RF2529(-3) \\ & - 0.00143*RF2529(-4) - 0.00121*RF2529(-5)) \end{aligned}$$

$$\begin{aligned} DPF3034_FE = & (0.00078*RF3034_FE + 0.00075*RF3034_FE(-1) + 0.00024*RF3034_FE(-2) \\ & - 0.00042*RF3034_FE(-3) - 0.00090*RF3034_FE(-4) - 0.00087*RF3034_FE(-5)) - \\ & (0.00078*RF3034 + 0.00075*RF3034(-1) + 0.00024*RF3034(-2) - 0.00042*RF3034(-3) \\ & - 0.00090*RF3034(-4) - 0.00087*RF3034(-5)) \end{aligned}$$

$$\begin{aligned} DPF3539_FE = & (0.00040*RF3539_FE + 0.00032*RF3539_FE(-1) - 0.00004*RF3539_FE(-2) \\ & - 0.00046*RF3539_FE(-3) - 0.00074*RF3539_FE(-4) - 0.00066*RF3539_FE(-5)) - \\ & (0.00040*RF3539 + 0.00032*RF3539(-1) - 0.00004*RF3539(-2) - 0.00046*RF3539(-3) \\ & - 0.00074*RF3539(-4) - 0.00066*RF3539(-5)) \end{aligned}$$

$$\begin{aligned} DPF4044_FE = & (0.00113*RF4044_FE + 0.00097*RF4044_FE(-1) + 0.00007*RF4044_FE(-2) \\ & - 0.00102*RF4044_FE(-3) - 0.00176*RF4044_FE(-4) - 0.00160*RF4044_FE(-5)) - \\ & (0.00113*RF4044 + 0.00097*RF4044(-1) + 0.00007*RF4044(-2) - 0.00102*RF4044(-3) \\ & - 0.00176*RF4044(-4) - 0.00160*RF4044(-5)) \end{aligned}$$

$$\begin{aligned} DPF4549_FE = & (0.00155*RF4549_FE + 0.00133*RF4549_FE(-1) + 0.00007*RF4549_FE(-2) \\ & - 0.00145*RF4549_FE(-3) - 0.00247*RF4549_FE(-4) - 0.00224*RF4549_FE(-5)) - \\ & (0.00155*RF4549 + 0.00133*RF4549(-1) + 0.00007*RF4549(-2) - 0.00145*RF4549(-3) \\ & - 0.00247*RF4549(-4) - 0.00224*RF4549(-5)) \end{aligned}$$

$$\begin{aligned} DPF5054_FE = & (0.00093*RF5054_FE + 0.00074*RF5054_FE(-1) - 0.00008*RF5054_FE(-2) \\ & - 0.00104*RF5054_FE(-3) - 0.00168*RF5054_FE(-4) - 0.00149*RF5054_FE(-5)) - \\ & (0.00093*RF5054 + 0.00074*RF5054(-1) - 0.00008*RF5054(-2) - 0.00104*RF5054(-3) \\ & - 0.00168*RF5054(-4) - 0.00149*RF5054(-5)) \end{aligned}$$

2.2 U.S. Earnings (MODSOL2)

Equation numbers identify the corresponding equations in the Fortran program EconModSol2EquationsMod.f90.

Quarterly Employment Equations

Agricultural Workers

$$\begin{aligned} EA &= \text{IF LONGRANGE} = 0 \\ &\quad \text{THEN GDPPF09} / (1.125 * 1.138 * \text{EXP}(-0.20541 + 0.03254 * \text{YEAR} - 0.07829 + 0.37854)) \\ &\quad \text{ELSE } E * EA.1/E.1 \end{aligned} \quad (20)$$

Nonagricultural workers

$$\text{ENA} = E - EA \quad (21)$$

Nonagricultural Self-employed workers

$$\text{EF1617NAS} = (0.12015 * \text{RTP.1} - 0.10551) * \text{EF1617} \quad (3)$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{ef1617nas/ef1617} = \begin{array}{r} 0.12015 * \text{rtp.1} - \\ (1.96868) \end{array} \quad \begin{array}{r} 0.10551 \\ (1.73441) \end{array}$$

Sum Sq 0.0000
Std Error 0.0030
LHS Mean 0.0142
R-Squared 0.5637
R Bar Squared 0.4182
F-stat 1, 3 3.8757
D.W. (1) 1.5620
D.W. (2) 2.3626

$$\text{EF1819NAS} = (0.11184 * \text{RTP.1} - 0.10241) * \text{EF1819} \quad (4)$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{ef1819nas/ef1819} = \begin{array}{r} 0.11184 * \text{rtp.1} - \\ (2.99537) \end{array} \quad \begin{array}{r} 0.10241 \\ (2.75170) \end{array}$$

Sum Sq 0.0000
Std Error 0.0018
LHS Mean 0.0090
R-Squared 0.7494
R Bar Squared 0.6659
F-STAT 1, 3 8.9722
D.W. (1) 3.2586
D.W. (2) 0.9766

$$\text{EF2024NAS} = (0.08908 * \text{RTP.1} - 0.07176) * \text{EF2024} \quad (5)$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{ef2024nas/ef2024} = \begin{array}{r} 0.08908 * \text{rtp.1} - \\ (2.54605) \end{array} \quad \begin{array}{r} 0.07176 \\ (2.05763) \end{array}$$

Sum Sq 0.0000
Std Error 0.0017
LHS Mean 0.0170
R-Squared 0.6836
R Bar Squared 0.5782
F-STAT 1, 3 6.4824
D.W. (1) 2.6600
D.W. (2) 1.5247

$$\text{EF2534NAS} = (0.00906 * \text{RTP.1} + 0.03539) * \text{EF2534} \quad (6)$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004
Date: 9 NOV 2005

$$\text{ef2534nas/ef2534} = \begin{matrix} 0.00906 * \text{rtp.1} + & 0.03539 \\ (0.34277) & (1.34366) \end{matrix}$$

Sum Sq 0.0000
Std Error 0.0013
LHS Mean 0.0444
R-Squared 0.0377
R Bar Squared 0.2831
F-STAT 1, 3 0.1175
D.W. (1) 3.0818
D.W. (2) 1.1094

$$\text{EF3544NAS} = (-0.01869 * \text{RTP.1} + 0.08087) * \text{EF3544} \quad (7)$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004
Date: 9 NOV 2005

$$\text{ef3544nas/ef3544} = \begin{matrix} -0.01869 * \text{rtp.1} + & 0.08087 \\ (0.70565) & (3.06320) \end{matrix}$$

Sum Sq 0.0000
Std Error 0.0013
LHS Mean 0.0622
R-Squared 0.1424
R Bar Squared 0.1435
F-STAT 1, 3 0.4979
D.W. (1) 2.2440
D.W. (2) 2.1852

$$\text{EF4554NAS} = (0.07232 * \text{RTP.1} - 0.00701) * \text{EF4554} \quad (8)$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004
Date: 9 NOV 2005

$$\text{ef4554nas/ef4554} = \begin{matrix} 0.07232 * \text{rtp.1} - & 0.00701 \\ (2.86756) & (0.27876) \end{matrix}$$

Sum Sq 0.0000
Std Error 0.0012
LHS Mean 0.0651
R-Squared 0.7327
R Bar Squared 0.6436
F-STAT 1, 3 8.2229
D.W. (1) 1.7821
D.W. (2) 2.7029

$$\text{EF5564NAS} = (0.07872 * \text{RTP.1} + 0.00466) * \text{EF5564} \quad (9)$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004
Date: 9 NOV 2005

$$\text{ef5564nas/ef5564} = \begin{matrix} 0.07872 * \text{rtp.1} + & 0.00466 \\ (1.38159) & (0.08196) \end{matrix}$$

Sum Sq 0.0000
Std Error 0.0028
LHS Mean 0.0831
R-Squared 0.3889
R Bar Squared 0.1851
F-STAT 1, 3 1.9088
D.W. (1) 2.6092
D.W. (2) 2.2686

$$\text{EF65ONAS} = (0.10940 * \text{EF6569} + 0.12265 * \text{EF7074} + 0.14137 * \text{EF750}) \quad (10)$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004
Date: 9 NOV 2005

$$\text{ef6569nas/ef6569} = 0.10940 \\ (37.7493)$$

Sum Sq 0.0002
 Std Error 0.0065
 LHS Mean 0.1094
 R-Squared 0.0000
 R Bar Squared 0.0000
 F 0, 4 NC
 D.W. (1) 3.0431
 D.W. (2) 1.2204

Ordinary Least Squares
 ANNUAL data for 5 periods from 2000 to 2004
 Date: 9 NOV 2005

$$\text{ef7074nas/ef7074} = 0.12265 \\ (16.4939)$$

Sum Sq 0.0011
 Std Error 0.0166
 LHS Mean 0.1226
 R-Squared 0.0000
 R Bar Squared 0.0000
 F 0, 4 NC
 D.W. (1) 1.0289
 D.W. (2) 1.7188

Ordinary Least Squares
 ANNUAL data for 5 periods from 2000 to 2004
 Date: 9 NOV 2005

$$\text{ef75onas/ef75o} = 0.14137 \\ (17.7500)$$

Sum Sq 0.0013
 Std Error 0.0178
 LHS Mean 0.1414
 R-Squared 0.0000
 R Bar Squared 0.0000
 F 0, 4 NC
 D.W. (1) 1.6889
 D.W. (2) 1.2345

$$\text{EM1617NAS} = (-0.23035 * \text{RTP.1} + 0.24985) * \text{EM1617} \quad (2)$$

Ordinary Least Squares
 ANNUAL data for 5 periods from 2000 to 2004
 Date: 9 NOV 2005

$$\text{em1617nas/em1617} = -0.23035 * \text{rtp.1} + 0.24985 \\ (5.08538) \quad (5.53372)$$

Sum Sq 0.0000
 Std Error 0.0022
 LHS Mean 0.0203
 R-Squared 0.8961
 R Bar Squared 0.8614
 F-STAT 1, 3 25.8611
 D.W. (1) 2.4658
 D.W. (2) 1.6839

$$\text{EM1819NAS} = (-0.05782 * \text{RTP.1} + 0.07265) * \text{EM1819} \quad (11)$$

Ordinary Least Squares
 ANNUAL data for 5 periods from 2000 to 2004
 Date: 9 NOV 2005

$$\text{em1819nas/em1819} = -0.05782 * \text{rtp.1} + 0.07265 \\ (3.43044) \quad (4.32458)$$

Sum Sq 0.0000
 Std Error 0.0008
 LHS Mean 0.0150
 R-Squared 0.7969
 R Bar Squared 0.7291
 F-STAT 1, 3 11.7679
 D.W. (1) 3.3262
 D.W. (2) 1.0399

$$\text{EM2024NAS} = (-0.09206 * \text{RTP.1} + 0.11567) * \text{EM2024} \quad (12)$$

Ordinary Least Squares
 ANNUAL data for 5 periods from 2000 to 2004
 Date: 9 NOV 2005

$$\text{em2024nas/em2024} = -0.09206 * \text{rtp.1} + 0.11567$$

(2.44839) (3.08618)

Sum Sq 0.0000
 Std Error 0.0018
 LHS Mean 0.0239
 R-Squared 0.6665
 R Bar Squared 0.5553
 F-STAT 1, 3 5.9946
 D.W. (1) 2.1493
 D.W. (2) 1.7046

$$\text{EM2534NAS} = (-0.09661 * \text{RTP.1} + 0.14843) * \text{EM2534} \quad (13)$$

Ordinary Least Squares
 ANNUAL data for 5 periods from 2000 to 2004
 Date: 9 NOV 2005

$$\text{em2534nas/em2534} = -0.09661 * \text{rtp.1} + 0.14843$$

(2.81478) (4.33847)

Sum Sq 0.0000
 Std Error 0.0017
 LHS Mean 0.0522
 R-Squared 0.7254
 R Bar Squared 0.6338
 F-STAT 1, 3 7.9230
 D.W. (1) 1.8300
 D.W. (2) 2.9632

$$\text{EM3544NAS} = (0.02739 * \text{RTP.1} + 0.05236) * \text{EM3544} \quad (14)$$

Ordinary Least Squares
 ANNUAL data for 5 periods from 2000 to 2004
 Date: 9 NOV 2005

$$\text{em3544nas/em3544} = 0.02739 * \text{rtp.1} + 0.05236$$

(0.61129) (1.17241)

Sum Sq 0.0000
 Std Error 0.0022
 LHS Mean 0.0797
 R-Squared 0.1108
 R Bar Squared 0.1857
 F-STAT 1, 3 0.3737
 D.W. (1) 2.5508
 D.W. (2) 2.2676

$$\text{EM4554NAS} = (0.06217 * \text{RTP.1} + 0.03411) * \text{EM4554} \quad (15)$$

Ordinary Least Squares
 ANNUAL data for 5 periods from 2000 to 2004
 Date: 9 NOV 2005

$$\text{em4554nas/em4554} = 0.06217 * \text{rtp.1} + 0.03411$$

(1.91738) (1.05544)

Sum Sq 0.0000
 Std Error 0.0016

LHS Mean 0.0961
 R-Squared 0.5507
 R Bar Squared 0.4009
 F-STAT 1, 3 3.6764
 D.W. (1) 2.5497
 D.W. (2) 1.5554

$$EM5564NAS = (-0.04776 * RTP.1 + 0.16626) * EM5564 \quad (16)$$

Ordinary Least Squares
 ANNUAL data for 5 periods from 2000 to 2004
 Date: 9 NOV 2005

$$em5564nas/em5564 = -0.04776 * rtp.1 + 0.16626 \\ (0.60480) \quad (2.11226)$$

Sum Sq 0.0000
 Std Error 0.0039
 LHS Mean 0.1187
 R-Squared 0.1087
 R Bar Squared 0.1884
 F-STAT 1, 3 0.3658
 D.W. (1) 2.9234
 D.W. (2) 1.9432

$$EM65ONAS = (0.16527 * EM6569 + 0.17798 * EM7074 + 0.19058 * EM75O) \quad (17)$$

Ordinary Least Squares
 ANNUAL data for 5 periods from 2000 to 2004
 Date: 9 NOV 2005

$$em6569nas/em6569 = 0.16527 \\ (53.9126)$$

Sum Sq 0.0002
 Std Error 0.0069
 LHS Mean 0.1653
 R-Squared 0.0000
 R Bar Squared 0.0000
 F-stat 0, 4 NC
 D.W. (1) 1.7716
 D.W. (2) 2.9645

Ordinary Least Squares
 ANNUAL data for 5 periods from 2000 to 2004
 Date: 9 NOV 2005

$$em7074nas/em7074 = 0.17798 \\ (22.9265)$$

Sum Sq 0.0012
 Std Error 0.0174
 LHS Mean 0.1780
 R-Squared 0.0000
 R Bar Squared 0.0000
 F-stat 0, 4 NC
 D.W. (1) 1.7116
 D.W. (2) 2.1991

Ordinary Least Squares
 ANNUAL data for 5 periods from 2000 to 2004
 Date: 9 NOV 2005

$$em75onas/em75o = 0.19058 \\ (20.1892)$$

Sum Sq 0.0018
 Std Error 0.0211
 LHS Mean 0.1906
 R-Squared 0.0000
 R Bar Squared 0.0000
 F-stat 0, 4 NC
 D.W. (1) 2.7330

D.W. (2) 0.9992

Nonagricultural Self-employed Workers:

“Raw” equations (before scaling the totals):

Female

$$\begin{aligned} \text{EF1617NAS_R} &= (0.12015 * \text{RTP.1} - 0.10551) * \text{EF1617} + & (3) \\ \text{EF1617NAS_ADJ} & & \\ \text{EF1819NAS_R} &= (0.11184 * \text{RTP.1} - 0.10241) * \text{EF1819} + & (4) \\ \text{EF1819NAS_ADJ} & & \\ \text{EF2024NAS_R} &= (0.08908 * \text{RTP.1} - 0.07176) * \text{EF2024} + & (5) \\ \text{EF2024NAS_ADJ} & & \\ \text{EF2534NAS_R} &= (0.00906 * \text{RTP.1} + 0.03539) * \text{EF2534} + & (6) \\ \text{EF2534NAS_ADJ} & & \\ \text{EF3544NAS_R} &= (-0.01869 * \text{RTP.1} + 0.08087) * \text{EF3544} + & (7) \\ \text{EF3544NAS_ADJ} & & \\ \text{EF4554NAS_R} &= (0.07232 * \text{RTP.1} - 0.00701) * \text{EF4554} + & (8) \\ \text{EF4554NAS_ADJ} & & \\ \text{EF5564NAS_R} &= (0.07872 * \text{RTP.1} + 0.00466) * \text{EF5564} + & (9) \\ \text{EF5564NAS_ADJ} & & \\ \text{EF65ONAS_R} &= (0.10940 * \text{EF6569} + 0.12265 * \text{EF7074} + 0.14137 * \text{EF75O}) + & \\ \text{EF65ONAS_ADJ} & & (10) \end{aligned}$$

Male

$$\begin{aligned} \text{EM1617NAS_R} &= (-0.23035 * \text{RTP.1} + 0.24985) * \text{EM1617} + & (2) \\ \text{EM1617NAS_ADJ} & & \\ \text{EM1819NAS_R} &= (-0.05782 * \text{RTP.1} + 0.07265) * \text{EM1819} + & (11) \\ \text{EM1819NAS_ADJ} & & \\ \text{EM2024NAS_R} &= (-0.09206 * \text{RTP.1} + 0.11567) * \text{EM2024} + & (12) \\ \text{EM2024NAS_ADJ} & & \\ \text{EM2534NAS_R} &= (-0.09661 * \text{RTP.1} + 0.14843) * \text{EM2534} + & (13) \\ \text{EM2534NAS_ADJ} & & \\ \text{EM3544NAS_R} &= (0.02739 * \text{RTP.1} + 0.05236) * \text{EM3544} + & (14) \\ \text{EM3544NAS_ADJ} & & \\ \text{EM4554NAS_R} &= (0.06217 * \text{RTP.1} + 0.03411) * \text{EM4554} + & (15) \\ \text{EM4554NAS_ADJ} & & \\ \text{EM5564NAS_R} &= (-0.04776 * \text{RTP.1} + 0.16626) * \text{EM5564} + & (16) \\ \text{EM5564NAS_ADJ} & & \\ \text{EM65ONAS_R} &= (0.16527 * \text{EM6569} + 0.17798 * \text{EM7074} + 0.19058 * \text{EM75O}) + & \\ \text{EM65ONAS_ADJ} & & (17) \end{aligned}$$

$$\begin{aligned} \text{ENAS_R} &= \text{EF1617NAS_R} + \text{EF1819NAS_R} + \text{EF2024NAS_R} + \text{EF2534NAS_R} + \text{EF3544NAS_R} + \text{EF4554NAS_R} + \text{EF5564NAS_R} + \\ & \text{EF65ONAS_R} + \text{EM1617NAS_R} + \text{EM1819NAS_R} + \text{EM2024NAS_R} + \text{EM2534NAS_R} + \text{EM3544NAS_R} + \\ & \text{EM4554NAS_R} + \text{EM5564NAS_R} + \text{EM65ONAS_R} \quad (18) \end{aligned}$$

Total nonagricultural SE workers:

$$\begin{aligned} \text{ENAS} &= \text{IF LONGRANGE} = 0 \\ & \text{THEN ENAS_R} \\ & \text{ELSE ENA} * (\text{ENAS.1}/\text{ENA.1}) \quad (22) \end{aligned}$$

Final (scaled) equations:

Male

$$\begin{aligned} \text{EM1617NAS} &= \text{EM1617NAS_R} * (\text{ENAS}/\text{ENAS_R}) \quad (23) \\ \text{EM1819NAS} &= \text{EM1819NAS_R} * (\text{ENAS}/\text{ENAS_R}) \quad (102) \\ \text{EM2024NAS} &= \text{EM2024NAS_R} * (\text{ENAS}/\text{ENAS_R}) \quad (109) \\ \text{EM2534NAS} &= \text{EM2534NAS_R} * (\text{ENAS}/\text{ENAS_R}) \quad (116) \\ \text{EM3544NAS} &= \text{EM3544NAS_R} * (\text{ENAS}/\text{ENAS_R}) \quad (123) \\ \text{EM4554NAS} &= \text{EM4554NAS_R} * (\text{ENAS}/\text{ENAS_R}) \quad (130) \\ \text{EM5564NAS} &= \text{EM5564NAS_R} * (\text{ENAS}/\text{ENAS_R}) \quad (137) \\ \text{EM65ONAS} &= \text{EM65ONAS_R} * (\text{ENAS}/\text{ENAS_R}) \quad (144) \end{aligned}$$

Female

$$\begin{aligned} \text{EF1617NAS} &= \text{EF1617NAS_R} * (\text{ENAS}/\text{ENAS_R}) \quad (157) \\ \text{EF1819NAS} &= \text{EF1819NAS_R} * (\text{ENAS}/\text{ENAS_R}) \quad (164) \\ \text{EF2024NAS} &= \text{EF2024NAS_R} * (\text{ENAS}/\text{ENAS_R}) \quad (171) \end{aligned}$$

$$\begin{aligned}
EF2534NAS &= EF2534NAS_R * (ENAS/ENAS_R) && (178) \\
EF3544NAS &= EF3544NAS_R * (ENAS/ENAS_R) && (185) \\
EF4554NAS &= EF4554NAS_R * (ENAS/ENAS_R) && (192) \\
EF5564NAS &= EF5564NAS_R * (ENAS/ENAS_R) && (199) \\
EF65ONAS &= EF65ONAS_R * (ENAS/ENAS_R) && (206) \\
\\
EFNAS &= EF1617NAS + EF1819NAS + EF2024NAS + EF2534NAS + EF3544NAS + EF4554NAS + EF5564NAS + \\
EF65ONAS &&& (212) \\
EMNAS &= EM1617NAS + EM1819NAS + EM2024NAS + EM2534NAS + EM3544NAS + EM4554NAS + EM5564NAS + \\
EM65ONAS &&& (150)
\end{aligned}$$

Nonagricultural Unpaid Family Workers

“Raw” equations (before scaling the totals):

$$\begin{aligned}
\text{Female} &&& (25-32) \\
EF1617NAU_R &= 0.00012 * ENAS + EF1617NAU.ADJ \\
EF1819NAU_R &= 0.00025 * ENAS + EF1819NAU.ADJ \\
EF2024NAU_R &= 0.00024 * ENAS + EF2024NAU.ADJ \\
EF2534NAU_R &= 0.00117 * ENAS + EF2534NAU.ADJ \\
EF3544NAU_R &= 0.00218 * ENAS + EF3544NAU.ADJ \\
EF4554NAU_R &= 0.00226 * ENAS + EF4554NAU.ADJ \\
EF5564NAU_R &= 0.00083 * ENAS + EF5564NAU.ADJ \\
EF65ONAU_R &= (0.00027 + 0.00021 + 0.00008) * ENAS + EF65ONAU.ADJ \\
\\
\text{Male} &&& \\
EM1617NAU_R &= 0.00028 * ENAS + EM1617NAU.ADJ && (24) \\
EM1819NAU_R &= 0.00033 * ENAS + EM1819NAU.ADJ && (33) \\
EM2024NAU_R &= 0.00050 * ENAS + EM2024NAU.ADJ && (34) \\
EM2534NAU_R &= 0.00044 * ENAS + EM2534NAU.ADJ && (35) \\
EM3544NAU_R &= 0.00043 * ENAS + EM3544NAU.ADJ && (36) \\
EM4554NAU_R &= 0.00052 * ENAS + EM4554NAU.ADJ && (37) \\
EM5564NAU_R &= 0.00037 * ENAS + EM5564NAU.ADJ && (38) \\
EM65ONAU_R &= (0.00023 + 0.00010 + 0.00011) * ENAS + EM65ONAU.ADJ && (39) \\
\\
ENAU_R &= EF1617NAU_R + EF1819NAU_R + EF2024NAU_R + EF2534NAU_R + EF3544NAU_R + EF4554NAU_R + \\
&EF5564NAU_R + EF65ONAU_R + EM1617NAU_R + EM1819NAU_R + EM2024NAU_R + EM2534NAU_R + \\
&EM3544NAU_R + EM4554NAU_R + EM5564NAU_R + EM65ONAU_R && (40)
\end{aligned}$$

$$\begin{aligned}
\text{Total Nonagricultural Unpaid Family Workers:} \\
ENAU &= ENAU_R && (41)
\end{aligned}$$

Final (scaled) equations:

$$\begin{aligned}
EM1617NAU &= EM1617NAU_R * (ENAU/ENAU_R) && (42) \\
EM1819NAU &= EM1819NAU_R * (ENAU/ENAU_R) && (103) \\
EM2024NAU &= EM2024NAU_R * (ENAU/ENAU_R) && (110) \\
EM2534NAU &= EM2534NAU_R * (ENAU/ENAU_R) && (117) \\
EM3544NAU &= EM3544NAU_R * (ENAU/ENAU_R) && (124) \\
EM4554NAU &= EM4554NAU_R * (ENAU/ENAU_R) && (131) \\
EM5564NAU &= EM5564NAU_R * (ENAU/ENAU_R) && (138) \\
EM65ONAU &= EM65ONAU_R * (ENAU/ENAU_R) && (145) \\
\\
EF1617NAU &= EF1617NAU_R * (ENAU/ENAU_R) && (158) \\
EF1819NAU &= EF1819NAU_R * (ENAU/ENAU_R) && (165) \\
EF2024NAU &= EF2024NAU_R * (ENAU/ENAU_R) && (172) \\
EF2534NAU &= EF2534NAU_R * (ENAU/ENAU_R) && (179) \\
EF3544NAU &= EF3544NAU_R * (ENAU/ENAU_R) && (186) \\
EF4554NAU &= EF4554NAU_R * (ENAU/ENAU_R) && (193) \\
EF5564NAU &= EF5564NAU_R * (ENAU/ENAU_R) && (200) \\
EF65ONAU &= EF65ONAU_R * (ENAU/ENAU_R) && (207) \\
\\
EFNAU &= EF1617NAU + EF1819NAU + EF2024NAU + EF2534NAU + EF3544NAU + EF4554NAU + EF5564NAU + \\
EF65ONAU &&& (213) \\
EMNAU &= EM1617NAU + EM1819NAU + EM2024NAU + EM2534NAU + EM3544NAU + EM4554NAU + EM5564NAU + \\
EM65ONAU &&& (151)
\end{aligned}$$

Agricultural Wage Workers

Total Agricultural Wage Workers

$$\begin{aligned} \text{EAW} &= \text{IF LONGRANGE} = 0 \\ &\quad \text{THEN EA} * (0.00893 * \text{YEAR} + 0.33159 * \text{RTP} - 0.67943) \\ &\quad \text{ELSE EA} * (\text{EAW.1} / \text{EA.1}) \end{aligned} \quad (43)$$

Raw Disaggregation of EAW:

Male

$$\begin{aligned} \text{EM1617AW_R} &= \text{MAX}(0, \text{EAW} * (-0.00594 - 0.09353 * \text{MOVAVG}(2, \text{RTP.1}) + 5.28754 * \text{EM1617/E} + 0.08116) + \\ &\quad \text{EM1617AW.ADJ}) \quad (44) \\ \text{EM1819AW_R} &= \text{MAX}(0, \text{EAW} * (-0.00131 - 0.18120 * \text{MOVAVG}(2, \text{RTP.1}) + 3.87151 * \text{EM1819/E} + 0.16636) + \text{EM1819AW.ADJ}) \\ &\quad (53) \\ \text{EM2024AW_R} &= \text{MAX}(0, \text{EAW} * (-0.00664 + 0.10493 * \text{MOVAVG}(2, \text{RTP.1}) + 2.00153 * \text{EM2024/E} - 0.08191) + \text{EM2024AW.ADJ}) \\ &\quad (54) \\ \text{EM2534AW_R} &= \text{MAX}(0, \text{EAW} * (-0.02065 + 0.38358 * \text{MOVAVG}(2, \text{RTP.1}) - 0.98380 * \text{EM2534/E} + 0.00751) + \text{EM2534AW.ADJ}) \\ &\quad (55) \\ \text{EM3544AW_R} &= \text{MAX}(0, \text{EAW} * (0.00402 - 0.15663 * \text{MOVAVG}(2, \text{RTP.1}) + 1.72119 * \text{EM3544/E} + 0.05679) + \text{EM3544AW.ADJ}) \\ &\quad (56) \\ \text{EM4554AW_R} &= \text{MAX}(0, \text{EAW} * (0.00834 + 0.03746 * \text{MOVAVG}(2, \text{RTP.1}) + 0.46522 * \text{EM4554/E} + 0.00144) + \text{EM4554AW.ADJ}) \\ &\quad (57) \\ \text{EM5564AW_R} &= \text{MAX}(0, \text{EAW} * (-0.00655 + 0.03521 * \text{MOVAVG}(2, \text{RTP.1}) + 0.46852 * \text{EM5564/E} - 0.00037) + \text{EM5564AW.ADJ}) \\ &\quad (58) \\ \text{EM65OAW_R} &= \text{MAX}(0, \text{EAW} * (-0.00114 + 0.07640 * \text{MOVAVG}(2, \text{RTP.1}) + 3.25911 * \text{EM65O/E} - 0.10058) + \text{EM65OAW.ADJ}) \\ &\quad (59) \end{aligned}$$

Female

$$\begin{aligned} \text{EF1617AW_R} &= \text{MAX}(0, \text{EAW} * (-0.00055 - 0.05470 * \text{MOVAVG}(2, \text{RTP.1}) + 1.41760 * \text{EF1617/E} + 0.04979) + \text{EF1617AW.ADJ}) \\ \text{EF1819AW_R} &= \text{MAX}(0, \text{EAW} * (0.00102 - 0.07375 * \text{MOVAVG}(2, \text{RTP.1}) + 0.78394 * \text{EF1819/E} + 0.07226) + \text{EF1819AW.ADJ}) \\ \text{EF2024AW_R} &= \text{MAX}(0, \text{EAW} * (0.00112 - 0.05971 * \text{MOVAVG}(2, \text{RTP.1}) + 0.57256 * \text{EF2024/E} + 0.05907) + \text{EF2024AW.ADJ}) \\ \text{EF2534AW_R} &= \text{MAX}(0, \text{EAW} * (0.00623 + 0.08868 * \text{MOVAVG}(2, \text{RTP.1}) + 1.00897 * \text{EF2534/E} - 0.15142) + \text{EF2534AW.ADJ}) \\ \text{EF3544AW_R} &= \text{MAX}(0, \text{EAW} * (0.00687 - 0.00259 * \text{MOVAVG}(2, \text{RTP.1}) + 0.51319 * \text{EF3544/E} - 0.00937) + \text{EF3544AW.ADJ}) \\ \text{EF4554AW_R} &= \text{MAX}(0, \text{EAW} * (0.00185 + 0.08747 * \text{MOVAVG}(2, \text{RTP.1}) + 0.28022 * \text{EF4554/E} - 0.08053) + \text{EF4554AW.ADJ}) \\ \text{EF5564AW_R} &= \text{MAX}(0, \text{EAW} * (-0.00140 - 0.03001 * \text{MOVAVG}(2, \text{RTP.1}) - 0.59383 * \text{EF5564/E} + 0.07088) + \text{EF5564AW.ADJ}) \\ \text{EF65OAW_R} &= \text{MAX}(0, \text{EAW} * (0.00096 + 0.06768 * \text{MOVAVG}(2, \text{RTP.1}) + 1.04213 * \text{EF65O/E} - 0.07359) + \text{EF65OAW.ADJ}) \end{aligned} \quad (45-52)$$

$$\begin{aligned} \text{EAW_R} &= \text{EF1617AW_R} + \text{EF1819AW_R} + \text{EF2024AW_R} + \text{EF2534AW_R} + \text{EF3544AW_R} + \text{EF4554AW_R} + \text{EF5564AW_R} + \\ &\quad \text{EF65OAW_R} + \text{EM1617AW_R} + \text{EM1819AW_R} + \text{EM2024AW_R} + \text{EM2534AW_R} + \text{EM3544AW_R} + \\ &\quad \text{EM4554AW_R} + \text{EM5564AW_R} + \text{EM65OAW_R} \end{aligned} \quad (60)$$

Final (scaled) equations:

$$\begin{aligned} \text{EM1617AW} &= \text{EM1617AW_R} * (\text{EAW} / \text{EAW_R}) \quad (61) \\ \text{EM1819AW} &= \text{EM1819AW_R} * (\text{EAW} / \text{EAW_R}) \quad (104) \\ \text{EM2024AW} &= \text{EM2024AW_R} * (\text{EAW} / \text{EAW_R}) \quad (111) \\ \text{EM2534AW} &= \text{EM2534AW_R} * (\text{EAW} / \text{EAW_R}) \quad (118) \\ \text{EM3544AW} &= \text{EM3544AW_R} * (\text{EAW} / \text{EAW_R}) \quad (125) \\ \text{EM4554AW} &= \text{EM4554AW_R} * (\text{EAW} / \text{EAW_R}) \quad (132) \\ \text{EM5564AW} &= \text{EM5564AW_R} * (\text{EAW} / \text{EAW_R}) \quad (139) \\ \text{EM65OAW} &= \text{EM65OAW_R} * (\text{EAW} / \text{EAW_R}) \quad (146) \end{aligned}$$

$$\begin{aligned} \text{EF1617AW} &= \text{EF1617AW_R} * (\text{EAW} / \text{EAW_R}) \quad (159) \\ \text{EF1819AW} &= \text{EF1819AW_R} * (\text{EAW} / \text{EAW_R}) \quad (166) \\ \text{EF2024AW} &= \text{EF2024AW_R} * (\text{EAW} / \text{EAW_R}) \quad (173) \\ \text{EF2534AW} &= \text{EF2534AW_R} * (\text{EAW} / \text{EAW_R}) \quad (180) \\ \text{EF3544AW} &= \text{EF3544AW_R} * (\text{EAW} / \text{EAW_R}) \quad (187) \\ \text{EF4554AW} &= \text{EF4554AW_R} * (\text{EAW} / \text{EAW_R}) \quad (194) \\ \text{EF5564AW} &= \text{EF5564AW_R} * (\text{EAW} / \text{EAW_R}) \quad (201) \\ \text{EF65OAW} &= \text{EF65OAW_R} * (\text{EAW} / \text{EAW_R}) \quad (208) \end{aligned}$$

$$\begin{aligned} \text{EFAW} &= \text{EF1617AW} + \text{EF1819AW} + \text{EF2024AW} + \text{EF2534AW} + \text{EF3544AW} + \text{EF4554AW} + \text{EF5564AW} + \\ &\quad \text{EF65OAW} \end{aligned} \quad (214)$$

$$\begin{aligned} \text{EMAW} &= \text{EM1617AW} + \text{EM1819AW} + \text{EM2024AW} + \text{EM2534AW} + \text{EM3544AW} + \text{EM4554AW} + \text{EM5564AW} + \\ &\quad \text{EM65OAW} \end{aligned} \quad (152)$$

Unpaid Agricultural Family Workers

Raw equations:

Male (71-78)

EM1617AU_R = MAX (0, 0.002 + EM1617AU.ADJ)
 EM1819AU_R = MAX (0, 0.001 + EM1819AU.ADJ)
 EM2024AU_R = MAX (0, 0.001 + EM2024AU.ADJ)
 EM2534AU_R = MAX (0, 0.003 + EM2534AU.ADJ)
 EM3544AU_R = MAX (0, 0.004 + EM3544AU.ADJ)
 EM4554AU_R = MAX (0, 0.005 + EM4554AU.ADJ)
 EM5564AU_R = MAX (0, 0.003 + EM5564AU.ADJ)
 EM65OAU_R = MAX (0, 0.001 + EM65OAU.ADJ)

Female

(63-70)

EF1617AU_R = MAX (0, 0.006 + EM1617AU.ADJ)
 EF1819AU_R = MAX (0, 0.005 + EF1819AU.ADJ)
 EF2024AU_R = MAX (0, 0.005 + EF2024AU.ADJ)
 EF2534AU_R = MAX (0, 0.002 + EF2534AU.ADJ)
 EF3544AU_R = MAX (0, 0.002 + EF3544AU.ADJ)
 EF4554AU_R = MAX (0, 0.001 + EF4554AU.ADJ)
 EF5564AU_R = MAX (0, 0.001 + EF5564AU.ADJ)
 EF65OAU_R = MAX (0, 0.002 + EF65OAU.ADJ)

EAU_R = EF1617AU_R + EF1819AU_R + EF2024AU_R + EF2534AU_R + EF3544AU_R + EF4554AU_R + EF5564AU_R +
 EF65OAU_R + EM1617AU_R + EM1819AU_R + EM2024AU_R + EM2534AU_R + EM3544AU_R + EM4554AU_R +
 EM5564AU_R + EM65OAU_R (79)

Total Unpaid Agricultural Family Workers :

EAU = IF LONGRANGE = 0
 THEN EAU_R
 ELSE EAU.I/EA.I * EA (80)

Final (scaled) equations :

EM1617AU = EM1617AU_R * (EAU/EAU_R) (99)
 EM1819AU = EM1819AU_R * (EAU/EAU_R) (106)
 EM2024AU = EM2024AU_R * (EAU/EAU_R) (113)
 EM2534AU = EM2534AU_R * (EAU/EAU_R) (120)
 EM3544AU = EM3544AU_R * (EAU/EAU_R) (127)
 EM4554AU = EM4554AU_R * (EAU/EAU_R) (134)
 EM5564AU = EM5564AU_R * (EAU/EAU_R) (141)
 EM65OAU = EM65OAU_R * (EAU/EAU_R) (148)

EF1617AU = EF1617AU_R * (EAU/EAU_R) (161)
 EF1819AU = EF1819AU_R * (EAU/EAU_R) (168)
 EF2024AU = EF2024AU_R * (EAU/EAU_R) (175)
 EF2534AU = EF2534AU_R * (EAU/EAU_R) (182)
 EF3544AU = EF3544AU_R * (EAU/EAU_R) (189)
 EF4554AU = EF4554AU_R * (EAU/EAU_R) (196)
 EF5564AU = EF5564AU_R * (EAU/EAU_R) (203)
 EF65OAU = EF65OAU_R * (EAU/EAU_R) (210)

EFAU = EF1617AU + EF1819AU + EF2024AU + EF2534AU + EF3544AU + EF4554AU + EF5564AU + EF65OAU (216)
 EMAU = EM1617AU + EM1819AU + EM2024AU + EM2534AU + EM3544AU + EM4554AU + EM5564AU + EM65OAU (154)

Self-employed Agricultural Workers

Total

EAS = EA - EAU - EAW (81)

Raw disaggregation:

EM1617AS_R = MAX (0, NM1617 * (0.00528 + 0.00404) + EM1617AS.ADJ) (62)
 EM1819AS_R = MAX (0, NM1819 * (0.00309 + 0.28448 * EA / (NMI6O+ NF16O) - 0.00165) +
 EM1819AS.ADJ) (90)
 EM2024AS_R = MAX (0, NM2024 * (-0.00181 + 0.97958 * EA / (NMI6O+ NF16O) - 0.01093) + EM2024AS.ADJ)
 (91)
 EM2534AS_R = MAX (0, NM2534 * (-0.00263 + 1.23186 * EA / (NMI6O+ NF16O) - 0.01021) + EM2534AS.ADJ)
 (92)
 EM3544AS_R = MAX (0, NM3544 * (-0.00151 + 1.66765 * EA / (NMI6O+ NF16O) - 0.01450) + EM3544AS.ADJ)
 (93)
 EM4554AS_R = MAX (0, NM4554 * (-0.00381 + 2.86654 * EA / (NMI6O+ NF16O) - 0.03175) + EM4554AS.ADJ)
 (94)
 EM5564AS_R = MAX (0, NM5564 * (-0.00460 + 2.78817 * EA / (NMI6O+ NF16O) - 0.02398) + EM5564AS.ADJ)
 (95)
 EM65OAS_R = MAX (0, NM65O * (0.00079 + 1.76904 * EA / (NMI6O+ NF16O) - 0.01437) + EM65OAS.ADJ) (96)

$$\begin{aligned}
EF1617AS_R &= \text{MAX}(0, NF1617 * (0.00181 + 0.00030) + EF1617AS.ADJ) && (82) \\
EF1819AS_R &= \text{MAX}(0, EM1819AS.1 * (-0.02393 + 0.63672 * \text{MOVAVG}(2, RTP.1) + 0.98791 * EF1819/EM1819 - 4.43926) + \\
&\quad EF1819AS.ADJ) && (83) \\
EF2024AS_R &= \text{MAX}(0, EM2024AS.1 * (0.07353 - 0.40207 * \text{MOVAVG}(2, RTP.1) + 0.57572 * EF2024/EM2024 - 0.01117) + \\
&\quad EF2024AS.ADJ) && (84) \\
EF2534AS_R &= \text{MAX}(0, EM2534AS.1 * (0.16575 + 0.16967 * \text{MOVAVG}(2, RTP.1) + 0.55503 * EF2534/EM2534 - 0.43412) + \\
&\quad EF2534AS.ADJ) && (85) \\
EF3544AS_R &= \text{MAX}(0, EM3544AS.1 * (0.15848 + 0.37839 * \text{MOVAVG}(2, RTP.1) + 0.37764 * EF3544/EM3544 - 0.45362) + \\
&\quad EF3544AS.ADJ) && (86) \\
EF4554AS_R &= \text{MAX}(0, EM4554AS.1 * (0.21947 + 0.29497 * \text{MOVAVG}(2, RTP.1) + 0.58974 * EF4554/EM4554 - 0.51966) + \\
&\quad EF4554AS.ADJ) && (87) \\
EF5564AS_R &= \text{MAX}(0, EM5564AS.1 * (0.20892 + 0.36294 * \text{MOVAVG}(2, RTP.1) + 0.65320 * EF5564/EM5564 - 0.66626) + \\
&\quad EF5564AS.ADJ) && (88) \\
EF65OAS_R &= \text{MAX}(0, EM65OAS.1 * (0.16242 + 0.54916 * \text{MOVAVG}(2, RTP.1) + 0.06199 * EF65O/EM65O - 0.47556) + \\
&\quad EF65OAS.ADJ) && (89) \\
EAS_R &= EF1617AS_R + EF1819AS_R + EF2024AS_R + EF2534AS_R + EF3544AS_R + EF4554AS_R + EF5564AS_R + \\
&\quad EF65OAS_R + EM1617AS_R + EM1819AS_R + EM2024AS_R + EM2534AS_R + EM3544AS_R + EM4554AS_R + \\
&\quad EM5564AS_R + EM65OAS_R && (97)
\end{aligned}$$

Final (scaled) equations:

$$\begin{aligned}
EM1617AS &= EM1617AS_R * (EAS/EAS_R) && (98) \\
EM1819AS &= EM1819AS_R * (EAS/EAS_R) && (105) \\
EM2024AS &= EM2024AS_R * (EAS/EAS_R) && (112) \\
EM2534AS &= EM2534AS_R * (EAS/EAS_R) && (119) \\
EM3544AS &= EM3544AS_R * (EAS/EAS_R) && (126) \\
EM4554AS &= EM4554AS_R * (EAS/EAS_R) && (133) \\
EM5564AS &= EM5564AS_R * (EAS/EAS_R) && (140) \\
EM65OAS &= EM65OAS_R * (EAS/EAS_R) && (147) \\
EF1617AS &= EF1617AS_R * (EAS/EAS_R) && (160) \\
EF1819AS &= EF1819AS_R * (EAS/EAS_R) && (167) \\
EF2024AS &= EF2024AS_R * (EAS/EAS_R) && (174) \\
EF2534AS &= EF2534AS_R * (EAS/EAS_R) && (181) \\
EF3544AS &= EF3544AS_R * (EAS/EAS_R) && (188) \\
EF4554AS &= EF4554AS_R * (EAS/EAS_R) && (195) \\
EF5564AS &= EF5564AS_R * (EAS/EAS_R) && (202) \\
EF65OAS &= EF65OAS_R * (EAS/EAS_R) && (209)
\end{aligned}$$

$$\begin{aligned}
EFAS &= EF1617AS + EF1819AS + EF2024AS + EF2534AS + EF3544AS + EF4554AS + EF5564AS + \\
EF65OAS &\quad (215) \\
EMAS &= EM1617AS + EM1819AS + EM2024AS + EM2534AS + EM3544AS + EM4554AS + EM5564AS + \\
EM65OAS &\quad (153)
\end{aligned}$$

Nonagricultural Private Household Wage Workers:

“Raw” equations (before scaling the totals): (229-244)

$$\begin{aligned}
EF1617NAWPH_R &= \text{MAX}(0.001, -0.20802 * \text{MOVAVG}(4, RTP.1) - 0.40988 * \text{MOVAVG}(4, RTP.5) + 0.01015 + 61.2465 * 1/YEAR - \\
&\quad 0.00965 * \text{MINW}/\text{CPIW_U} + 0.01561 * \text{NU10}/\text{NF1617} - 0.13398) * EF1617 + EF1617NAWPH.ADJ \\
EF1819NAWPH_R &= \text{MAX}(0.001, -0.03363 * \text{MOVAVG}(4, RTP.1) - 0.12989 * \text{MOVAVG}(4, RTP.5) - 0.00661 + 8.44701 * 1/YEAR - \\
&\quad 0.00539 * \text{MINW}/\text{CPIW_U} + 0.00345 * \text{NU10}/\text{NF1819} + 0.07597) * EF1819 + EF1819NAWPH.ADJ \\
EF2024NAWPH_R &= \text{MAX}(0.001, -0.18707 * \text{MOVAVG}(20, RTP.1) - 0.00223 + 2.12060 * 1/YEAR + 0.00820 * \text{NU10}/\text{NF2024} + \\
&\quad 0.14537) * EF2024 + EF2024NAWPH.ADJ \\
EF2534NAWPH_R &= \text{MAX}(0.001, 0.01874 * \text{MOVAVG}(4, RTP.1) - 0.04167 * \text{MOVAVG}(20, RTP.5) - 0.00090 + 1.55167 * 1/YEAR + \\
&\quad 0.01021 * \text{NU10}/\text{NF2534} - 0.00170) * EF2534 + EF2534NAWPH.ADJ \\
EF3544NAWPH_R &= (0.00622 * \text{MOVAVG}(4, RTP.1) - 0.06062 * \text{MOVAVG}(20, RTP.5) + 0.00008 + 0.29372 * \text{MOVAVG}(12, \\
&\quad EF2534NAWPH.36/EF2534.36) + 0.06187) * EF3544 + EF3544NAWPH.ADJ \\
EF4554NAWPH_R &= (0.02788 * \text{MOVAVG}(4, RTP.1) - 0.10996 * \text{MOVAVG}(20, RTP.5) - 0.00349 + 0.53068 * \text{MOVAVG}(12, \\
&\quad EF3544NAWPH.36/EF3544.36) + 0.08883) * EF4554 + EF4554NAWPH.ADJ \\
EF5564NAWPH_R &= (0.05939 * \text{MOVAVG}(4, RTP.1) - 0.10618 * \text{MOVAVG}(8, RTP.5) - 0.00579 + 0.66195 * \text{MOVAVG}(12, \\
&\quad EF4554NAWPH.36/EF4554.36) + 0.05966) * EF5564 + EF5564NAWPH.ADJ \\
EF65ONAWPH_R &= (0.22642 * \text{MOVAVG}(4, RTP.1) - 0.02069 + 0.33505 * \text{MOVAVG}(12, EF5564NAWPH.36) - 0.19707) + \\
&\quad EF65ONAWPH.ADJ \\
EM1617NAWPH_R &= \text{MAX}(0.001, -0.05284 * \text{MOVAVG}(4, RTP.1) - 0.17833 * \text{MOVAVG}(4, RTP.5) - 0.00768 + 9.19738 * 1/YEAR - \\
&\quad 0.00588 * \text{MINW}/\text{CPIW_U} + 0.16862) * EM1617 + EM1617NAWPH.ADJ \\
EM1819NAWPH_R &= \text{MAX}(0.001, -0.07122 * \text{MOVAVG}(4, RTP.1) - 0.03737 * \text{MOVAVG}(4, RTP.5) - 0.00282 + 3.76796 * 1/YEAR -
\end{aligned}$$

$$\begin{aligned}
EM2024NAWPH_R &= 0.00499 * MINW/CPIW_U + 0.08727) * EM1819 + EM1819NAWPH.ADJ \\
&MAX (0.001, -0.00450 * MOVAVG (4, RTP.1) - 0.02345 * MOVAVG (4, RTP.5) - 0.00113 - 0.00057 * \\
&MINW/CPIW_U + 0.03265) * EM2024 + EM2024NAWPH.ADJ \\
EM2534NAWPH_R &= MAX (0.001, -0.00490 * MOVAVG (4, RTP.5) - 0.00054 - 0.00051 * MINW/CPIW_U + 0.00789) * EM2534 + \\
&EM2534NAWPH.ADJ \\
EM3544NAWPH_R &= (-0.00446 * MOVAVG (4, RTP.5) - 0.00041 - 0.00053 * MINW/CPIW_U + 0.00726) * EM3544 + \\
&EM3544NAWPH.ADJ \\
EM4554NAWPH_R &= (-0.00039 + 0.00129) * EM4554 + EM4554NAWPH.ADJ \\
EM5564NAWPH_R &= (-0.00015 + 0.00200) * EM5564 + EM5564NAWPH.ADJ \\
EM65ONAWPH_R &= (-0.00679 + 0.64405 * MOVAVG (12, EM5564NAWPH.36) + 0.00231) + EM65ONAWPH.ADJ \\
ENAWPH_R &= EF1617NAWPH_R + EF1819NAWPH_R + EF2024NAWPH_R + EF2534NAWPH_R + EF3544NAWPH_R + \\
&EF4554NAWPH_R + EF5564NAWPH_R + EF65ONAWPH_R + EM1617NAWPH_R + EM1819NAWPH_R + \\
&EM2024NAWPH_R + EM2534NAWPH_R + EM3544NAWPH_R + EM4554NAWPH_R \\
&+ EM5564NAWPH_R + EM65ONAWPH_R \quad (245)
\end{aligned}$$

Total Private Household Wage Workers:

$$\begin{aligned}
ENAWPH &= IF LONGRANGE = 0 \\
&THEN ENAWPH_R \\
&ELSE ENAWPH.I * (E_FE/E_FE.1) \quad (246)
\end{aligned}$$

Final (scaled) equations:

Male (260-267)

$$\begin{aligned}
EM1617NAWPH &= EM1617NAWPH_R * (ENAWPH/ENAWPH_R) \\
EM1819NAWPH &= EM1819NAWPH_R * (ENAWPH/ENAWPH_R) \\
EM2024NAWPH &= EM2024NAWPH_R * (ENAWPH/ENAWPH_R) \\
EM2534NAWPH &= EM2534NAWPH_R * (ENAWPH/ENAWPH_R) \\
EM3544NAWPH &= EM3544NAWPH_R * (ENAWPH/ENAWPH_R) \\
EM4554NAWPH &= EM4554NAWPH_R * (ENAWPH/ENAWPH_R) \\
EM5564NAWPH &= EM5564NAWPH_R * (ENAWPH/ENAWPH_R) \\
EM65ONAWPH &= EM65ONAWPH_R * (ENAWPH/ENAWPH_R)
\end{aligned}$$

Female (247-254)

$$\begin{aligned}
EF1617NAWPH &= EF1617NAWPH_R * (ENAWPH/ENAWPH_R) \\
EF1819NAWPH &= EF1819NAWPH_R * (ENAWPH/ENAWPH_R) \\
EF2024NAWPH &= EF2024NAWPH_R * (ENAWPH/ENAWPH_R) \\
EF2534NAWPH &= EF2534NAWPH_R * (ENAWPH/ENAWPH_R) \\
EF3544NAWPH &= EF3544NAWPH_R * (ENAWPH/ENAWPH_R) \\
EF4554NAWPH &= EF4554NAWPH_R * (ENAWPH/ENAWPH_R) \\
EF5564NAWPH &= EF5564NAWPH_R * (ENAWPH/ENAWPH_R) \\
EF65ONAWPH &= EF65ONAWPH_R * (ENAWPH/ENAWPH_R)
\end{aligned}$$

$$\begin{aligned}
EFNAWPH &= EF1617NAWPH + EF1819NAWPH + EF2024NAWPH + EF2534NAWPH + EF3544NAWPH + EF4554NAWPH + \\
&EF5564NAWPH + EF65ONAWPH \quad (255)
\end{aligned}$$

$$\begin{aligned}
EMNAWPH &= EM1617NAWPH + EM1819NAWPH + EM2024NAWPH + EM2534NAWPH + EM3544NAWPH + EM4554NAWPH + \\
&EM5564NAWPH + EM65ONAWPH \quad (268)
\end{aligned}$$

OTHER EMPLOYMENT MEASURES

Federal Civilian Government and Government Enterprises

$$\begin{aligned}
EGFC &= IF LONGRANGE = 0 \\
&THEN (EGFC.1 * 1.0094^{0.25}) \\
&ELSE (EGFC.1 * (E_FE/E_FE.1)) \quad (257)
\end{aligned}$$

$$\begin{aligned}
EGFCPS &= IF LONGRANGE = 0 \\
&THEN (EGFCPS.1 * 1.0075^{0.25}) \\
&ELSE (EGFCPS.1 * (E_FE/E_FE.1)) \quad (256)
\end{aligned}$$

$$\begin{aligned}
EGGEFC &= EGFC + EGFCPS \quad (258)
\end{aligned}$$

State and Local Government and Government Enterprises

$$\begin{aligned}
EGGESL &= IF LONGRANGE = 0 \\
&THEN EGGESL.1 * (LC_FE/LC_FE.4)^{0.25} \\
&ELSE EGGESL.1 * (E_FE/E_FE.1) \quad (259)
\end{aligned}$$

Military

Decrease (if any) in EDMIL compared to a year ago

$$\text{DNEDMIL} = \begin{cases} \text{IF (EDMIL-EDMIL.4) < 0} \\ \text{THEN (EDMIL-EDMIL.4)} \\ \text{ELSE 0} \end{cases} \quad (228)$$

Private employment

$$\text{EP} = \text{E-EGGESL} - \text{EGGEFC} - \text{EAS} - \text{ENAS} \quad (269)$$

Compensation and Output Sectors

Price Deflator for Medical Services

$$\text{CPIWMS} = \text{CPIWMS.1} * (1 + ((\text{CPIW_U}/\text{CPIW_U.4})^{0.25} - 1) * \text{CPIWMSWT}) \quad (287)$$

Unemployment Insurance and Workers Compensation Effective Tax Rates

$$\text{TMAXUI_SL} = \text{TMAXUI_SL.1} * \text{AWSUI.1}/\text{AWSUI.2} \quad (404)$$

$$\text{RELMAX_UI} = \text{TMAXUI_SL}/\text{AWSUI.1}/1000 \quad (405)$$

$$\text{CR_UI} = 0.775 \quad (402)$$

$$\text{TRATIO_UI} = 0.96996 * \text{RELMAX_UI} - 0.13744 * \text{MOVAVG}(4, \text{RTP.1}) + 0.10368 * \text{MOVAVG}(4, \text{RTP.5}) + 0.04887 \quad (406)$$

$$\text{TRATE_UI} = 0.00143 * \text{MOVAVG}(4, \text{RU.5}) + 0.00128 * \text{MOVAVG}(4, \text{RU.9}) + 0.00057 * \text{MOVAVG}(4, \text{RU.13}) + 0.00356 \quad (407)$$

$$\text{RUIWS1} = \text{CR_UI} * \text{TRATIO_UI} * \text{TRATE_UI} \quad (408)$$

$$\text{RUIWS2} = 0.32476 * \text{MOVAVG}(4, \text{RUIWS1.8} * (\text{WSP.8} - \text{WSPRRB.8} + \text{WSGGESL.8})) / (\text{WSP.1} - \text{WSPRR2.1} + \text{WSGGESL.1}) \quad (409)$$

Workers' Compensation

$$\text{RWCWS} = \text{RWCWS.1} - (\text{RWCWS.1} - 0.0144)/12 \quad (311)$$

Wages

Average lagged private-sector wage

$$\text{AWSPL} = \text{MOVAVG}(8, \text{AWSP.1}) \quad (272)$$

Average lagged private-sector compensation

$$\text{AWSSPL} = \text{MOVAVG}(8, \text{AWSSP.1}) \quad (343)$$

Average wage in state & local government (incl. gov't enterprises)

$$\text{AWSGGESL} = \begin{cases} \text{IF LONGRANGE} = 0 \\ \text{THEN AWSGGESL.1} * \text{AWSPL}/\text{AWSPL.1} \\ \text{ELSE AWSGGESL.1} * \text{AVG_GDP}/\text{AVG_GDP.1} * (1 + \text{WS_TO_WSS_D}/100)^{0.25} \end{cases} \quad (273)$$

Total wages in state & local government (incl. gov't enterprises)

$$\text{WSGGESL} = \text{AWSGGESL} * \text{EGGESL} \quad (274)$$

Employer Contribution for Government Social Insurance in State & Local Government Sector

$$\text{OASDISL_L} = (\text{EMPTRASI} + \text{EMPTRDI}) * 0.978 * \text{CSLA} * \text{WSGGESL} \quad (307)$$

$$\text{HISL_L} = \text{EMPTRHI} * 1.0 * \text{CSLHI} * \text{WSGGESL} \quad (308)$$

$$\text{SOC_UISL} = (-0.02821 * \text{MOVAVG}(4, \text{RTP.2}) + 0.03145) * \text{WSGGESL} \quad (309)$$

$$\text{RSOCSL_WC} = \text{RSOCSL_WC.1} - (\text{RSOCSL_WC.1} - 0.176)/12 \quad (310)$$

$$\text{SOC_WCSL} = \text{RSOCSL_WC} * \text{RWCWS} * \text{WSGGESL} \quad (312)$$

$$\text{SOC_SL} = (\text{OASDISL_L} + \text{HISL_L} + \text{SOC_UISL} + \text{SOC_WCSL}) \quad (313)$$

Employer Contributions for Employee Pension and Insurance funds in State & Local Government Sector

Workers' Compensation - employees and annuitants

$$\text{OLI_WCSL} = (1 - \text{RSOCSL_WC}) * \text{RWCWS} * \text{WSGGESL} \quad (316)$$

Pensions

$$\text{OLI_RETSL} = \text{WSGGESL} * (\text{OLI_RETSL.1}/\text{WSGGESL.1}) \quad (317)$$

Life Insurance - employees and annuitants
 $OLI_GLI_SL = 2.0 * EGGESL * ((WSGGESL/EGGESL) + 2.0) * 0.075 * 26/1000$ (314)

Health Insurance - employees and annuitants
 $OLI_GHI_SL = (OLI_GHI_SL.1 / EGGESL.1) * CPIWMS/CPIWMS.1 * EGGESL * RGR_GHI$ (315)

Total
 $OLI_SL = (OLI_GLI_SL + OLI_GHI_SL + OLI_WCSL + OLI_RETSL)$ (318)

$RCWSSL = (1 + (SOC_SL + OLI_SL)/WSGGESL)$ (319)

$WSGGESL = IF\ LONGRANGE = 0$
 THEN $RCWSSL * WSGGESL$
 ELSE $(WSGGESL.1/EGGESL.1) * AVG_GDP/AVG_GDP.1 * EGGESL$ (320)

$WSSGSL = WSGGESL * WSSGSL.1/WSGGESL.1$ (321)

$WSSGESL = WSGGESL - WSSGSL$ (324)

$CFCGSL = IF\ LONGRANGE = 0$
 THEN $WSSGSL * RCFCGSL$
 ELSE $CFCGSL.1 * WSGGESL/WSSGGESL.1$ (322)

$GDPGSL = WSSGSL + CFCGSL$ (323)

$CFCGESL = IF\ LONGRANGE = 0$
 THEN $WSSGESL * RCFCGESL$
 ELSE $CFCGESL.1 * WSGGESL/WSSGGESL.1$ (325)

$GDPGESL = WSSGESL + CFCGESL$ (326)

$GDPGGESL = GDPGSL + GDPGESL$ (327)

Federal Civilian General Government and Government Enterprises

Wages

General Government and Government Enterprises

Civilian pay raise

$CRAZI = IF\ LONGRANGE = 0$
 THEN $((IF\ QTR = 1\ THEN\ (0.82429 * (AWSP.6/AWSP.10 - 1) - 0.005)\ ELSE\ 0))$
 ELSE $(IF\ QTR = 1\ THEN\ (AWSP.6/AWSP.10 - 1)\ ELSE\ 0)$ (270)

Military pay raise

$MRAZ = IF\ LONGRANGE = 0$
 THEN $((IF\ QTR = 1\ THEN\ (0.82429 * (AWSP.6/AWSP.10 - 1) - 0.005)\ ELSE\ 0))$
 ELSE $(IF\ QTR = 1\ THEN\ (AWSP.6/AWSP.10 - 1)\ ELSE\ 0)$ (277)

Average wage in Federal Civilian Government

$AWSGGEFC = IF\ LONGRANGE = 0$
 THEN $(AWSGGEFC.1 * (1 + 1.0 * CRAZI + 0.0015))$
 ELSE $AWSGGEFC.1 * AVG_GDP/AVG_GDP.1 * (1 + WS_TO_WSS_D/100)^{0.25}$ (275)

Total wages in FCG

$WSGGEFC = AWSGGEFC * EGGEFC$ (276)

CSRS workers

$AWEFC_N = IF\ LONGRANGE = 0$
 THEN $(AWEFC_N.1 * (1 + 1.0 * CRAZI + 0.00082))$
 ELSE $AWEFC_N.1 * AVG_GDP/AVG_GDP.1 * (1 + WS_TO_WSS_D/100)^{0.25}$ (271)

$WEFC_N = AWEFC_N * TEFC_N$ (283)

Government Enterprises (Mostly U.S. Postal Service)

$AWSGEFC = IF\ LONGRANGE = 0$
 THEN $(AWSGEFC.1 * (1 + 1.0 * CRAZI + 0.0015))$
 ELSE $AWSGEFC.1 * AVG_GDP/AVG_GDP.1 * (1 + WS_TO_WSS_D/100)^{0.25}$ (290)

$WSGEFC = AWSGEFC * EGEFCPS$ (291)

General Government

$WSGFC = WSGGEFC - WSGEFC$ (292)

$AWSGFC = WSGFC/EGFC$ (378)

Employer Contribution for Government Social Insurance

General Government and Government Enterprises

$$\begin{aligned} \text{OASDIFC_L} &= (\text{EMPTRASI} + \text{EMPTRDI}) * 1.04 * (\text{WSGGFC} - \text{WEFC_N}) * \text{ADJ_FSA_FC} & (284) \\ \text{HIFC_L} &= \text{EMPTRHI} * 1.055 * \text{WSGGFC} * \text{ADJ_FSA_FC} & (285) \\ \text{SOCF_UIFC} &= (-0.05934 * \text{RTP} + 0.06165) * \text{WSGGFC} & (281) \\ \text{SOCF_WC} &= 0.0159 * \text{WSGGFC} & (282) \\ \text{SOC_FC} &= (\text{SOCF_UIFC} + \text{SOCF_WC} + \text{OASDIFC_L} + \text{HIFC_L}) & (286) \end{aligned}$$

Employer Contributions for Employee Pension and Insurance funds

General Government and Government Enterprises

Pensions

$$\begin{aligned} \text{OLI_CSRS1} &= ((0.174 * \text{WSGGFC} + 0.07 * \text{WSGFC}) / \text{WSGGFC}) * \text{WEFC_N} & (293) \\ \text{OLI_FERS1} &= 0.107 * (\text{WSGGFC} * 0.9 - \text{WEFC_N}) & (294) \\ \text{OLI_FERSFC} &= 0.048 * (\text{WSGGFC} * 0.9 - \text{WEFC_N}) & (295) \\ \text{OLI_RETFC} &= \text{OLI_CSRS1} + \text{OLI_FERS1} + \text{OLI_FERSFC} + \text{OLIF_RETFCO} & (296) \end{aligned}$$

Life Insurance - employees and annuitants

$$\text{OLI_GLI_FC} = 2.0 * \text{EGGFC} * ((\text{WSGGFC}/\text{EGGFC}) + 2.0) * 0.075 * 26/1000 \quad (289)$$

Health Insurance - employees and annuitants

$$\begin{aligned} \text{OLI_GHI_FC} &= (\text{OLI_GHI_FC.1} / \text{EGGFC.1}) * \text{CPIWMS}/\text{CPIWMS.1} * \text{EGGFC} * \text{RGR_GHI} & (288) \\ \text{OLI_FC} &= (\text{OLI_GHI_FC} + \text{OLI_GLI_FC} + \text{OLI_RETFC}) & (297) \end{aligned}$$

Compensation

General Government and Government Enterprises

$$\begin{aligned} \text{RCWSF} &= (1 + (\text{SOC_FC} + \text{OLI_FC})/\text{WSGGFC}) & (298) \\ \text{WSSGGFC} &= \text{IF LONGRANGE} = 0 & \\ & \quad \text{THEN RCWSF} * \text{WSGGFC} & \\ & \quad \text{ELSE } (\text{WSSGGFC.1}/\text{EGGFC.1}) * \text{AVG_GDP}/\text{AVG_GDP.1} * \text{EGGFC} & (300) \\ \text{WSSGFC} &= \text{IF LONGRANGE} = 0 & \\ & \quad \text{THEN RCWSF} * \text{WSGFC} & \\ & \quad \text{ELSE } (\text{WSSGFC.1} / (\text{EGGFC.1} - \text{EGEFCPS.1})) * \text{AVG_GDP}/\text{AVG_GDP.1} * (\text{EGGFC} - \text{EGEFCPS}) & (299) \\ \text{WSSGEFC} &= \text{IF LONGRANGE} = 0 & \\ & \quad \text{THEN RCWSF} * \text{WSGEFC} & \\ & \quad \text{ELSE } (\text{WSSGEFC.1}/\text{EGEFCPS.1}) * \text{AVG_GDP}/\text{AVG_GDP.1} * \text{EGEFCPS} & (303) \end{aligned}$$

Consumption of Fixed Capital

General Government and Government Enterprises

$$\begin{aligned} \text{CFCGFC} &= \text{IF LONGRANGE} = 0 & \\ & \quad \text{THEN WSSGFC} * \text{RCFCGFC} & \\ & \quad \text{ELSE } \text{CFCGFC.1} * \text{WSSGGFC} / \text{WSSGGFC.1} & (301) \\ \text{CFCGEFC} &= \text{IF LONGRANGE} = 0 & \\ & \quad \text{THEN WSSGEFC} * \text{RCFCGEFC} & \\ & \quad \text{ELSE } \text{CFCGEFC.1} * \text{WSSGGFC} / \text{WSSGGFC.1} & (304) \end{aligned}$$

Gross Domestic Product

General Government and Government Enterprises

$$\begin{aligned} \text{GDPGFC} &= \text{WSSGFC} + \text{CFCGFC} & (302) \\ \text{GDPGEFC} &= \text{WSSGEFC} + \text{CFCGEFC} & (305) \\ \text{GDPGGEFC} &= \text{GDPGFC} + \text{GDPGEFC} & (306) \end{aligned}$$

Federal Government Military

Wages

$$\begin{aligned} \text{AWSGFM} &= \text{IF LONGRANGE} = 0 & \\ & \quad \text{THEN } (\text{AWSGFM.1} * (1.0027 + 1.0 * \text{MRAZ})) & \\ & \quad \text{ELSE } \text{AWSGFM.1} * \text{AVG_GDP}/\text{AVG_GDP.1} * (1 + \text{WS_TO_WSS_D}/100)^{0.25} & (278) \end{aligned}$$

$$\text{WSGFM} = \text{AWSGFM} * (\text{EDMIL} + \text{EDMIL_R}) \quad (279)$$

Employer Contribution for Government Social Insurance

$$\text{OASDIFM_L} = (\text{EMPTRASI} + \text{EMPTRDI}) * 0.9975 * \text{CML} * \text{WSGFM} \quad (331)$$

$$\text{HIFM_L} = \text{EMPTRHI} * 1.0 * \text{CML} * \text{WSGFM} \quad (332)$$

$$\text{SOCF_UIFM} = \text{MAX}(0.001, (-0.05263 * \text{DIFF}(\text{EDMIL} + \text{EDMIL_R}) - 0.03079 * \text{RTP} + 0.03310)) * \text{WSGFM} \quad (329)$$

$$\text{SOCF_MIFM} = 0.30 * \text{CPIWMS} * (\text{EDMIL} + \text{EDMIL_R}) \quad (330)$$

$$\text{SOC_FM} = (\text{SOCF_UIFM} + \text{SOCF_MIFM} + \text{OASDIFM_L} + \text{HIFM_L}) \quad (333)$$

Employer Contributions for Employee Pension and Insurance funds

$$\text{OLI_RETFM} = (\text{OLI_RETFM.1} / \text{WSGFM.1} - (\text{OLI_RETFM.1} / \text{WSGFM.1} - 0.472) / 12) * \text{WSGFM} \quad (328)$$

Compensation

$$\text{RCWSM} = (1 + (\text{OLI_RETFM} + \text{SOC_FM}) / \text{WSGFM}) \quad (334)$$

$$\text{WSSGFM} = \begin{cases} \text{IF LONGRANGE} = 0 \\ \text{THEN RCWSM} * \text{WSGFM} \\ \text{ELSE } (\text{WSSGFM.1} / \text{EDMIL.1}) * \text{AVG_GDP} / \text{AVG_GDP.1} * \text{EDMIL} \end{cases} \quad (335)$$

Consumption of Fixed Capital

$$\text{CFCGFM} = \begin{cases} \text{IF LONGRANGE} = 0 \\ \text{THEN WSSGFM} * \text{RCFCGFM} \\ \text{ELSE CFCGFM.1} * \text{WSSGFM} / \text{WSSGFM.1} \end{cases} \quad (336)$$

Gross Domestic Product

$$\text{GDPGFM} = \text{WSSGFM} + \text{CFCGFM} \quad (337)$$

$$\text{GDPGF} = \text{GDPGFC} + \text{GDPGFM} \quad (387)$$

$$\text{GDPGGE} = \text{GDPGGEFC} + \text{GDPGGESE} + \text{GDPGFM} \quad (338)$$

Total (Civilian and Military) Federal General Government and Government Enterprises

$$\text{WSSGF} = \text{WSSGFC} + \text{WSSGFM} \quad (392)$$

$$\text{WSSGE} = \text{WSSGEFC} + \text{WSSGESL} \quad (394)$$

$$\text{WSSG} = \text{WSSGF} + \text{WSSGSL} \quad (393)$$

$$\text{GDPGE} = \text{GDPGEFC} + \text{GDPGESL} \quad (389)$$

$$\text{GDPG} = \text{GDPGF} + \text{GDPGSL} \quad (388)$$

NIPA Farm Output and Earnings

Real farm output

$$\text{GDPPF09} = \begin{cases} \text{IF LONGRANGE} = 0 \\ \text{THEN EXP}(-3.52340 + 0.02055 * \text{YEAR}) * \text{N_SSA} * 1.125 * 1.138 \\ \text{ELSE GDPPF09.1} * \text{GDP09} / \text{GDP09.1} \end{cases} \quad (19)$$

Farm sector deflator

$$\text{PGDPAF} = \begin{cases} \text{IF LONGRANGE} = 0 \\ \text{THEN PGDPAF.1} * ((\text{PGDP} / \text{PGDP.1})^4 - 0.01)^{0.25} \\ \text{ELSE PGDPAF.1} * ((\text{PGDP} / \text{PGDP.1})^4)^{0.25} \end{cases} \quad (223)$$

Nominal farm output

$$\text{GDPPF} = \text{GDPPF09} * \text{PGDPAF} \quad (339)$$

Farm compensation and wages

$$\text{WSSPF} = \begin{cases} \text{IF LONGRANGE} = 0 \\ \text{THEN EAW} * \text{MOVAVG}(4, \text{WSSP.2} / \text{EP.2}) * (3.15749 / (\text{YEAR} - 65) - 0.43419 * \text{RTP} + 0.68725) \\ \text{ELSE } (\text{WSSPF.1} / \text{EAW.1}) * \text{AVG_GDP} / \text{AVG_GDP.1} * \text{EAW} \end{cases} \quad (360)$$

$$\text{WSPF} = \begin{cases} \text{IF LONGRANGE} = 0 \\ \text{THEN WSSPF} * (\text{MOVAVG}(12, (\text{WSP.1} / \text{WSSP.1})) + 0.015) \\ \text{ELSE } (\text{WSPF.1} / \text{WSSPF.1}) * (\text{WSP.1} / \text{WSSP.1}) / (\text{WSP.2} / \text{WSSP.2}) * \text{WSSPF} \end{cases} \quad (381)$$

$$\text{AWSPF} = \text{WSPF} / \text{EAW} \quad (382)$$

Farm proprietors' income

$$\text{AYF_K} = ((\text{YF.1} / \text{EAS.1}) / (\text{WSSPF.1} / \text{EAW.1}) - 5.0) * 0.8 + 5.0 \quad (395)$$

$$\text{YF} = \text{AYF_K} * (\text{WSSPF} / \text{EAW}) * \text{EAS} \quad (396)$$

GDP, WSS and WS, Private Households & Nonprofit Institutions

Private Households

Compensation & Wages

$$\begin{aligned} \text{WSSPH} &= \text{IF LONGRANGE} = 0 \\ &\quad \text{THEN } (((\text{WSSPH.1}/\text{ENAWPH.1})/\text{MOVAVG}(4, \text{WSSP.3}/\text{EP.3}) - 0.41) * 0.875 + 0.41) \\ &\quad \quad * \text{MOVAVG}(4, \text{WSSP.2}/\text{EP.2}) * \text{ENAWPH} \\ &\quad \text{ELSE } (\text{AVG_GDP}/\text{AVG_GDP.1}) * \text{ENAWPH} * (\text{WSSPH.1}/\text{ENAWPH.1}) \end{aligned} \quad (340)$$

$$\begin{aligned} \text{WSPH} &= \text{IF LONGRANGE} = 0 \\ &\quad \text{THEN } \text{WSSPH} / (1 + \text{CPH} * 1 * (\text{EMPTROASI} + \text{EMPTRDI} + \text{EMPTRHI})) \\ &\quad \text{ELSE } (\text{AWSPH.1} * \text{ENAWPH.1}/\text{WSSPH.1}) * (1 + \text{WS_TO_WSS_D}/100)^{0.25} * \text{WSSPH} \end{aligned} \quad (383)$$

$$\text{AWSPH} = \text{WSPH} / \text{ENAWPH} \quad (384)$$

Owner Occupied Housing

$$\text{OOH} = \text{OOH.1} * (\text{KGDP09} * \text{PGDP}) / (\text{KGDP09.1} * \text{PGDP.1}) \quad (341)$$

Gross Value Added

$$\begin{aligned} \text{GDPPH} &= \text{IF LONGRANGE} = 0 \\ &\quad \text{THEN } \text{WSSPH} + \text{OOH} \\ &\quad \text{ELSE } (\text{AVG_GDP}/\text{AVG_GDP.1}) * \text{ENAWPH} * (\text{GDPPH.1}/\text{ENAWPH.1}) \end{aligned} \quad (342)$$

Nonprofit Institutions

Health Services

$$\begin{aligned} \text{EPHS_EST} &= \text{IF LONGRANGE} = 0 \\ &\quad \text{THEN } \text{EPHS_EST.1} + 0.275/4 \\ &\quad \text{ELSE } \text{EPHS_EST.1} * (\text{E_FE}/\text{E_FE.1}) \end{aligned} \quad (345)$$

$$\begin{aligned} \text{AWSSPHS} &= \text{IF LONGRANGE} = 0 \\ &\quad \text{THEN } \text{AWSSPHS.1} * \text{AWSSPL}/\text{AWSSPL.1} \\ &\quad \text{ELSE } \text{AWSSPHS.1} * \text{AVG_GDP}/\text{AVG_GDP.1} \end{aligned} \quad (344)$$

$$\text{WSSPHS} = \text{AWSSPHS} * \text{EPHS_EST} \quad (346)$$

Educational Services

$$\begin{aligned} \text{EPES_EST} &= \text{IF LONGRANGE} = 0 \\ &\quad \text{THEN } \text{EPES_EST.1} + 0.075/4 \\ &\quad \text{ELSE } \text{EPES_EST.1} * (\text{E_FE}/\text{E_FE.1}) \end{aligned} \quad (348)$$

$$\begin{aligned} \text{AWSSPES} &= \text{IF LONGRANGE} = 0 \\ &\quad \text{THEN } \text{AWSSPES.1} * \text{AWSSPL}/\text{AWSSPL.1} \\ &\quad \text{ELSE } \text{AWSSPES.1} * \text{AVG_GDP}/\text{AVG_GDP.1} \end{aligned} \quad (347)$$

$$\text{WSSPES} = \text{AWSSPES} * \text{EPES_EST} \quad (349)$$

Social Services

$$\begin{aligned} \text{EPSS_EST} &= \text{IF LONGRANGE} = 0 \\ &\quad \text{THEN } \text{EPSS_EST.1} + 0.075/4 \\ &\quad \text{ELSE } \text{EPSS_EST.1} * (\text{E_FE}/\text{E_FE.1}) \end{aligned} \quad (351)$$

$$\begin{aligned} \text{AWSSPSS} &= \text{IF LONGRANGE} = 0 \\ &\quad \text{THEN } \text{AWSSPSS.1} * \text{AWSSPL}/\text{AWSSPL.1} \\ &\quad \text{ELSE } \text{AWSSPSS.1} * \text{AVG_GDP}/\text{AVG_GDP.1} \end{aligned} \quad (350)$$

$$\text{WSSPSS} = \text{AWSSPSS} * \text{EPSS_EST} \quad (352)$$

Gross Value Added

$$\begin{aligned} \text{WSSPNI} &= \text{WSSPNI.1} * (\text{WSSPHS} + \text{WSSPES} + \text{WSSPSS}) / (\text{WSSPHS.1} + \text{WSSPES.1} + \text{WSSPSS.1}) \end{aligned} \quad (353)$$

$$\begin{aligned} \text{WSPNI} &= \text{IF LONGRANGE} = 0 \\ &\quad \text{THEN } \text{WSSPNI} * (\text{WSPNI.1}/\text{WSSPNI.1}) * ((\text{WSP.1}/\text{WSSP.1}) / (\text{WSP.9}/\text{WSSP.9}))^{(1/8)} \\ &\quad \text{ELSE } \text{WSSPNI} * (\text{WSPNI.1}/\text{WSSPNI.1}) * (1 + \text{WS_TO_WSS_D}/100)^{0.25} \end{aligned} \quad (391)$$

$$\begin{aligned} \text{GDPPNI} &= \text{IF LONGRANGE} = 0 \\ &\quad \text{THEN } \text{WSSPNI} / ((\text{WSSPNI.1}/\text{GDPPNI.1} - 0.866) * 0.8 + 0.866) \\ &\quad \text{ELSE } \text{WSSPNI} / 0.866 \end{aligned} \quad (354)$$

Private Output and Compensation

$$\begin{aligned} \text{ROADIP_L} &= (\text{EMPTROASI} + \text{EMPTRDI}) * \text{TXRP} * \text{CP} \quad (365) \\ \text{RHIP_L} &= \text{EMPTRHI} * 1.0 * \text{CP} \quad (366) \\ \text{RSOC_UIP} &= 0.00109 * \text{MOVAVG}(4, \text{RU.2}) + 0.00045 * \text{MOVAVG}(4, \text{RU.10}) + 0.00048 * \text{MOVAVG}(4, \text{RU.18}) - 0.00331 \quad (367) \end{aligned}$$

RSOC_WCP = RWCWS * RSOC_SL_WC (368)
 RSOCF_PBG = 0.00022 (369)

OLI
 ROLI_WCP = RWCWS * (1 - RSOC_SL_WC) (370)
 ROLI_SU = 0.0005 (371)
 OLI_GLI_P = 0.0025 * EP * AWSP.1 (363)
 OLI_GHI_P = (OLI_GHI_P.1 / EP.1) * CPIWMS / CPIWMS.1 * EP * RGR_GHI (364)
 ROLI_PPPS = MAX (ROLI_PPPS.1, 0.00031 * YEAR + 0.00866) (372)

Employee Compensation and Nonfarm Proprietor Income (WSS and YNF)

WSSGGE = (WSSGGESL + WSSGGEFC + WSSGFM) (374)
 GDPPBNFXGE = (GDP - GDPGGE - GDPPF - GDPPH - GDPPNI) (355)
 RWSSPBNFXGE = 0.30026 * RTP.1 + 0.31936 + (0.6038 - (0.30026 * 1.0 + 0.31936)) + RWSSPBNFXGE_ADJ (280)
 ENAW = ENA - ENAS - ENAU (356)
 ENAWPBXGE = ENAW - (ENAWPH + EGGEFC + EGGESL + WSSPNI / (WSSPHS + WSSPES + WSSPSS) * (EPHS_EST + EPES_EST + EPSS_EST)) (357)
 ENAWSPBXGE = ENAWPBXGE + ENAS (403)
 AYNF_K = ((YNF.1 / ENAS.1 / (WSSPBNFXGE.1 / ENAWPBXGE.1)) - 1.65) * 0.9 + 1.65 (401)
 AYF = YF / EAS (399)
 AWSSPF = WSSPF / EAW (398)
 AYNF = YNF / ENAS (400)
 AWSSPBNFXGE = WSSPBNFXGE / ENAWPBXGE (397)
 YNF = YNF.1 * (GDPPBNFXGE / GDPPBNFXGE.1) * (ENAS / (ENAS + ENAWPBXGE)) / (ENAS.1 / (ENAS.1 + ENAWPBXGE.1)) (358)
 WSSPBNFXGE = RWSSPBNFXGE * (GDPPBNFXGE - YNF) (359)
 WSSP = WSSPBNFXGE + WSSPF + WSSPH + WSSPNI (361)
 RCWSP = WSSP / (WSSP - SOCF_RETRR - OLI_GLI_P - OLI_GHI_P) * (1 + ROASDIP_L + RHIP_L + RSOC_UIP + RSOC_WCP + RSOCF_PBG + ROLI_WCP + ROLI_SU + ROLI_PPPS) (373)
 WS = IF WS_TO_WSS_DYR = 0
 THEN (WSGGESL + WSGGEFC + WSGFM + WSSP / RCWSP)
 ELSE WSS * WS.1 / WSS.1 * (1 + WS_TO_WSS_D / 100)^{0.25} (376)
 WSD = WS (410)
 WSP = (WS - WSGGESL - WSGGEFC - WSGFM) (379)
 AWSP = WSP / EP (380)
 AWSSP = WSSP / EP (385)

Other Variables

WSDP = (WSD - WSGGESL - WSGGEFC - WSGFM) (411)
 AWSE = WS / (E + EDMIL - EAS - ENAS) (377)
 AWSUI = (WS - WSGGEFC - WSGFM) / (E - EGGEFC - EAS - ENAS) (386)
 WSS = (WSSP + WSSGGE) (375)
 OLI_GGE = OLI_FC + OLI_SL + OLI_RETFM (412)
 OLI_WCP = ROLI_WCP * WSP (413)
 OLI_SU = ROLI_SU * WSP (414)
 OLI_PPPS = ROLI_PPPS * WSP (415)
 OLI_P = OLI_WCP + OLI_SU + OLI_GHI_P + OLI_GLI_P + OLI_PPPS (416)
 OLI = OLI_GGE + OLI_P (417)
 SOC_GGE = SOC_FC + SOC_FM + SOC_SL (418)
 SOC_UIP = RSOC_UIP * WSP (419)
 SOC_WCP = RSOC_WCP * WSP (420)
 SOCF_PBG = RSOCF_PBG * WSP (423)
 SOCF_RETRR = 0.20 * WSPRRB (362)
 SOC_P = SOC_UIP + SOC_WCP + OASDIP_L + HIP_L + SOCF_PBG + SOCF_RETRR (424)
 SOC = SOC_GGE + SOC_P (425)
 OASDIP_L = ROASDIP_L * WSP (421)
 HIP_L = RHIP_L * WSP (422)
 OLI_PPS = OLI_PPPS + OLI_RETFC + OLI_RETFM + OLI_RETSL (426)
 OLI_GHI = OLI_GHI_P + OLI_GHI_FC + OLI_GHI_SL (428)
 OLI_GLI = OLI_GLI_P + OLI_GLI_FC + OLI_GLI_SL (429)
 OLI_WC = OLI_WCP + OLI_WCSL (427)
 SOCSL_WC = SOC_WCSL + SOC_WCP (430)
 SOCF_UIFED = SOCF_UIFC + SOCF_UIFM (431)
 SOCF_UIS = (SOC_UIP + SOC_UISL) * RUIWS1 / (RUIWS1 + RUIWS2) (432)
 SOCF_UIF = (SOC_UIP + SOC_UISL) - SOCF_UIS (433)
 SOCF_OASDI = OASDIP_L + OASDISL_L + OASDIFC_L + OASDIFM_L (434)

```
SOCF_HI = HIP_L + HISL_L + HIFC_L + HIFM_L (435)
TAXMAX = IF (first quarter of the year) THEN
          300 *NINT(0.5+MOVAVG(4,AWSE.5)/MOVAVG(4,AWSE.9)*1000*TAXMAX.1/300)/1000
        ELSE TAXMAX.1 (390)
```


2.3 OASDI Covered Employment and Earnings (MODSOLA)

Total At-Any-Time Employment (Equations 1–52)

Ages 0 through 15, where s=sex;a=age 0,1,2,3,...15; i=calendar year (1–10, 21, 22, 27–36, 47, 48)

$$\begin{aligned} \text{he_m_sy}(s,a,i) = & (\text{he_m_sy}(s,a,\text{histend}) / \text{nsy_a}(s,a,\text{histend}) \& \\ & + \text{he_m_sy}(s,a,\text{histend}-1) / \text{nsy_a}(s,a,\text{histend}-1) \& \\ & + \text{he_m_sy}(s,a,\text{histend}-2) / \text{nsy_a}(s,a,\text{histend}-2)) / 3 \& \\ & * \text{nsy_a}(s,a,i) \end{aligned}$$

HI covered workers age groups 10-13 and 14-15, by sex and calendar year (21, 22, 47, 48)

$$\begin{aligned} \text{he_m_1013}(s,i) = & \text{sum}(\text{he_m_sy}(s,10:13,i)) \\ \text{he_m_1415}(s,i) = & \text{sum}(\text{he_m_sy}(s,14:15,i)) \end{aligned}$$

HI covered workers age group 15u, by sex and calendar year (23, 49)

$$\text{he_m_15u}(s,i) = \text{sum}(\text{he_m_sy}(s,0:9,i)) + \text{he_m_1013}(s,i) + \text{he_m_1415}(s,i)$$

OASDI covered = HI covered, by sex, single year of age, and calendar year (11–22, 37–48)

$$\text{ce_m_sy}(s,a,i) = \text{he_m}(s,a,i)$$

OASDI covered workers age groups 10-13 and 14-15, by sex and calendar year (24, 25, 50, 51)

$$\begin{aligned} \text{ce_m_1013}(s,i) = & \text{sum}(\text{ce_m_sy}(s,10:13,i)) \\ \text{ce_m_1415}(s,i) = & \text{sum}(\text{ce_m_sy}(s,14:15,i)) \end{aligned}$$

OASDI covered workers age group 15u, by sex and calendar year (26, 52)

$$\text{ce_m_15u}(s,i) = \text{sum}(\text{ce_m_sy}(s,0:9,i)) + \text{ce_m_1013}(s,i) + \text{ce_m_1415}(s,i)$$

Male Disaggregates Aged 16 and Over

Preliminary

Average Weeks Worked

$$\begin{aligned} \text{AWWM1617_P} &= 0.35417 * \text{TREND_TE} - 0.17568 * \text{RM1617} + 8.80229; & (53) \\ \text{AWWM1819_P} &= 0.28021 * \text{TREND_TE} - 0.16633 * \text{RM1819} + 15.4522; & (58) \\ \text{AWWM2024_P} &= 0.17338 * \text{TREND_TE} - 0.32843 * \text{RM2024} + 31.4779; & (63) \end{aligned}$$

$$\begin{aligned} \text{AWWM2529_P} &= 0.04353 * \text{TREND_TE} - 0.32898 * \text{RM2529} + 46.8304; & (68) \\ \text{AWWM3034_P} &= 0.04353 * \text{TREND_TE} - 0.32898 * \text{RM3034} + 46.8304; & (73) \\ \text{AWWM3539_P} &= 0.03256 * \text{TREND_TE} - 0.26573 * \text{RM3539} + 47.9512; & (78) \\ \text{AWWM4044_P} &= 0.03256 * \text{TREND_TE} - 0.26573 * \text{RM4044} + 47.9512; & (83) \\ \text{AWWM4549_P} &= 0.03923 * \text{TREND_TE} - 0.23146 * \text{RM4549} + 46.8025; & (88) \\ \text{AWWM5054_P} &= 0.03923 * \text{TREND_TE} - 0.23146 * \text{RM5054} + 46.8025; & (93) \end{aligned}$$

$$\begin{aligned} \text{AWWM5559_P} &= 0.04481 * \text{TREND_TE} - 0.28537 * \text{RM5559} + 45.0196; & (98) \\ \text{AWWM6064_P} &= 0.14488 * \text{TREND_TE} + 0.01910 * \text{RM6064} + 29.5797; & (103) \\ \text{AWWM6569_P} &= 0.21483 * \text{TREND_TE} - 0.23366 * \text{RM6569} + 21.1343; & (108) \\ \text{AWWM700_P} &= 0.09069 * \text{TREND_TE} + 0.43206 * \text{RM700} + 28.6903; & (113) \end{aligned}$$

$$\begin{aligned} \text{AWWM1617_PL} &= 0.35417 * \text{TREND_TE.1} - 0.17568 * \text{RM1617.1} + 8.80229; & (55) \\ \text{AWWM1819_PL} &= 0.28021 * \text{TREND_TE.1} - 0.16633 * \text{RM1819.1} + 15.4522; & (60) \\ \text{AWWM2024_PL} &= 0.17338 * \text{TREND_TE.1} - 0.32843 * \text{RM2024.1} + 31.4779; & (65) \end{aligned}$$

$$\begin{aligned} \text{AWWM2529_PL} &= 0.04353 * \text{TREND_TE.1} - 0.32898 * \text{RM2529.1} + 46.8304; & (70) \\ \text{AWWM3034_PL} &= 0.04353 * \text{TREND_TE.1} - 0.32898 * \text{RM3034.1} + 46.8304; & (75) \\ \text{AWWM3539_PL} &= 0.03256 * \text{TREND_TE.1} - 0.26573 * \text{RM3539.1} + 47.9512; & (80) \\ \text{AWWM4044_PL} &= 0.03256 * \text{TREND_TE.1} - 0.26573 * \text{RM4044.1} + 47.9512; & (85) \\ \text{AWWM4549_PL} &= 0.03923 * \text{TREND_TE.1} - 0.23146 * \text{RM4549.1} + 46.8025; & (90) \\ \text{AWWM5054_PL} &= 0.03923 * \text{TREND_TE.1} - 0.23146 * \text{RM5054.1} + 46.8025; & (95) \end{aligned}$$

$$\text{AWWM5559_PL} = 0.04481 * \text{TREND_TE.1} - 0.28537 * \text{RM5559.1} + 45.0196; \quad (100)$$

$AWWM6064_PL = 0.14488 * TREND_TE.1 + 0.01910 * RM6064.1 + 29.5797;$ (105)
 $AWWM6569_PL = 0.21483 * TREND_TE.1 - 0.23366 * RM6569.1 + 21.1343;$ (110)
 $AWWM700_PL = 0.09069 * TREND_TE.1 + 0.43206 * RM700.1 + 28.6903;$ (115)

Work Experience

$WEM1617_3_P = EM1617 * 52 / AWWM1617_P;$ (54)
 $WEM1819_3_P = EM1819 * 52 / AWWM1819_P;$ (59)
 $WEM2024_3_P = EM2024 * 52 / AWWM2024_P;$ (64)

 $WEM2529_3_P = EM2529 * 52 / AWWM2529_P;$ (69)
 $WEM3034_3_P = EM3034 * 52 / AWWM3034_P;$ (74)
 $WEM3539_3_P = EM3539 * 52 / AWWM3539_P;$ (79)
 $WEM4044_3_P = EM4044 * 52 / AWWM4044_P;$ (84)
 $WEM4549_3_P = EM4549 * 52 / AWWM4549_P;$ (89)
 $WEM5054_3_P = EM5054 * 52 / AWWM5054_P;$ (94)

 $WEM5559_3_P = EM5559 * 52 / AWWM5559_P;$ (99)
 $WEM6064_3_P = EM6064 * 52 / AWWM6064_P;$ (104)
 $WEM6569_3_P = EM6569 * 52 / AWWM6569_P;$ (109)
 $WEM700_3_P = EM700 * 52 / AWWM700_P;$ (114)

$WEM1617_3_PL = EM1617.1 * 52 / AWWM1617_PL;$ (56)
 $WEM1819_3_PL = EM1819.1 * 52 / AWWM1819_PL;$ (61)
 $WEM2024_3_PL = EM2024.1 * 52 / AWWM2024_PL;$ (66)

$WEM2529_3_PL = EM2529.1 * 52 / AWWM2529_PL;$ (71)
 $WEM3034_3_PL = EM3034.1 * 52 / AWWM3034_PL;$ (76)
 $WEM3539_3_PL = EM3539.1 * 52 / AWWM3539_PL;$ (81)
 $WEM4044_3_PL = EM4044.1 * 52 / AWWM4044_PL;$ (86)
 $WEM4549_3_PL = EM4549.1 * 52 / AWWM4549_PL;$ (91)
 $WEM5054_3_PL = EM5054.1 * 52 / AWWM5054_PL;$ (96)

$WEM5559_3_PL = EM5559.1 * 52 / AWWM5559_PL;$ (101)
 $WEM6064_3_PL = EM6064.1 * 52 / AWWM6064_PL;$ (106)
 $WEM6569_3_PL = EM6569.1 * 52 / AWWM6569_PL;$ (111)
 $WEM700_3_PL = EM700.1 * 52 / AWWM700_PL;$ (116)

Total Employed

$TEM1617_P = ((WEM1617_3_P / WEM1617_3_PL) * (TEM1617.1 - NM1617M.1) + NM1617M) * MULT1_TEM1617 * MULT2_TEM1617;$ (57)
 $TEM1819_P = ((WEM1819_3_P / WEM1819_3_PL) * (TEM1819.1 - NM1819M.1) + NM1819M) * MULT1_TEM1819 * MULT2_TEM1819;$ (62)
 $TEM2024_P = ((WEM2024_3_P / WEM2024_3_PL) * (TEM2024.1 - NM2024M.1) + NM2024M) * MULT1_TEM2024 * MULT2_TEM2024;$ (67)

 $TEM2529_P = ((WEM2529_3_P / WEM2529_3_PL) * (TEM2529.1 - NM2529M.1) + NM2529M) * MULT1_TEM2529 * MULT2_TEM2529;$ (72)
 $TEM3034_P = ((WEM3034_3_P / WEM3034_3_PL) * (TEM3034.1 - NM3034M.1) + NM3034M) * MULT1_TEM3034 * MULT2_TEM3034;$ (77)
 $TEM3539_P = ((WEM3539_3_P / WEM3539_3_PL) * (TEM3539.1 - NM3539M.1) + NM3539M) * MULT1_TEM3539 * MULT2_TEM3539;$ (82)
 $TEM4044_P = ((WEM4044_3_P / WEM4044_3_PL) * (TEM4044.1 - NM4044M.1) + NM4044M) * MULT1_TEM4044 * MULT2_TEM4044;$ (87)
 $TEM4549_P = ((WEM4549_3_P / WEM4549_3_PL) * (TEM4549.1 - NM4549M.1) + NM4549M) * MULT1_TEM4549 * MULT2_TEM4549;$ (92)
 $TEM5054_P = ((WEM5054_3_P / WEM5054_3_PL) * (TEM5054.1 - NM5054M.1) + NM5054M) * MULT1_TEM5054 * MULT2_TEM5054;$ (97)

 $TEM5559_P = ((WEM5559_3_P / WEM5559_3_PL) * (TEM5559.1 - NM5559M.1) + NM5559M) * MULT1_TEM5559 * MULT2_TEM5559;$ (102)
 $TEM6064_P = ((WEM6064_3_P / WEM6064_3_PL) * (TEM6064.1)) * MULT1_TEM6064 * MULT2_TEM6064;$ (107)
 $TEM6569_P = ((WEM6569_3_P / WEM6569_3_PL) * (TEM6569.1)) * MULT1_TEM6569 * MULT2_TEM6569;$ (112)
 $TEM700_P = ((WEM700_3_P / WEM700_3_PL) * (TEM700.1)) * MULT1_TEM700 * MULT2_TEM700;$ (117)

$$\begin{aligned} \text{WEM160_3_P} &= \text{WEM1617_3_P} + \text{WEM1819_3_P} + \text{WEM2024_3_P} + \text{WEM2529_3_P} + \text{WEM3034_3_P} + \text{WEM3539_3_P} + \\ &\quad \text{WEM4044_3_P} + \text{WEM4549_3_P} \\ &\quad + \text{WEM5054_3_P} + \text{WEM5559_3_P} + \text{WEM6064_3_P} + \text{WEM6569_3_P} + \text{WEM700_3_P}; \end{aligned} \quad (190)$$

$$\text{AWWM160_P} = \text{EM160} * 52 / \text{WEM160_3_P}; \quad (191)$$

$$\begin{aligned} \text{TEM160_P} &= \text{TEM1617_P} + \text{TEM1819_P} + \text{TEM2024_P} + \text{TEM2529_P} + \text{TEM3034_P} + \text{TEM3539_P} + \text{TEM4044_P} + \text{TEM4549_P} + \\ &\quad \text{TEM5054_P} + \text{TEM5559_P} + \text{TEM6064_P} + \text{TEM6569_P} + \text{TEM700_P}; \end{aligned} \quad (118)$$

Final (Pre-TE.ADD) (192-230)

Average Weeks Worked

$$\begin{aligned} \text{AWWM1617} &= \text{AWWM1617_P}; \\ \text{AWWM1819} &= \text{AWWM1819_P}; \\ \text{AWWM2024} &= \text{AWWM2024_P}; \end{aligned}$$

$$\begin{aligned} \text{AWWM2529} &= \text{AWWM2529_P}; \\ \text{AWWM3034} &= \text{AWWM3034_P}; \\ \text{AWWM3539} &= \text{AWWM3539_P}; \\ \text{AWWM4044} &= \text{AWWM4044_P}; \\ \text{AWWM4549} &= \text{AWWM4549_P}; \\ \text{AWWM5054} &= \text{AWWM5054_P}; \end{aligned}$$

$$\begin{aligned} \text{AWWM5559} &= \text{AWWM5559_P}; \\ \text{AWWM6064} &= \text{AWWM6064_P}; \\ \text{AWWM6569} &= \text{AWWM6569_P}; \\ \text{AWWM700} &= \text{AWWM700_P}; \end{aligned}$$

Work Experience

$$\begin{aligned} \text{WEM1617_3} &= \text{WEM1617_3_P}; \\ \text{WEM1819_3} &= \text{WEM1819_3_P}; \\ \text{WEM2024_3} &= \text{WEM2024_3_P}; \end{aligned}$$

$$\begin{aligned} \text{WEM2529_3} &= \text{WEM2529_3_P}; \\ \text{WEM3034_3} &= \text{WEM3034_3_P}; \\ \text{WEM3539_3} &= \text{WEM3539_3_P}; \\ \text{WEM4044_3} &= \text{WEM4044_3_P}; \\ \text{WEM4549_3} &= \text{WEM4549_3_P}; \\ \text{WEM5054_3} &= \text{WEM5054_3_P}; \end{aligned}$$

$$\begin{aligned} \text{WEM5559_3} &= \text{WEM5559_3_P}; \\ \text{WEM6064_3} &= \text{WEM6064_3_P}; \\ \text{WEM6569_3} &= \text{WEM6569_3_P}; \\ \text{WEM700_3} &= \text{WEM700_3_P}; \end{aligned}$$

Total Employed

$$\begin{aligned} \text{TEM1617} &= \text{TEM1617_P}; \\ \text{TEM1819} &= \text{TEM1819_P}; \\ \text{TEM2024} &= \text{TEM2024_P}; \end{aligned}$$

$$\begin{aligned} \text{TEM2529} &= \text{TEM2529_P}; \\ \text{TEM3034} &= \text{TEM3034_P}; \\ \text{TEM3539} &= \text{TEM3539_P}; \\ \text{TEM4044} &= \text{TEM4044_P}; \\ \text{TEM4549} &= \text{TEM4549_P}; \\ \text{TEM5054} &= \text{TEM5054_P}; \end{aligned}$$

$$\begin{aligned} \text{TEM5559} &= \text{TEM5559_P}; \\ \text{TEM6064} &= \text{TEM6064_P}; \\ \text{TEM6569} &= \text{TEM6569_P}; \\ \text{TEM700} &= \text{TEM700_P}; \end{aligned}$$

$$\text{WEM160_3} = \text{WEM160_3_P};$$

$AWWM16O = AWWM16O_P;$ (119)
 $TEM16O = TEM16O_P;$ (120)
 $TEM = TEM16O + HE_M_15U(1, YEAR)$

Female Disaggregates Aged 16 and Over

Preliminary

Average Weeks Worked

$AWWF1617_P = 0.44829 * TREND_TE - 0.06786 * RF1617 + 0.37331;$ (121)
 $AWWF1819_P = 0.24262 * TREND_TE - 0.00950 * RF1819 + 17.0791;$ (126)
 $AWWF2024_P = 0.18291 * TREND_TE - 0.06453 * RF2024 + 26.6823;$ (131)

 $AWWF2529_P = 0.07674 * TREND_TE - 0.10131 * RF2529 + 40.0210;$ (136)
 $AWWF3034_P = 0.07674 * TREND_TE - 0.10131 * RF3034 + 40.0210;$ (141)
 $AWWF3539_P = 0.05004 * TREND_TE - 0.10322 * RF3539 + 43.9974;$ (146)
 $AWWF4044_P = 0.05004 * TREND_TE - 0.10322 * RF4044 + 43.9974;$ (151)
 $AWWF4549_P = 0.08209 * TREND_TE - 0.17307 * RF4549 + 41.1649;$ (156)
 $AWWF5054_P = 0.08209 * TREND_TE - 0.17307 * RF5054 + 41.1649;$ (161)

 $AWWF5559_P = 0.04868 * TREND_TE + 0.17072 * RF5559 + 41.9580;$ (166)
 $AWWF6064_P = 0.14339 * TREND_TE + 0.01918 * RF6064 + 29.1567;$ (171)
 $AWWF6569_P = 0.01857 * TREND_TE + 0.64199 * RF6569 + 36.1193;$ (176)
 $AWWF700_P = 0.20193 * TREND_TE + 0.92866 * RF700 + 14.2412;$ (181)

$AWWF1617_PL = 0.44829 * TREND_TE.1 - 0.06786 * RF1617.1 + 0.37331;$ (123)
 $AWWF1819_PL = 0.24262 * TREND_TE.1 - 0.00950 * RF1819.1 + 17.0791;$ (128)
 $AWWF2024_PL = 0.18291 * TREND_TE.1 - 0.06453 * RF2024.1 + 26.6823;$ (133)
 $AWWF2529_PL = 0.07674 * TREND_TE.1 - 0.10131 * RF2529.1 + 40.0210;$ (138)
 $AWWF3034_PL = 0.07674 * TREND_TE.1 - 0.10131 * RF3034.1 + 40.0210;$ (143)
 $AWWF3539_PL = 0.05004 * TREND_TE.1 - 0.10322 * RF3539.1 + 43.9974;$ (148)
 $AWWF4044_PL = 0.05004 * TREND_TE.1 - 0.10322 * RF4044.1 + 43.9974;$ (153)
 $AWWF4549_PL = 0.08209 * TREND_TE.1 - 0.17307 * RF4549.1 + 41.1649;$ (158)
 $AWWF5054_PL = 0.08209 * TREND_TE.1 - 0.17307 * RF5054.1 + 41.1649;$ (163)

 $AWWF5559_PL = 0.04868 * TREND_TE.1 + 0.17072 * RF5559.1 + 41.9580;$ (168)
 $AWWF6064_PL = 0.14339 * TREND_TE.1 + 0.01918 * RF6064.1 + 29.1567;$ (173)
 $AWWF6569_PL = 0.01857 * TREND_TE.1 + 0.64199 * RF6569.1 + 36.1193;$ (178)
 $AWWF700_PL = 0.20193 * TREND_TE.1 + 0.92866 * RF700.1 + 14.2412;$ (183)

Work Experience

$WEF1617_3_P = EF1617 * 52 / AWWF1617_P;$ (122)
 $WEF1819_3_P = EF1819 * 52 / AWWF1819_P;$ (127)
 $WEF2024_3_P = EF2024 * 52 / AWWF2024_P;$ (132)

 $WEF2529_3_P = EF2529 * 52 / AWWF2529_P;$ (137)
 $WEF3034_3_P = EF3034 * 52 / AWWF3034_P;$ (142)
 $WEF3539_3_P = EF3539 * 52 / AWWF3539_P;$ (147)
 $WEF4044_3_P = EF4044 * 52 / AWWF4044_P;$ (152)
 $WEF4549_3_P = EF4549 * 52 / AWWF4549_P;$ (157)
 $WEF5054_3_P = EF5054 * 52 / AWWF5054_P;$ (162)

 $WEF5559_3_P = EF5559 * 52 / AWWF5559_P;$ (167)
 $WEF6064_3_P = EF6064 * 52 / AWWF6064_P;$ (172)
 $WEF6569_3_P = EF6569 * 52 / AWWF6569_P;$ (177)
 $WEF700_3_P = EF700 * 52 / AWWF700_P;$ (182)

 $WEF1617_3_PL = EF1617.1 * 52 / AWWF1617_PL;$ (124)
 $WEF1819_3_PL = EF1819.1 * 52 / AWWF1819_PL;$ (129)
 $WEF2024_3_PL = EF2024.1 * 52 / AWWF2024_PL;$ (134)

 $WEF2529_3_PL = EF2529.1 * 52 / AWWF2529_PL;$ (139)
 $WEF3034_3_PL = EF3034.1 * 52 / AWWF3034_PL;$ (144)

WEF3539_3_PL = EF3539.1 * 52 / AWWF3539_PL; (149)
 WEF4044_3_PL = EF4044.1 * 52 / AWWF4044_PL; (154)
 WEF4549_3_PL = EF4549.1 * 52 / AWWF4549_PL; (159)
 WEF5054_3_PL = EF5054.1 * 52 / AWWF5054_PL; (164)

 WEF5559_3_PL = EF5559.1 * 52 / AWWF5559_PL; (169)
 WEF6064_3_PL = EF6064.1 * 52 / AWWF6064_PL; (174)
 WEF6569_3_PL = EF6569.1 * 52 / AWWF6569_PL; (179)
 WEF700_3_PL = EF700.1 * 52 / AWWF700_PL; (184)

Total Employed

TEF1617_P = ((WEF1617_3_P / WEF1617_3_PL) * (TEF1617.1 - NF1617M.1) + NF1617M) * MULT1_TEF1617 * MULT2_TEF1617; (125)
 TEF1819_P = ((WEF1819_3_P / WEF1819_3_PL) * (TEF1819.1 - NF1819M.1) + NF1819M) * MULT1_TEF1819 * MULT2_TEF1819; (130)
 TEF2024_P = ((WEF2024_3_P / WEF2024_3_PL) * (TEF2024.1 - NF2024M.1) + NF2024M) * MULT1_TEF2024 * MULT2_TEF2024; (135)

 TEF2529_P = ((WEF2529_3_P / WEF2529_3_PL) * (TEF2529.1 - NF2529M.1) + NF2529M) * MULT1_TEF2529 * MULT2_TEF2529; (140)
 TEF3034_P = ((WEF3034_3_P / WEF3034_3_PL) * (TEF3034.1 - NF3034M.1) + NF3034M) * MULT1_TEF3034 * MULT2_TEF3034; (145)
 TEF3539_P = ((WEF3539_3_P / WEF3539_3_PL) * (TEF3539.1 - NF3539M.1) + NF3539M) * MULT1_TEF3539 * MULT2_TEF3539; (150)
 TEF4044_P = ((WEF4044_3_P / WEF4044_3_PL) * (TEF4044.1 - NF4044M.1) + NF4044M) * MULT1_TEF4044 * MULT2_TEF4044; (155)
 TEF4549_P = ((WEF4549_3_P / WEF4549_3_PL) * (TEF4549.1 - NF4549M.1) + NF4549M) * MULT1_TEF4549 * MULT2_TEF4549; (160)
 TEF5054_P = ((WEF5054_3_P / WEF5054_3_PL) * (TEF5054.1 - NF5054M.1) + NF5054M) * MULT1_TEF5054 * MULT2_TEF5054; (165)

 TEF5559_P = ((WEF5559_3_P / WEF5559_3_PL) * (TEF5559.1 - NF5559M.1) + NF5559M) * MULT1_TEF5559 * MULT2_TEF5559; (170)
 TEF6064_P = ((WEF6064_3_P / WEF6064_3_PL) * (TEF6064.1)) * MULT1_TEF6064 * MULT2_TEF6064; (175)
 TEF6569_P = ((WEF6569_3_P / WEF6569_3_PL) * (TEF6569.1)) * MULT1_TEF6569 * MULT2_TEF6569; (180)
 TEF700_P = ((WEF700_3_P / WEF700_3_PL) * (TEF700.1)) * MULT1_TEF700 * MULT2_TEF700; (185)

 WEF160_3_P = WEF1617_3_P + WEF1819_3_P + WEF2024_3_P + WEF2529_3_P + WEF3034_3_P + WEF3539_3_P + WEF4044_3_P + WEF4549_3_P + WEF5054_3_P + WEF5559_3_P + WEF6064_3_P + WEF6569_3_P + WEF700_3_P; (233)
 AWWF160_P = EF160 * 52 / WEF160_3_P; (234)
 TEF160_P = TEF1617_P + TEF1819_P + TEF2024_P + TEF2529_P + TEF3034_P + TEF3539_P + TEF4044_P + TEF4549_P + TEF5054_P + TEF5559_P + TEF6064_P + TEF6569_P + TEF700_P; (186)

Final (Pre-TE.ADD) (235-273)

Average Weeks Worked

AWWF1617 = AWWF1617_P;
 AWWF1819 = AWWF1819_P;
 AWWF2024 = AWWF2024_P;

 AWWF2529 = AWWF2529_P;
 AWWF3034 = AWWF3034_P;
 AWWF3539 = AWWF3539_P;
 AWWF4044 = AWWF4044_P;
 AWWF4549 = AWWF4549_P;
 AWWF5054 = AWWF5054_P;

 AWWF5559 = AWWF5559_P;
 AWWF6064 = AWWF6064_P;
 AWWF6569 = AWWF6569_P;
 AWWF700 = AWWF700_P;

Work Experience

WEF1617_3 = WEF1617_3_P;
WEF1819_3 = WEF1819_3_P;
WEF2024_3 = WEF2024_3_P;

WEF2529_3 = WEF2529_3_P;
WEF3034_3 = WEF3034_3_P;
WEF3539_3 = WEF3539_3_P;
WEF4044_3 = WEF4044_3_P;
WEF4549_3 = WEF4549_3_P;
WEF5054_3 = WEF5054_3_P;

WEF5559_3 = WEF5559_3_P;
WEF6064_3 = WEF6064_3_P;
WEF6569_3 = WEF6569_3_P;
WEF700_3 = WEF700_3_P;

Total Employed

TEF1617 = TEF1617_P;
TEF1819 = TEF1819_P;
TEF2024 = TEF2024_P;

TEF2529 = TEF2529_P;
TEF3034 = TEF3034_P;
TEF3539 = TEF3539_P;
TEF4044 = TEF4044_P;
TEF4549 = TEF4549_P;
TEF5054 = TEF5054_P;

TEF5559 = TEF5559_P;
TEF6064 = TEF6064_P;
TEF6569 = TEF6569_P;
TEF700 = TEF700_P;

WEF16O_3 = WEF16O_3_P; (274)
AWWF16O = AWWF16O_P; (275)
TEF16O = TEF16O_P; (187)
TEF = TEF16O + HE_M_15U(2, YEAR) (188)

Combined, Age 16 and Over

WE16O_3_P = WEM16O_3_P + WEF16O_3_P; (276)
AWW16O_P = E16O * 52 / WE16O_3_P; (277)

WE16O_3 = WE16O_3_P; (278)
AWW16O = AWW16O_P; (279)

TE = TEM + TEF (189)

Underground Economy and the Earnings Suspense File

TE_U = TEO_UND (312)
TEL_SO = TEL_SO.1 * (TE - TEO_UND - TEO_ESF) / (TE.1 - TEO_UND.1 - TEO_ESF.1) (313)
TE_S = TEL_SO + TEO_ESF (314)
TE_SFO_LRP = TEL_SO (389)
WE_SFO_LRP = TE_SFO_LRP * ACWA (390)
TE_SFM_LRP = TE_SFO_LRP (391)
WE_SFM_LRP = TE_SFM_LRP * ACWA * 0.5 (392)
WE_SF_LRP = WE_SFO_LRP + WE_SFM_LRP (393)
WE_SF = WS_EO_ESF + WE_SF_LRP (394)

MEF

TE_M = TE - TE_U - TE_S (315)
TE_MN = TE_RRO_M + TE_SLOO_M + TE_SLOS_M + TE_SLOE_M + TE+PS_M + TE_PH_M + TEO_ASF1 + TEO_ASJ1 + TEO_AWH (316)
HE_M = TE_M - TE_MN (317)
WS_MEF = WSCAHI - WE_SF + WSPRRB + WESL_N_NHI +
(WESL_N_NHI_S / TESL_N_S) * TE_PS_M +
(0.5 * 1.8 / 44.32167) * (AIW.1 / 1000 * PROD / PROD.1 * AHRS / AHRS.1 * PGDP / PGDP.1) * TE_PH_M +
WS_EO_ASF1 + WS_EO_ASJ1 + WS_EO_AWJ + WS_EO_AWH (395)

Self-Employed Only

SEOCMB = WSW_HIO_OTH_SE + TEFC_N_N_SE + TESL_N_N_HI_SE (308)
SEOCMBL1 = WSW_HIO_OTH_SE + TEFC_N_N_SE + TESL_N_N_HI_SE (309)
SEO = (SEO.1 * (EAS + ENAS) / (EAS.1 + ENAS.1) + (SEOCMB - SEOCMBL1)) * MULTSEO (310)

Combination Workers

CMB_TOT = ((-0.01468 + 0.06227 * RTP.1 - 0.0008) * WSWA - SEOCMB) * MULTCMB (322)
CSW_TOT = SEO + CMB_TOT
AW_CMBTOT = 1.4953 * ACWA (337)
W_CMBTOT = AW_CMBTOT * CMB_TOT (338)
CMB_WRELMAX = TAXMAX / AW_CMBTOT (344)

CMB Wage Andover Curve

CMB_WAO1 = IF (CMB_WRELMAX < 0.0543009)
THEN 1 - 0.722659 * CMB_WRELMAX^{0.65} - 0.461913 * CMB_WRELMAX^{0.8}
ELSE IF (CMB_WRELMAX < 0.1086018)
THEN -1.02884 * CMB_WRELMAX^{0.6} + 0.324761 * CMB_WRELMAX^{1.6} + 1.02015
ELSE IF (CMB_WRELMAX < 0.1629027)
THEN -0.906607 * CMB_WRELMAX^{0.7} + 0.947662
ELSE IF (CMB_WRELMAX < 0.2172037)
THEN -0.813951 * CMB_WRELMAX^{0.55} + 0.991722
ELSE IF (CMB_WRELMAX < 0.3258055)
THEN -0.755135 * CMB_WRELMAX^{0.55} + 0.964593
ELSE 0 (345-349)

CMB_WAO2 = IF (CMB_WRELMAX < 0.5430091)
THEN -0.649755 * CMB_WRELMAX^{0.6} + 0.886467
ELSE IF (CMB_WRELMAX < 0.7059119)
THEN -0.573205 * CMB_WRELMAX^{0.7} + 0.810122
ELSE IF (CMB_WRELMAX < 0.9231155)
THEN - 5.22264 * CMB_WRELMAX^{0.06} + 5.47514
ELSE IF (CMB_WRELMAX < 1.0860183)
THEN - 2.02619 * CMB_WRELMAX^{0.15} + 2.27963
ELSE IF (CMB_WRELMAX < 1.5204256)
THEN 0.605192 * EXP (-0.2 * CMB_WRELMAX) - 0.827158 * EXP (-0.8 * CMB_WRELMAX)
+ 1.52918 * EXP (- 1.5 * CMB_WRELMAX) - 0.212269
ELSE 0

CMB_WAO3 = IF (CMB_WRELMAX < 1.8462311)
THEN 0.19139 * EXP (-0.6 * CMB_WRELMAX) + 0.764408 * EXP (- 1.8 * CMB_WRELMAX) + 0.0194903
ELSE IF (CMB_WRELMAX < 2.3077888)
THEN 0.12964 * EXP (-0.5 * CMB_WRELMAX) + 0.644861 * EXP (- 1.5 * CMB_WRELMAX) + 0.0183343
ELSE IF (CMB_WRELMAX < 2.9865502)
THEN 0.361318 * EXP (-0.8 * CMB_WRELMAX) + 0.0219491
ELSE IF (CMB_WRELMAX < 4.3440731)
THEN 0.193202 * EXP (-0.45 * CMB_WRELMAX) + 0.00425171
ELSE IF (CMB_WRELMAX < 5.4300913)
THEN 0.0560412 * EXP (-0.25 * CMB_WRELMAX) + 0.311286 * EXP (-0.8 * CMB_WRELMAX) +
0.00297316
ELSE 0

CMB_WAO4 = IF (CMB_WRELMAX < 13.5752283)
THEN 0.0995677 * EXP (-0.32 * CMB_WRELMAX) + 0.00355234
ELSE IF (CMB_WRELMAX < 21.7203653)
THEN 0.041159 * EXP (-0.19 * CMB_WRELMAX) + 0.00156765
ELSE IF (CMB_WRELMAX < 678.7614168)

```

THEN 0.265022 * CMB_WRELMAX(-1.555)
ELSE 0

CMB_WAO = IF (CMB_WRELMAX < 0.3258055)
THEN CMB_WAO1
ELSE IF (CMB_WRELMAX < 1.5204256)
THEN CMB_WAO2
ELSE IF (CMB_WRELMAX < 5.4300913)
THEN CMB_WAO3
ELSE CMB_WAO4

CMB = (1 - (CMB_WAO - 0.019)) * CMB_TOT (350)
CSW = SEO + CMB (353)
SEOCMB_HI = 0.039 * (TEFC_N_N + TESL_N_N_HI)
SEO_HI = SEO - SEOCMB_HI (311)
CMB_HI = CMB_TOT + SEOCMB_HI (351)
CSW_TOT = SEO + CMB_TOT (352)
CSW_HI = SEO_HI + CMB_HI (354)

NIPA Wages

Private Residual Sector
WSDPB = WSDP - WSPH - WSPF - WSPRRB - TIPS_SR (327)
TIPS_SR = (0.000508328 * RTP - 0.000481700) * GDP * 1.26393 + TIPS_SR_ADD (326)

OASDI Wages

Covered Employment and Wages – Federal Civilian Government
TEFC = (TEFC.1 / EGGEFC.1) * EGGEFC (362)
TEFC_N = IF (CSRS.1 > 0) THEN TEFC_N.1 / CSRS.1 * CSRS - TEFC_N_SW ELSE 0
TEFC_N_N = HE_WOF_M (319)
TEFC_N_N_SE = HE_WOSF_M (302)
TEFC_N_O = (TEFC_N - TEFC_N_N) (365)
TEFC_O = (TEFC - TEFC_N) (364)
WEFC = (WEFC.1 / WSGGEFC.1) * WSGGEFC (331)
WEFC_O = (WEFC - WEFC_N) * ADJ_FSA_FC (332)

Covered Employment and Wages - State and Local Govt.
TESL = (TESL.1 / EGGESLMAX.1) * EGGESLMAX (280)
TESL_O = (TESL_O.1 / TESL.1) * TESL (281)
TESL_N = (TESL - TESL_O) (282)
TESL_N_O = (TESL_N_O_HI + TESL_N_O_NHI) (293)
TESL_N_O_HI = (TESL_N - TESL_N_O_NHI - TESL_N_N_NHI) * CER_MQGE_O (292)
TESL_N_O_NHI = (TESL_N_O_NHI_S + TESL_N_O_NHI_E + TESL_N_O_NHI_NS) (288)
TESL_N_S = TESL_N_S.1 * (NF1819 + NF2024 + NM1819 + NM2024) / (NF1819.1 + NF2024.1 + NM1819.1 + NM2024.1) (355)
TESL_N_E = TESL_N_E.1 * (TESL / TESL.1) (285)
TESL_N_O_NHI_S = TESL_N_S * (TESL_N_O_NHI_S.1 / TESL_N_S.1) (284)
TESL_N_O_NHI_E = TESL_N_E * 0.6 (286)
TESL_N_O_NHI_NS = TESL_N_O_NHI_NS.1 * ESR_NS (287)
TESL_N_N = (TESL_N - TESL_N_O) (294)
TESL_N_N_HI = (TESL_N_N - TESL_N_N_NHI) (295)
TESL_N_N_HI_SE = (TESL_N_N_HI_SE.1 / TESL_N_N_HI.1) * TESL_N_N_HI (303)
TESL_N_N_NHI = (TESL_N_N_NHI_S + TESL_N_N_NHI_E + TESL_N_N_NHI_NS) (291)
TESL_N_N_NHI_S = (TESL_N_S - TESL_N_O_NHI_S) (283)
TESL_N_N_NHI_E = (TESL_N_E - TESL_N_O_NHI_E) (289)
TESL_N_N_NHI_NS = TESL_N_N_NHI_NS.1 * ESR_NS (290)
WESL = (WESL.1 / WSGGESL.1) * WSGGESL (297)
WESL_O = (WESL_O.1 / WSGGESL.1) * WSGGESL (298)
WESL_N = (WESL - WESL_O) (299)
WESL_N_HI = (WESL_N - WESL_N_NHI) (361)
WESL_N_NHI = (WESL_N_NHI_S + WESL_N_NHI_E + WESL_N_NHI_NS) (360)
WESL_N_NHI_S = WESL_N_NHI_S.1 * (TESL_N_S / TESL_N_S.1) * (AWSGGESL / AWSGGESL.1) (356)
WESL_N_NHI_E = WESL_N_NHI_E.1 * (TESL_N_E / TESL_N_E.1) * (AWSGGESL / AWSGGESL.1) (357)

```


RAWR_NS = IF (AWR_NS = 0) THEN 0 ELSE AWR_NS/AWR_NS.1 (358)
 WESL_N_NHI_NS = IF (ESR_NS = 0) THEN 0 ELSE WESL_N_NHI_NS.1 * (TESL_N_O_NHI_NS + TESL_N_N_NHI_NS) /
 (TESL_N_O_NHI_NS.1 + TESL_N_N_NHI_NS.1) * (AWSGGESL/AWSGGESL.1) * RAWR_NS (359)

 WSW_HIO_OTH = 0 (296)
 WSCA_HIO_OTH = WSW_HIO_OTH*(WESL_N/TESL_N) (300)
 WSW_HIO_OTH_SE = 0 (301)

 HE_WOL_M = TESL_N_N_HI (304)
 HE_WOR_M = 0 (305)
 HE_WOSL_M = (HE_WOSL_M.1 / HE_WOL_M.1) * HE_WOL_M (306)
 HE_WOSR_M = 0 (307)

Self-Employed Earnings Sector

Covered SENE (366-374)
 CSE_TOT = (YF + YNF) / (YF.1 + YNF.1) * CSE_TOT.1
 CSE_CMB_N = (CSE_TOT / (CMB_TOT + SEO)) / (CSE_TOT.1 / (CMB_TOT.1 + SEO.1)) * (CSE_CMB_N.1 / (CMB_TOT.1 -
 CMB.1)) * (CMB_TOT - CMB)
 CSE = CSE_TOT - CSE_CMB_N
 ACSE_SEO = (CSE_TOT / (SEO + 0.416488 * CMB_TOT))
 ACSE_CMB_TOT = 0.416488 * ACSE_SEO
 CSE_SEO = ACSE_SEO * SEO
 CSE_CMB_TOT = ACSE_CMB_TOT * CMB_TOT
 CSE_CMB = CSE_CMB_TOT - CSE_CMB_N
 ACSE_CMB = CSE_CMB/CMB

Present Law OASDI and HI Covered Wages and Earnings

WSGMLC = CML * WSGFM (334)
 WSGFCA = WEFC_O (333)
 CFCA = WSGFCA/WSGGEFCA (375)
 CSLHI = (WESL_O+WESL_N_HI)/WSGGESL (376)
 WSGSLCA = WESL_O (330)
 WSPH_O = CPH * WSPH (323)
 WSPF_O = WSPF_O.1 * WSPF/WSPF.1 (324)
 CPF = WSPF_O/WSPF (377)
 WSPRR_O = CPRR * WSPRRB (325)
 WSPC = WSPH_O + WSPF_O + WSPRR_O + TIPS_SR + WSPB_O (329)
 CP = WSPC/WSDP (378)
 WSCA = (WSPC + WSGSLCA + WSGFCA + WSGMLC) (335)
 COVERNA = (WSCA + CSE) (379)
 ACWA = WSCA/WSWA (336)
 ASE = CSE/CSW (380)
 ASEHI = CSE_TOT/CSW_HI (381)
 ACEA = COVERNA/TCEA (382)
 ACSLW = WESL_O/TESL_O * MULTACSLW (383)
 ACMW = ACMW.1 * AWSGFM/AWSGFM.1 * CML/CML.1 (384)
 ACFCW = WEFC_O/TEFC_O (385)
 ACFMW = ACFMW.1 * (AIW.1/AIW.3)^{0.5} (386)
 TEPH_N = ENAWPH * (1 - CPH)
 TEP_N_N_S = TEP_N_N_S.1 * (NF1819 + NF2024 + NM1819 + NM2024) / (NF1819.1 + NF2024.1 + NM1819.1 + NM2024.1)
 WSWA = (TCEA-SEO) (321)

Present Law HI Covered Wages and Earnings

WSCAHI_ADD = WSCA * WSCAHI_ADD.1/WSCA.1
 TCEAHI = HE_M + TE_S (318)
 TCEA = TCEAHI - ((TESL_N_N_HI + TEFC_N_N + WSW_HIO_OTH) -
 (TESL_N_N_HI_SE + TEFC_N_N_SE + WSW_HIO_OTH_SE)) (320)
 WSWAHI = TCEAHI - SEO_HI
 WSCAHI = WSCA + WEFC_N + WESL_N_HI + WSCA_HIO_OTH (388)
 ACWAHI = WSCAHI/WSWAHI
 COVERNHI = WSCAHI + CSE_TOT
 ACEAHI = COVERNHI/TCEAHI

Complete Coverage concepts

WSWC = (WSWAHI + TEPH_N + EPRRB + TEP_N_N_S + TEPO_N + TESL_N_N_NHI) + LOST_MF
 ACWC = WSD/WSWC

AIW = IF AIW_GR_YR = 0
 THEN AIW.1 * ACWC / ACWC.1 * MULTAIW
 ELSE AIW.1 * (1 + AIW_GR / 100)

Taxable Maximums

RAIW = AIW.2 / AIWBASE (339)
 TAXMAXB1 = RAIW * TMAXBASE * 1000 / 300 (340)
 TAXMAXB2 = IF TAXMAXB1 - ROUND (TAXMAXB1) >= 0.5
 THEN ROUND (TAXMAXB1) + 1
 ELSE ROUND (TAXMAXB1) (341)
 TAXMAXB3 = IF TAXMAXB2 < TAXMAX.1
 THEN TAXMAX.1 * 1000 / 300
 ELSE TAXMAXB2 (342)
 TAXMAX = IF BENINC.1 <= 0.001
 THEN TAXMAX.1
 ELSE 300 * TAXMAXB3 / 1000 (343)

Deemed Military Wage Credits

EDMILAF = EDMIL * 1.1
 EDMILT = (2.00303 - 50.7517 / YEAR) * EDMILAF
 EDMILR = EDMILT - EDMILAF
 MWC_ED_O = 1.2 * EDMILAF * 0.997
 MWC_ED_HI = 1.2 * EDMILAF
 AMWC_GO2 = MIN (1.2, AWSGFM * (2/52) * (1/3))
 MWC_EDR_O = AMWC_GO2 * EDMILR * (1 - 0.017)
 MWC_EDR_HI = MWC_EDR_O + ((1.2 + AMWC_GO2) * 0.5) * EDMILR * 0.017
 MWC_O = MWC_ED_O + MWC_EDR_O
 MWC_HI = MWC_ED_HI + MWC_EDR_HI

2.4 Effective Taxable Payroll (TAXPAY)

2.4.1 Ratio of taxable employee to total covered OASDI wages (RWTEE)

```
if (relmax > 15.01754965d0) then
  RWTEE = -(0.200178d0 / 0.74d0) * relmax**(-0.74d0) + &
    1.000209697d0
else if (relmax > 2.79052673d0) then
  RWTEE = -(0.009321d0 / 0.15d0) * exp(-0.15d0 * relmax) - &
    (0.055530d0 / 0.40d0) * exp(-0.40d0 * relmax) - &
    (0.809959d0 / 1.25d0) * exp(-1.25d0 * relmax) + &
    0.000581d0 * relmax + 0.961924965d0
else if (relmax > 1.68599158d0) then
  RWTEE = -(0.104668d0 / 0.30d0) * exp(-0.30d0 * relmax) - &
    (1.276789d0 / 1.55d0) * exp(-1.55d0 * relmax) - &
    0.0126230d0 * relmax + 1.054563461d0
else if (relmax > 0.89508398d0) then
  RWTEE = (0.046180d0 / 0.25d0) * exp(-0.25d0 * relmax) - &
    (1.197836d0 / 1.3d0) * exp(-1.3d0 * relmax) + &
    0.040800d0 * relmax + 0.675476436d0
else if (relmax > 0.39794128d0) then
  RWTEE = (0.371366d0 / 1.5d00) * relmax**1.50d0 - &
    (1.793426d0 / 0.69d0) * exp(-0.69d0 * relmax) - &
    0.939072d0 * relmax + 2.604319058d0
else
  RWTEE = relmax - &
    (0.290789d0 / 1.5d0) * relmax**1.5d0 - &
    (0.398961d0 / 2.00d0) * relmax**2d0
end if
```

Where

relmax = OASDI taxable maximum / average covered OASDI wage
RWTEE = Ratio of OASDI taxable employee to covered wages

2.4.2 Taxable employee OASDI wages (WTEE)

WTEE = RWTEE * WSC

Where

RWTEE = Ratio of OASDI taxable employee to covered wages
WSC = OASDI total covered wages
WTEE = OASDI taxable employee wages

2.4.3 Ratio of multi-employer refund wages to total OASDI covered wages (RMER)

RMER = (MER(-1) / WSC(-1)) - 0.03217 * (RWTEE - RWTEE(-1)) - 0.00024 * (RU - RU(-1))

Where

MER(-1) = Multi-employer refund wages in prior year
RMER = Ratio of multi-employer refund wages to total OASDI covered wages
RU = Annual average civilian unemployment rate
RWTEE = Ratio of OASDI taxable employee to covered wages
WSC(-1) = OASDI total covered wages in prior year

2.4.4 Multi-employer refund wages (MER)

MER = RMER * WSC

Where

MER = OASDI multi-employer refund wages
RMER = Ratio of multi-employer refund wages to total OASDI covered wages
WSC = OASDI total covered wages

2.4.5 Taxable employer OASDI wages (WTER)

WTER = WTEE + MER

Where

MER = OASDI multi-employer refund wages

WTEE = OASDI taxable employee wages
 WTER = OASDI taxable employer wages

2.4.6 Ratio of taxable to covered self-employment earnings (RSET)

Preliminary

BASECT = 77074.88
 BASECW = 59157.14
 BASEO = 38188.22

Self-employed only

SECSEO = CSE - SECCMB
 ASESEO = SECSEO / SEO
 ASEO17 = ASESEO(2017)
 ASESEO = ASESEO * BASEO / ASEO17
 O = TAXMAX / ASESEO

```

if( o < 0.026186088d0 ) then
  otr = o - (9.574544d0 / 2.5d0) * o**2.5d0
else if( o < 0.104744354d0 ) then
  otr = -(1.04227d0 / 1.5d0) * o**1.5d0 + (0.67587d0 / 2.15d0) * o**2.15d0 - &
    (3.471502d0 / 3.3d0) * o**3.3d0 + 1.110432d0 * o - 0.00049085d0
else if( o < 0.209488707d0 ) then
  otr = (2.185075d0 / 1.3d0) * o**1.3d0 - (4.972755d0 / 1.75d0) * o**1.75d0 + &
    (3.184795d0 / 2.4d0) * o**2.4d0 + 0.475027d0 * o + 0.00377668d0
else if( o < 0.471349592d0 ) then
  otr = (7.17629d0 / 1.25d0) * o**1.25d0 - (0.28816d0 / 1.6d0) * o**1.6d0 - &
    (4.650629d0 / 2.4d0) * dexp(-2.4d0 * o) - 6.893912d0 * o + 1.987803109d0
else if( o < 1.021257449d0 ) then
  otr = -(0.52704d0 / 0.85d0) * dexp(-0.85d0 * o) + (0.369144d0 / 1.55d0) * &
    dexp(-1.55d0 * o) - (0.845421d0 / 3.1d0) * dexp(-3.1d0 * o) - &
    0.001447d0 * o + 0.665888025d0
else if( o < 1.309304422d0 ) then
  otr = -(0.174971d0 / 0.55d0) * dexp(-0.55 * o) - (0.393885d0 / 1.75d0) * &
    dexp(-1.75d0 * o) - (0.003202d0 / 2.6d0) * dexp(-2.6d0 * o) + &
    0.013703d0 * o + 0.64673395d0
else if( o < 1.728281837d0 ) then
  otr = (0.338456d0 / 0.3d0) * dexp(-0.3d0 * o) - (0.904979d0 / 1.15d0) * &
    dexp(-1.15d0 * o) + (0.51182d0 / 2.5d0) * dexp(-2.5d0 * o) + &
    0.185928d0 * o - 0.351278731d0
else if( o < 2.17344534d0 ) then
  otr = (0.358360d0 / 0.55d0) * dexp(-0.55d0 * o) - (5.475731d0 / 1.85d0) * &
    dexp(-1.85d0 * o) + (9.368262d0 / 2.6d0) * dexp(-2.6d0 * o) + &
    0.121142d0 * o + 0.156160373d0
else if( o < 3.142330612d0 ) then
  otr = -(0.067751d0 / 0.2d0) * dexp(-0.2d0 * o) - (2.812835d0 / 1.9d0) * &
    dexp(-1.9d0 * o) + (8.107091d0 / 2.75d0) * dexp(-2.75d0 * o) + &
    0.00956d0 * o + 0.79107633d0
else if( o < 52.372176875d0 ) then
  otr = -(0.024629d0 / 0.1d0) * dexp(-0.1d0 * o) - (0.11274d0 / 0.4d0) * &
    dexp(-0.4d0 * o) + 0.001128d0 * o + 0.893694435d0
else if( o < 1000d0 ) then
  otr = -(1.22364d0 / 0.85d0) * o**(-0.85d0) + 1.001234831d0
else
  otr = 1d0
end if

```

SETSEO=OTR*SECSEO

OASDI taxable wages of workers with both wages and self-employment earnings

AWSCMB=WSCCMB/CMBNT
 AWSCMB17=AWSCMB(2017)
 AWSCMB=AWSCMB*BASECW/AWSCMB17
 CW=TAXMAX/AWSCMB

```

if( cw < 0.0190413d0 ) then
  cwtr = cw - (1.269097d0 / 1.72d0) * cw**1.72d0

```

```

else if ( cw < 0.059164455d0 ) then
  cwtr = (0.733496d0 / 1.45d0) * cw**1.45d0 - (1.932995d0 / 1.6d0) * cw**1.6d0 + &
    0.982734d0 * cw + 0.00003324d0
else if ( cw < 0.160589235d0 ) then
  cwtr = -(1.777645d0 / 1.8d0) * cw**1.8d0 + (1.737345d0 / 2.65d0) * cw**2.65d0 + &
    1.002541d0 * cw - 0.00013743d0
else if ( cw < 0.236657820d0 ) then
  cwtr = -(1.264913d0 / 1.3d0) * cw**1.3d0 + (0.225457d0 / 2.2d0) * cw**2.2d0 + &
    1.381608d0 * cw - 0.00414044d0
else if ( cw < 0.388794990d0 ) then
  cwtr = -(1.257299d0 / 1.2d0) * cw**1.2d0 - (0.286311d0 / 6.5d0) * cw**6.5d0 + &
    1.542586d0 * cw - 0.00150659d0
else if ( cw < 0.540932161d0 ) then
  cwtr = -(1.607077d0 / 1.2d0) * cw**1.2d0 - (1.475696d0 / 1.55d0) * &
    dexp(-1.55d0 * cw) - 1.637803d0 * cw + 0.98777752d0
else if ( cw < 0.862110631d0 ) then
  cwtr = -(0.687757d0 / 0.35d0) * dexp(-0.35d0 * cw) - (0.424043d0 / 1.35d0) * &
    dexp(-1.35d0 * cw) - 0.351856d0 * cw + 2.29858919d0
else if ( cw < 1.081864321d0 ) then
  cwtr = -(0.51421d0 / 0.4d0) * dexp(-0.4d0 * cw) - (0.877948d0 / 2.9d0) * &
    dexp(-2.9d0 * cw) - 0.147171d0 * cw + 1.50626904d0
else if ( cw < 1.572084092d0 ) then
  cwtr = -(4.289368d0 / 0.25d0) * dexp(-0.25d0 * cw) + (4.153888d0 / 0.55d0) * &
    dexp(-0.55d0 * cw) - (2.186503d0 / 1.3d0) * dexp(-1.3d0 * cw) - &
    1.292888d0 * cw + 11.23676668d0
else if ( cw < 3.211784704d0 ) then
  cwtr = -(0.102092d0 / 0.3d0) * dexp(-0.3d0 * cw) - (0.647197d0 / 1.4d0) * &
    dexp(-1.4d0 * cw) + 0.00085d0 * cw + 0.84811899d0
else if ( cw < 16.904130019d0 ) then
  cwtr = -(0.028188d0 / 0.15d0) * dexp(-0.15d0 * cw) - (0.221879d0 / 0.63d0) * &
    dexp(-0.63d0 * cw) + 0.000527d0 * cw + 0.8767982d0
else if ( cw < 1d3 ) then
  cwtr = -(0.088498d0 / 0.254d0) * cw**(-0.254d0) + 1.0407112d0
else
  cwtr = 1d0
end if

```

WSTCMB=CWTR*WSCCMB

OASDI taxable earnings of workers with both wages and self-employment earnings

```

TECCMB=SECCMB+WSCCMB
ATECMB=TECCMB/CMBNT
ATECMB17=ATECMB(2017)
ATECMB=ATECMB*BASECT/ATECMB17
CT=TAXMAX/ATECMB

```

```

if ( ct < 0.025948791d0 ) then
  ctr = ct - (40.88231d0 / 3.3d0) * ct**3.3d0
else if ( ct < 0.051897583d0 ) then
  ctr = -(0.015962d0 / 1.4d0) * ct**1.4d0 - (2.849709d0 / 2.5d0) * ct**2.5d0 + &
    1.005757d0 * ct - 0.00002946d0
else if ( ct < 0.097307968d0 ) then
  ctr = -(1.740332d0 / 2.25d0) * ct**2.25d0 - (4.39659d0 / 4.3d0) * ct**4.3d0 + &
    1.010732d0 * ct - 0.00017087d0
else if ( ct < 0.233539123d0 ) then
  ctr = -(3.007061d0 / 2.35d0) * ct**2.35d0 + (29.74953d0 / 5.5d0) * ct**5.5d0 + &
    1.047821d0 * ct - 0.00257006d0
else if ( ct < 0.376257476d0 ) then
  ctr = -(17.9483d0 / 1.85d0) * ct**1.85d0 + (56.48945d0 / 0.34d0) * dexp(-0.34d0 * ct) + &
    58.06407d0 * ct - 166.16320626d0
else if ( ct < 0.518975828d0 ) then
  ctr = (0.35103d0 / 0.65d0) * dexp(-0.65d0 * ct) - (1.220332d0 / 1.25d0) * &
    dexp(-1.25d0 * ct) + 0.050683d0 * ct + 0.455703736d0
else if ( ct < 0.700617368d0 ) then
  ctr = -(0.158062d0 / 0.45d0) * dexp(-0.45d0 * ct) - (0.865629d0 / 1.25d0) * &
    dexp(-1.25d0 * ct) - 0.139568d0 * ct + 1.069620633d0
else if ( ct < 0.869284513d0 ) then
  ctr = -(10.956926d0 / 0.45d0) * dexp(-0.45d0 * ct) + (13.02327d0 / 1.35d0) * &
    dexp(-1.35d0 * ct) - (8.498083d0 / 2.45d0) * dexp(-2.45d0 * ct) - &

```

```

4.127007d0 * ct + 17.95991367d0
else if ( ct < 1.076874844d0 ) then
  ctr = -(2.015557d0 / 0.35d0) * dexp(-0.35d0 * ct) + (3.017185d0 / 1.2d0) * &
  dexp(-1.2d0 * ct) - (3.225627d0 / 2.0d0) * dexp(-2.0d0 * ct) - &
  0.725524d0 * ct + 4.75386607d0
else if ( ct < 7.784637426d0 ) then
  ctr = -(0.100861d0 / 0.35d0) * dexp(-0.35d0 * ct) - (0.166637d0 / 1d0) * &
  dexp(-1d0 * ct) - (0.732639d0 / 2.15d0) * dexp(-2.15d0 * ct) + &
  0.004175d0 * ct + 0.809186759d0
else if ( ct < 25.948791419d0 ) then
  ctr = -(0.036719d0 / 0.18d0) * dexp(-0.18d0 * ct) - (0.780182d0 / 0.9d0) * &
  dexp(-0.9d0 * ct) + 0.001067d0 * ct + 0.865444162d0
else if ( ct < 1d3 ) then
  ctr = -(0.142018d0 / 0.34d0) * ct**(-0.34d0) + 1.029280103d0
else
  ctr = 1d0
end if

```

```

TETCMB=CTTR*TECCMB
SETCMB=TETCMB-WSTCMB

```

Ratio OASDI taxable to covered self-employment earnings

RSET = (SETCMB+SETSEO)/CSE

Where

ASEO17	=	Average self-employment earnings of workers with no OASDI taxable wages in 2017
ATECMB	=	Average OASDI covered earnings of workers with both OASDI covered wages and self-employment earnings
ATECMB17	=	Average OASDI covered earnings of workers with both OASDI covered wages and self-employment earnings in 2015
AWSCMB	=	Average OASDI covered wage of workers with both wages and self-employment earnings
AWSCMB17	=	Average OASDI covered wage of workers with both wages and self-employment earnings in 2017
ASESEO	=	Average self-employment earnings of workers with no OASDI taxable wages
AWSCMB	=	Average OASDI covered wage of workers with both wages and self-employment earnings
BASECT	=	Average total earnings of workers with both self-employment earnings and wages in 1% sample data for 2017 used to produce equations
BASECW	=	Average OASDI covered wages of workers with both self-employment earnings and wages in 1% sample data for 2017 used to produce equations
BASEO	=	Average self-employment earnings of workers with no OASDI taxable wages in 1% sample data for 2017 used to produce equations
CMBNT	=	Number of workers with both OASDI taxable wages and self-employment earnings
CSE	=	OASDI covered self-employment earnings
CT	=	Ratio OASDI taxable maximum to average earnings of workers with both self-employment earnings and OASDI taxable wages
CW	=	Ratio OASDI taxable maximum to average self-employment earnings of workers with both self-employment earnings and OASDI taxable wages
CTTR	=	Ratio of OASDI taxable to covered earnings for workers with both wages and self-employment earnings
CWTR	=	Ratio of OASDI taxable to covered wages for workers with both wages and self-employment earnings
O	=	Ratio OASDI taxable maximum to average self-employment earnings of workers with no OASDI taxable wages
OTR	=	Ratio of OASDI taxable self-employment to covered earnings for workers with no OASDI taxable wages
SECCMB	=	OASDI covered self-employment earnings of workers with both self-employment earnings and OASDI taxable wages
SECSEO	=	OASDI covered self-employment earnings of workers with no OASDI taxable wages
SEO	=	Number of workers with OASDI covered self-employment earnings and no OASDI taxable wages
SETCMB	=	OASDI taxable self-employment earnings of workers with both OASDI taxable wages and self-employment earnings
SETSEO	=	OASDI taxable self-employment earnings of workers with no OASDI taxable wages
TAXMAX	=	OASDI taxable maximum
TECCMB	=	OASDI covered earnings of workers with both wages and self-employed earnings
TETCMB	=	OASDI taxable earnings of workers with both wages and self-employed earnings
WSCCMB	=	OASDI covered wages of workers with both wages and self-employed earnings
WSTCMB	=	OASDI taxable wages of workers with both wages and self-employed earnings

2.4.7 OASDI taxable self-employment earnings (SET)

SET = RSET * CSE

Where

CSE	=	OASDI covered self-employment earnings
SET	=	OASDI taxable self-employment earnings
RSET	=	Ratio of OASDI taxable to covered self-employment earnings

2.4.8 OASDI effective taxable payroll (ETP)

$$\text{ETP} = \text{WTER} + \text{SET} - 0.5 * \text{MER}$$

Where

ETP	=	OASDI effective taxable payroll
MER	=	OASDI multi-employer refund wages
SET	=	OASDI taxable self-employment earnings
WTER	=	Annual OASDI taxable employer wages

2.5 Revenues (REVENUES)

2.5.1 OASDI taxable wage liability (WTL)

$$WTL = WTER * TRW$$

Where

TRW	=	OASDI combined employee-employer tax rate
WTL	=	Annual OASDI taxable wage liabilities
WTER	=	Annual OASDI taxable employer wages

2.5.2 OASDI taxable self-employment liability (SEL)

$$SEL = SET * TRSE$$

Where

SEL	=	OASDI taxable self-employment earnings liabilities
SET	=	OASDI taxable self-employment earnings
TRSE	=	OASDI self-employment tax rate

2.5.3 OASDI multi-employer refund wage liability (MERL)

$$MERL = MER * TRWEE$$

Where

MERL	=	OASDI multi-employer refund wage liabilities
MER	=	OASDI multi-employer refund wages
TRWEE	=	OASDI employee tax rate

2.5.4 OASDI quarterly taxable wage liability (WTLQ)

Federal Civilian

Annual total wages (OASDI + MQGE)

$$BAFCW = 34198.84$$

$$AWCFC = WCFC / ECFC * BAFCW / AWCFCOT97$$

$$T = \text{MAX} / AWCFC$$

IF(T.LT.0.014620379)THEN

$$FCTR = T - (1.04262 / 1.73) * T ** 1.73$$

ELSE IF(T.LT.0.292407578)THEN

$$FCTR = -(1.22471 / 1.6) * T ** 1.6 + (.826746 / 1.8) * \text{DEXP}(-1.8 * T) + 1.8535 * T - .459368449$$

ELSE IF(T.LT.0.760259704)THEN

$$FCTR = -(.635082 / 2D0) * T ** 2 + (.604884 / 2.9) * T ** 2.9 - (.403213 / 4.6) * T ** 4.6 + .910343 * T + .002291358$$

ELSE IF(T.LT.1.228111829)THEN

$$FCTR = -(1.162181 / 1.7) * T ** 1.7 + (.143632 / 2.7) * T ** 2.7 - (.312012 / 3.4) * T ** 3.4 + .841165 * T + .011332647$$

ELSE IF(T.LT.1.520519407)THEN

$$FCTR = -(1.34084 / 3.5) * T ** 3.5 + (1.09868 / 5D0) * T ** 5 - (.404253 / 5.8) * T ** 5.8 + 1.17397 * T - .222555715$$

ELSE IF(T.LT.2.339260627)THEN

$$FCTR = (.671304 / .5) * \text{DEXP}(-.5 * T) - (3.27076 / 1.4) * \text{DEXP}(-1.4 * T) + .126626 * T + .353367869$$

ELSE IF(T.LT.3.50889094)THEN

$$FCTR = (.0571643 / .95) * \text{DEXP}(-.95 * T) - (3.17633 / 1.8) * \text{DEXP}(-1.8 * T) + .000623031 * T + .996284293$$

ELSE IF(T.LT.4.970928832)THEN

$$FCTR = -(12.3148 / 2.25) * \text{DEXP}(-2.25 * T) + .0000698013 * T + .999222265$$

ELSE

$$FCTR = -(0.0285502 / 2D0) * T ** (-2D0) + 1.00007094$$

END IF

$$WTFCTOT = FCTR * WCFC$$

Where

AWCFC	=	Average covered Federal Civilian wages (OASDI plus MQGE)
AWCFCOT97	=	Average covered Federal Civilian wages (OASDI plus MQGE) for 1997
BAFCW	=	Average Federal Civilian wages (OASDI plus MQGE) in 1% sample data for 1997 used to produce equations
ECFC	=	Covered Federal Civilian employment (OASDI plus MQGE)
FCTR	=	Ratio of taxable to covered Federal Civilian wages (OASDI plus MQGE)
MAX	=	OASDI taxable maximum
T	=	Ratio of the OASDI taxable maximum to average covered Federal Civilian wages (OASDI plus MQGE)

WCFC = Covered Federal Civilian wages (OASDI plus MQGE)
WTFCTOT = Taxable Federal Civilian wages (OASDI plus MQGE)

Annual MQGE wages

BAFCW = 50147.72
AWCFC = WCFC / ECFC * BAFCW / AWCFCO97
T = MAX / AWCFC

```
IF(T.LT.0.019941085)THEN
  FCTR=T-(0.0450661/1.47)*T**1.47
ELSE IF(T.LT.0.099705424)THEN
  FCTR=(-.0518044/1.9)*T**1.9-(.0368056/2.3)*T**2.3+.99479*T+.0000248091
ELSE IF(T.LT.0.358939528)THEN
  FCTR=(-.05907/1.25)*T**1.25-(.0746657/2.9)*T**2.9+1.02092*T-.00032173
ELSE IF(T.LT.0.558350377)THEN
  FCTR=(-2.4664/1.4)*T**1.4+(4.82919/2.3)*T**2.3-(3.97473/3)*T**3+1.83998*T-.026694932
LSE IF(T.LT.0.797643395)THEN
  FCTR=(.609091/2.1)*T**2.1-(1.16086/4)*T**4+.788373*T+.043208139
ELSE IF(T.LT.1.196465093)THEN
  FCTR=(2.35647/4)*DEXP(-.4*T)-(3.87811/1.2)*DEXP(-1.2*T)-(1.1179/2.5)*DEXP(-2.5*T)+.738296*T-2.83402534
ELSE IF(T.LT.1.694992215)THEN
  FCTR=(-.422884/1.3)*DEXP(-1.3*T)-(6.90241/3D0)*DEXP(-3*T)-.0229917*T+1.068147457
ELSE IF(T.LT.2.592341034)THEN
  FCTR=(-.557032/1.2)*DEXP(-1.2*T)-(5.40739/2.2)*DEXP(-2.2*T)+.0102014*T+.960037325
ELSE
  FCTR=(-(32.3187/3.5)*DEXP(-3.5*T)+1.000030482
END IF
```

WTFCHO=FCTR*WCFC

Where

AWCFC = Average covered Federal Civilian MQGE wages
AWCFCO97 = Average covered Federal Civilian MQGE wages for 1997
BAFCW = Average Federal Civilian MQGE wages in 1% sample data for 1997 used to produce equations
ECFC = Covered Federal Civilian MQGE employment
FCTR = Ratio of taxable to covered Federal Civilian MQGE wages
MAX = OASDI taxable maximum
T = Ratio of the OASDI taxable maximum to average covered Federal Civilian MQGE wages
WCFC = Covered Federal Civilian MQGE wages
WTFCHO = Taxable Federal Civilian MQGE wages

Annual OASDI taxable wages

WTFC = WTFCTOT - WTFCHO

Where

WTFC = Annual OASDI taxable Federal Civilian wages
WTFCHO = Taxable Federal Civilian MQGE wages
WTFCTOT = Taxable Federal Civilian wages (OASDI plus MQGE)

Quarterly OASDI covered wages

CFCQD(1) = .98357 * TCFCD(I,1) + FCPD(I,1)
CFCQD(2) = .98909 * TCFCD(I,2) + FCPD(I,2)
CFCQD(3) = 1.01833 * TCFCD(I,3) + FCPD(I,3)
CFCQD(4) = 1.00814 * TCFCD(I,4) + FCPD(I,4)

QWFCOD(J) = CFCQD(J) * WTFC

Where

CFCQD = Proportion of annual OASDI covered Federal Civilian wages paid in each quarter
FCPD = Payday variable for Federal Civilian wages based on calendar
I = Calendar year
J = Quarter
TCFCD = Proportion of annual NIPA Federal Civilian wages paid in each quarter
QWFCOD = Quarterly OASDI covered Federal Civilian wages
WTFC = Annual OASDI taxable Federal Civilian wages

Quarterly OASDI taxable wages

```
IF(FCTR.LE.0.928)FCQD(2)=CFCQD(2)+.27522*(1.-FCTR)-.15127*(1.-FCTR)**2+.35146*(1.-FCTR)**3
IF(FCTR.LE.0.993)THEN
  FCQD(3)=CFCQD(3)+.28047*(1.-FCTR)-4.73021*(1.-FCTR)**2+25.3606*(1.-FCTR)**3-58.1741*(1.-FCTR)**4+45.1465*(1.-FCTR)**5
  FCQD(4)=CFCQD(4)-.75095*(1.-FCTR)+3.65109*(1.-FCTR)**2-16.9355*(1.-FCTR)**3+23.9578*(1.-FCTR)**4
END IF
First quarter is always 100 percent taxable.
QWTFC(I,1)=QWCFC(I,1)
IF(FCTR.LE.0.928)THEN
  Compute taxable for 2nd-4th quarter.
  FCQ=FCQD(2)+FCQD(3)+FCQD(4)
  WTFC2=WTFC-QWTFC(I,1)
  FCQD(2:4)=FCQD(2:4)/FCQ
  QWTFC(I,2:4)=FCQD(2:4)*WTFC2
ELSE IF(FCTR.LE.0.993)THEN
  Second quarter covered is completely taxable.
  QWTFC(I,2)=QWCFC(I,2)
  QWTFC(I,3)=FCQD(3)*WTFC
  QWTFC(I,4)=WTFC-QWTFC(I,1)-QWTFC(I,2)-QWTFC(I,3)
ELSE
  Second and third quarter covered is completely taxable.
  QWTFC(I,2)=QWCFC(I,2)
  QWTFC(I,3)=QWCFC(I,3)
  QWTFC(I,4)=WTFC-QWTFC(I,1)-QWTFC(I,2)-QWTFC(I,3)
END IF
```

Where

CFCQD	=	Proportion of annual OASDI covered Federal Civilian wages paid in each quarter
FCQ	=	Sum of proportions of annual OASDI covered Federal Civilian wages paid in each quarter for quarters two to four
FCQD	=	Proportion of annual OASDI taxable Federal Civilian wages paid in each quarter
FCTR	=	Ratio annual OASDI taxable to covered Federal Civilian wages
I	=	Calendar year
TCFCD	=	Proportion of annual NIPA Federal Civilian wages paid in each quarter
QWCFC	=	Quarterly OASDI covered Federal Civilian wages
QWTFC	=	Quarterly OASDI taxable Federal Civilian wages
WTFC	=	Annual OASDI taxable Federal Civilian wages
WTFC2	=	Total OASDI taxable Federal Civilian wages paid in quarters two to four

Quarterly OASDI taxable wage liabilities

```
WTLQFCEE(I, J) = QWTFC(I, J) * TRWEE(I)
WTLQFCER(I, J) = QWTFC(I, J) * TRWER(I)
WTLQFC(I, J) = WTLQFCEE(I, J) + WTLQFCER(I, J)
```

Where

I	=	Calendar year
J	=	Quarter
TRWEE	=	OASDI employee tax rate
TRWER	=	OASDI employer tax rate
WTLQFC	=	Quarterly OASDI taxable Federal Civilian combined employee-employer wage liabilities
WTLQFCEE	=	Quarterly OASDI taxable Federal Civilian employee wage liabilities
WTLQFCER	=	Quarterly OASDI taxable Federal Civilian employer wage liabilities

Military wages

Annual OASDI taxable wages

```
BACMW = 16439.95
ACMW = AWCML * BACMW / AWCML97
T = MAX / ACMW
```

```
IF(T.LT.0.060827432)THEN
  MTR=T-(.712875/2)*T**2
ELSE IF(T.LT.0.182482295)THEN
  MTR=(.71197/1.8)*T**1.8-(1.59752/2D0)*T**2+.97587*T+0.000542413
ELSE IF(T.LT.0.608274315)THEN
  MTR=-(1.75026/2D0)*T**2+(2.86837/3D0)*T**3-(1.90346/4D0)*T**4+1.10056*T-.006441373
ELSE IF(T.LT.1.094893767)THEN
  MTR=-(.700864/1.4)*T**1.4-(.40042/3.3)*T**3.3+(.197091/4.1)*T**4.1+1.33615*T-.056637087
```

```

ELSE IF(T.LT.1.703168082)THEN
  MTR=(21.3527/.3)*DEXP(-.3*T)-(21.1277/0.5)*DEXP(-.5*T)+(2.73027/1.1)*DEXP(-1.1*T)+4.34833*T-31.56802874
ELSE IF(T.LT.2.311442397)THEN
  MTR=(33.3894/1.2)*T**1.2+(14.9436/1.6)*T**1.6-(2.58041/2.1)*T**2.1+21.3365*T-.872981629
ELSE IF(T.LT.3.163026438)THEN
  MTR=(-.076094/.3)*DEXP(-.3*T)-(1.59668/1.4)*DEXP(-1.4*T)-.0271355*T+1.182946986
ELSE IF(T.LT.4.257920205)THEN
  MTR=(.482918/1.5)*T**1.5-(9.21141/.9)*DEXP(-.9*T)+(25.93/1.5)*DEXP(-1.5*T)-1.14706*T+3.246003821
ELSE
  MTR=(-9.00723/1.8)*DEXP(-1.8*T)+1.000285789
END IF
WTML=MTR*WCML

```

Where

ACMW	=	Average OASDI covered military wages adjusted for level used to produce equations
AWCML	=	Average OASDI covered military wages
AWCML97	=	Average OASDI covered military wages in 1997
BACMW	=	Average OASDI covered military wages in 1% sample data for 1997 used to produce equations
MAX	=	OASDI taxable maximum
MTR	=	Ratio of OASDI taxable to covered military wages
T	=	Ratio of the OASDI taxable maximum to average covered military wages
WCML	=	Annual OASDI covered military wages
WTML	=	Annual OASDI taxable military wages

Quarterly OASDI covered wages

CMLQD(1)	=	.97978*TCMLD(I,1)*MLPD(I,1)
CMLQD(2)	=	1.002*TCMLD(I,2)*MLPD(I,2)
CMLQD(3)	=	1.02145*TCMLD(I,3)*MLPD(I,3)
CMLQD(4)	=	.99689*TCMLD(I,4)*MLPD(I,4)
QWCML	=	CMLQD(J)*WCML

Where

CMLQD	=	Proportion of annual OASDI covered military wages paid in each quarter
I	=	Calendar year
J	=	Quarter
MLPD	=	Payday variable for military wages based on calendar
QWCML	=	Quarterly OASDI covered military wages
TCMLD	=	Proportion of annual NIPA military wages paid in each quarter
WCML	=	Annual OASDI covered military wages

Quarterly OASDI taxable wages

```

T=MAX/AWCML
IF(MLTR.LT.0.776)QML(1)=CMLQD(1)+.393565-.018307*T-3.44641/T+15.6381/T**2-40.0168/T**3+62.0449/T**4-
57.525/T**5+30.2498/T**6-7.8664/T**7+.674629/T**8
IF(MLTR.LT.0.952)QML(2)=CMLQD(2)+.844748-.0401062*T-7.24247/T+32.4957/T**2-83.3328/T**3+129.374/T**4-
122.526/T**5+68.2737/T**6-20.1479/T**7+2.34289/T**8
IF(MLTR.LT.0.985)QML(3)=CMLQD(3)-2.62266+.125592*T+22.5832/T-105.727/T**2+300.027/T**3-540.915/T**4+622.304/T**5-
441.658/T**6+175.722/T**7-29.8987/T**8
IF(MLTR.LT.1.)QML(4)=CMLQD(4)+2.37295-.111565*T-21.1954/T+106.049/T**2-330.637/T**3+658.869/T**4-
835.626/T**5+648.641/T**6-279.392/T**7+50.9246/T**8
IF(MLTR.LT.0.776)THEN
  QWTML(I,1:4)=QML(1:4)*WTML
ELSE IF(MLTR.LT.0.952)THEN
  QWTML(I,1)=QWCML(I,1)
  TOTWG1=WTML-QWTML(I,1)
  Q1=QML(2)+QML(3)+QML(4)
  QML(2:4)=QML(2:4)/Q1
  QWTML(I,2:4)=QML(2:4)*TOTWG1
ELSE IF(MLTR.LT.0.985)THEN
  QWTML(I,1)=QWCML(I,1)
  QWTML(I,2)=QWCML(I,2)
  TOTWG1=WTML-QWTML(I,1)-QWTML(I,2)
  Q1=QML(3)+QML(4)
  QML(2:4)=QML(2:4)/Q1
  QWTML(I,2:4)=QML(2:4)*TOTWG1
ELSE IF(MLTR.LT.1.)THEN
  QWTML(I,1)=QWCML(I,1)
  QWTML(I,2)=QWCML(I,2)

```

$QWTML(I,3)=QWCML(I,3)$
 $QWTML(I,4)=WTML-QWTML(I,1)-QWTML(I,2)-QWTML(I,3)$
 END IF

Where

AWCML = Average OASDI covered military wages
 CMLQD = Proportion of annual OASDI covered military wages paid in each quarter
 MLTR = Ratio of OASDI taxable to covered military wages
 MAX = OASDI taxable maximum
 I = Calendar year
 Q1 = Sum of proportions of annual OASDI taxable military wages paid in each quarter for last three or two quarters in year
 QML = Proportion of annual OASDI taxable military wages paid in each quarter
 QWCML = Quarterly OASDI covered military wages
 QWTML = Quarterly OASDI taxable military wages
 T = Ratio of the OASDI taxable maximum to average covered military wages
 TOTWG1 = Annual OASDI taxable military wages for all quarters except first or first and second
 WTML = Annual OASDI taxable military wages

Quarterly OASDI taxable wage liabilities

$WTLQMLEE(I, J) = QWTML(I, J) * TRWEE(I)$
 $WTLQMLER(I, J) = QWTML(I, J) * TRWER(I)$
 $WTLQML(I, J) = WTLQMLEE(I, J) + WTLQMLER(I, J)$

Where

I = Calendar year
 J = Quarter
 TRWEE = OASDI employee tax rate
 TRWER = OASDI employer tax rate
 WTLQML = Quarterly OASDI taxable military combined employee-employer wage liabilities
 WTLQMLEE = Quarterly OASDI taxable military employee wage liabilities
 WTLQMLER = Quarterly OASDI taxable military employer wage liabilities

Federal

$WCF = WCFC + WCML$
 $QWCF = QWCFC + QWCML$
 $WTF = WTFC + WTML$
 $QWTF = QWTFC + QWTML$
 $WTLQFEE(I,J) = QWTF(I,J) * TRWEE(I)$
 $WTLQFER(I,J) = QWTF(I,J) * TRWER(I)$
 $WTLQF(I,J) = WTLQFEE(I,J) + WTLQFER(I,J)$

Where

I = Calendar year
 J = Quarter
 QWCF = Quarterly OASDI covered Federal wages
 QWCF C = Quarterly OASDI covered Federal Civilian wages
 QWCML = Quarterly OASDI covered military wages
 QWTF = Quarterly OASDI taxable Federal wages
 QWTFC = Quarterly OASDI taxable Federal Civilian wages
 QWTML = Quarterly OASDI taxable military wages
 WCF = Annual OASDI covered Federal wages
 WCFC = Annual OASDI covered Federal Civilian wages
 WCML = Annual OASDI covered military wages
 WTF = Annual OASDI taxable Federal wages
 WTFC = Annual OASDI taxable Federal Civilian wages
 WTLQF = Quarterly OASDI taxable Federal combined employee-employer wage liabilities
 WTLQFEE = Quarterly OASDI taxable Federal employee wage liabilities
 WTLQFER = Quarterly OASDI taxable Federal employer wage liabilities
 WTML = Annual OASDI taxable military wages

State and Local wages

Annual OASDI taxable wages

$BACW = 21583.61$
 $AWCSL = WCSL / ESLC * BACW / AWCSLOD97$
 $S = MAX / ASLC$
 IF(S.LT.0.02316573)THEN

```

SLTR=S-(1.1803/1.71)*S**1.71
ELSE IF(S.LT.0.463314609)THEN
SLTR=(1.54738/1.6)*S**1.6-(.421147/2.5)*S**2.5+(3.34881/5)*DEXP(-.5*S)+4.39012*S-6.697774474
ELSE IF(S.LT.0.833966296)THEN
SLTR=(-.756943/1.8)*S**1.8+(.485982/2.3)*S**2.3-(.175681/3.2)*S**3.2+.88749*S+.004652169
ELSE IF(S.LT.1.945921357)THEN
SLTR=(3.4167/3)*DEXP(-.3*S)-(7.26467/9)*DEXP(-.9*S)+(4.57049/1.5)*DEXP(-1.5*S)+1.0378*S-6.245057503
ELSE IF(S.LT.3.243202261)THEN
SLTR=(-2.40293/2)*DEXP(-.2*S)+(6.44952/4)*DEXP(-.4*S)-(.564852/6)*DEXP(-.6*S)-.278204*S+.099074279
ELSE IF(S.LT.5.559775305)THEN
SLTR=(-.0434955/6)*DEXP(-.6*S)-(4.00403/1.7)*DEXP(-1.7*S)+.00006219*S+.997065459
ELSE IF(S.LT.18.53258435)THEN
SLTR=(-.0272758/5)*DEXP(-.5*S)+.0000671826*S+.997657785
ELSE
SLTR=(-.00861948/7)*S**(-.7)+1.000492941
END IF
WTSL=SLTR*WCSL

```

Where

AWCSL	=	Average OASDI covered State and Local wages adjusted for average wage used to produce equations
AWCSL097	=	Average OASDI covered State and Local wages for 1997
BACW	=	Average OASDI covered State and Local wages in 1% sample data for 1997 used to produce equations
ESLC	=	OASDI covered State and Local employment
MAX	=	OASDI taxable maximum
S	=	Ratio of the OASDI taxable maximum to average covered State and Local wages
SLTR	=	Ratio of OASDI taxable to covered State and Local wages
WCSL	=	OASDI covered State and Local wages
WTSL	=	OASDI taxable State and Local wages

Quarterly OASDI covered wages

```

CSLQD(1)=1.0131455*TCSLD(I,1)+SLPD(I,1)
CSLQD(2)=1.0431906*TCSLD(I,2)+SLPD(I,2)
CSLQD(3)=.9060524*TCSLD(I,3)+SLPD(I,3)
CSLQD(4)=1.0365866*TCSLD(I,4)+SLPD(I,4)
QWCSL=CSLQD(I,4)*WCSL

```

Where

CSLQD	=	Proportion of annual OASDI covered State and Local wages paid in each quarter
I	=	Calendar year
QWCSL	=	Quarterly OASDI covered State and Local wages
SLPD	=	Payday variable for State and Local wages based on calendar
TCSLD	=	Proportion of annual NIPA State and Local wages paid in each quarter
WCSL	=	Annual OASDI covered State and Local wages

Quarterly OASDI taxable wages

```

QSL(1)=(CSLQD(1)-.24087*(1.-/SLTR))
QSL(2)=(CSLQD(2)-1.0492*(1.-/SLTR)+.51259*(1.-/SLTR**2)-.07643*(1.-/SLTR**3))
QSL(3)=(CSLQD(3)-5.99032*(1.-SLTR**2)+13.238*(1.-SLTR**3)-11.3291*(1.-SLTR**4)+3.52237*(1.-SLTR**5))
QSL(4)=(CSLQD(4)+8.99897*(1.-SLTR**25)-5.48866*(1.-SLTR**5))
TQSL=QSL(2)+QSL(3)+QSL(4)
QSL(2:4)=QSL(2:4)/TQSL
QWTSL(I,1)=QSL(1)*WTSL
QWTSL(I,2:4)=QSL(2:4)*(WTSL- QWTSL(I,1))

```

Where

CSLQD	=	Proportion of annual OASDI covered State and Local wages paid in each quarter
I	=	Calendar year
QSL	=	Proportion of annual OASDI taxable State and Local wages paid in each quarter
QWTSL	=	Quarterly OASDI taxable State and Local wages
SLTR	=	Ratio of OASDI taxable to covered State and Local wages
WTSL	=	OASDI taxable State and Local wages

Quarterly OASDI taxable wage liabilities

```

WTLQSL(I,J) = QWTSL(I,J) * TRW(I)

```

Where

I	=	Calendar year
---	---	---------------

J = Quarter
 TRW = OASDI combined employee-employer tax rate
 WTLQSL = Quarterly OASDI taxable State and Local combined employee-employer wage liabilities

Private household quarterly OASDI taxable wages and liabilities

$QWTPHH(I,J) = WCPHH(I) * QDPHH(J)$
 $WTLQPHH(I,J) = QWTPHH(I,J) * TRW(I)$

Where
 I = Calendar year
 J = Quarter
 QDPHH = Proportion of annual OASDI taxable private household wages paid in each quarter
 QWTPHH = Quarterly OASDI taxable private household wages
 TRW = OASDI combined employee-employer tax rate
 WCPHH = Annual OASDI covered private household wages
 WTLQPHH = Quarterly OASDI taxable private household combined employee-employer wage liabilities

Farm taxable wages

Annual OASDI

BAFMW = 7467.91
 AWCfm97 = ACFMW(1997)
 $F = \text{MAX} / (\text{ACFMW} * \text{BAFMW} / \text{AWCFM97})$

IF(F.LT.0.066953142)THEN
 FMTR=F- (1.30211/1.75)*F**1.75
 ELSE IF(F.LT.0.401718855)THEN
 FMTR=-(1.18244/1.35)*F**1.35+(.25412/1.75)*F**1.75+1.24681*F-.001598087
 ELSE IF(F.LT.0.669531425)THEN
 FMTR=-(.508764/.6)*DEXP(-.6*F)-(.300083/2.8)*DEXP(-2.8*F)+.0188542*F+.966550312
 ELSE IF(F.LT.1.87468799)THEN
 FMTR=-(.638146/.6)*DEXP(-.6*F)-(.0322774/1.5)*DEXP(-1.5*F)-.033706*F+1.133974442
 ELSE IF(F.LT.2.41031313)THEN
 FMTR=-(2.64644/1.1)*DEXP(-1.1*F)+(17.4638/2)*DEXP(-2*F)-(26.4191/2.5)*DEXP(-2.5*F)+.00686748*F+.909154345
 ELSE IF(F.LT.4.82062626)THEN
 FMTR=-(1.06567/1.3)*F**1.3+(-.073837/2.1)*F**2.1+1.31021*F-.007628879
 ELSE IF(F.LT.6.427501679)THEN
 FMTR=-(.178355/.5)*DEXP(-.5*F)-(1.70356/1.3)*DEXP(-1.3*F)+.00115171*F+.959096096
 ELSE IF(F.LT.10.7125028)THEN
 FMTR=-(.0474377/0.35)*DEXP(-.35*F)-(1.32456/1)*DEXP(-1*F)+.0016146*F+.957903052
 ELSE IF(F.LT.11.38203422)THEN
 FMTR=-(.0581938/.35)*DEXP(-.35*F)+.00130453*F+.961918378
 ELSE IF(F.LT.24.1031313)THEN
 FMTR=-(.0492564/.3)*DEXP(-.3*F)+.000761577*F+.97040299
 ELSE
 FMTR=-(.00304904/.06)*DEXP(-.06*F)+1.000606299
 END IF
 TFMW=FMTR*WCFM

Where
 ACFMW = Annual average OASDI covered farm wages
 AWCfm97 = Annual average OASDI covered farm wages for 1997
 BAFMW = Average farm wage in 1% sample data for 1997 used to produce equations
 F = Ratio of taxable maximum to annual average OASDI covered farm wages adjusted for average wage used in equations
 FMTR = Ratio of OASDI taxable to covered farm wages
 MAX = OASDI taxable maximum
 TFMW = Annual OASDI taxable farm wages

Quarterly OASDI wages and liabilities

$QWTFM(I,J) = TTFMD(I,J) * TFMW$
 $WTLQFM(I,J) = QWTFM(I,J) * TRW(I)$

Where
 I = Calendar year
 J = Quarter
 QWTFM = Quarterly OASDI taxable farm wages

TFMW = Annual OASDI taxable farm wages
 TRW = OASDI com
 TTFMD = Proportion of annual OASDI taxable farm wages paid in each quarter
 WTLQFM = Quarterly OASDI taxable farm combined employee-employer wage liabilities

Quarterly OASDI taxable employee tips

QWTTIPSEE(L,J) = QDTIP(J) * WTTIPSEE(I)
 QWTTIPSEE(L,2) = QWTTIPSEE(L,2) + WTTIPSSR(I)
 WTLQTIPSEE(L,J) = QWTTIPSEE(L,J) * TRW(I)

Where

I = Calendar year
 J = Quarter
 QDTIP = Proportion of annual OASDI taxable tips received in each quarter
 QWTTIPSEE = Quarterly OASDI taxable tips received by employees
 WTLQTIPSEE = Quarterly OASDI combined employee-employer wage liabilities on taxable tips received by employees
 TRW = OASDI combined employee-employer tax rate
 WTTIPSEE = Annual OASDI taxable tips received by employees reported by employers
 WTTIPSSR = Annual OASDI taxable tips received by employees self-reported on income tax returns

Private nonfarm OASDI taxable wages and liabilities

Annual

WTPNF = WTER – WTFC – WTML – WTSL – TFMW – WTTIPSEE – WTTIPSSR

Where

TFMW = Annual OASDI taxable farm wages
 WTSL = Annual OASDI taxable State and Local wages
 WTFC = Annual OASDI taxable Federal Civilian wages
 WTPNF = Annual OASDI taxable private nonfarm wages excluding tips
 WTTIPSEE = Annual OASDI taxable tips received by employees reported by employers
 WTTIPSSR = Annual OASDI taxable tips received by employees self-reported on income tax returns
 WTER = Annual OASDI taxable employer wages

Quarterly

BACW93 = 21912.00
 NACW = BACW93 / ACW93 * AWC
 X = MAX / NACW
 IF(X.LT.0.91274)THEN
 TWTR=1D0+.990751*DEXP(X)**(-1)/(-1)-.013904602
 ELSE IF(X.LT.2.05367)THEN
 TWTR=1D0+(-.003129*X+(1.167562*DEXP(X)**(-1.17)/(-1.17)))-.065747345
 ELSE IF(X.LT.4.791895)THEN
 TWTR=1D0+(.003962*X+(.770093*X**(-1.85053))/(-1.85053))-0.6071106
 ELSE
 TWTR=1D0+(.267708*X**(-.94))/(-.94)+.00066
 END IF
 IF(TWTR.LT.0.70)THEN
 QP(1)=-(-0.000575+0.18692*DLOG(TWTR)-0.23133*DLOG(TWTR)**2-
 0.10453*DLOG(TWTR)**3+0.04306*DLOG(TWTR)**4+0.01906*DLOG(TWTR)**5)-0.0325201+PD(1)+TCPD(I,1)
 QP(2)=-(-0.00657+1.7015*TWTR-8.60615*TWTR**2+14.444*TWTR**3-9.97171*TWTR**4+2.42519*TWTR**5)-
 0.0080956+PD(2)+TCPD(I,2)
 QP(3)=-(-0.12167+1.31142*TWTR**3-6.31672*TWTR**4+8.03785*TWTR**5-3.15412*TWTR**6)+0.019325+PD(3)+TCPD(I,3)
 QP(4)=-(-0.1548-0.41354*TWTR**5+0.25874*TWTR**7)+0.0197767+PD(4)+TCPD(I,4)
 ELSE IF(TWTR.LT.0.88)THEN
 QP(1)=0.224763-0.237056*TWTR+PD(1)+TCPD(I,1)
 QP(2)=0.190385-0.209676*TWTR+0.00176*(TWTR-0.7)/(0.88-0.7)+PD(2)+TCPD(I,2)
 QP(3)=-0.052523+0.05309*TWTR+PD(3)+TCPD(I,3)
 QP(4)=-0.354571+0.38249*TWTR+PD(4)+TCPD(I,4)
 ELSE
 QP(1)=0.968092-1.877574*TWTR+0.904348*TWTR**2+PD(1)+TCPD(I,1)
 QP(2)=-0.468266+1.148107*TWTR-0.690132*TWTR**2+PD(2)+TCPD(I,2)
 QP(3)=-0.850885+1.824094*TWTR-0.981557*TWTR**2+PD(3)+TCPD(I,3)
 QP(4)=-0.350767-1.093966*TWTR+0.766972*TWTR**2+PD(4)+TCPD(I,4)
 END IF
 IF(PTR.LT.0.86)THEN
 QP(J)=QP(J)+ADJTP(J)

ELSE
 IF((ADJCP(J)-ADJTP(J)).NE.0D0)QP(J)=QP(J)+ADJTP(J)+((PTR-BPTR)/(1.-BPTR))**4*(ADJCP(J)-ADJTP(J))
 END IF
 QWTPNF(I, J) = QP(J) * WTPNF(I) + QWTTIPSEE(I, J) + QWTPHH(I, J)
 QWTPNF(I, 2) = QWTPNF(I, 2) + WTTIPSSR(I)

Where

ACW93 = Annual average OASDI covered wage for 1993
 AWC = Annual average OASDI covered wage for current year
 BACW93 = Annual average OASDI covered wage for 1993 from actual data used to determine taxable to covered wage equations
 I = Calendar year
 J = Quarter
 MAX = Annual OASDI taxable maximum
 NACW = Annual average OASDI covered wage for current year adjusted for average from actual data used to determine equations
 PD = Payday variable for private nonfarm based on calendar
 QP = Proportion of annual OASDI taxable private nonfarm wages excluding taxable tips paid in each quarter
 QWTPNF = Quarterly OASDI taxable private nonfarm wages including tips
 TCPD = Proportion of annual NIPA private wages paid in each quarter
 TWTR = Ratio of OASDI taxable to covered wages computed using equations based on data for 1993
 X = Ratio of annual OASDI taxable maximum to adjusted annual average OASDI covered wage (NACW)

Quarterly OASDI wage liabilities

WTLQPNF(I,J) = (QWTPNF(I,J) - QWTPHH(I,J)) * TRW(I)

Where

QWTPHH = Quarterly OASDI taxable private household wages
 QWTPNF = Quarterly OASDI taxable private nonfarm wages including tips
 TRW = OASDI combined employee-employer tax rate
 WTLQPNF = Quarterly OASDI tax liabilities from taxable private nonfarm wages including tips, excluding private household taxable wages

Total quarterly OASDI taxable wages and wage liabilities

QWT(I,J) = QWTPNF(I,J) + QWTF(I,J) + QWTSL(I,J) + QWTFM(I,J)
 WTLQ(I,J) = QWT(I,J) * TRW(I)

Where

I = Calendar year
 J = Quarter
 QWT = Quarterly OASDI taxable wages
 QWTF = Quarterly OASDI taxable Federal wages
 QWTFM = Quarterly OASDI taxable farm wages
 QWTPNF = Quarterly OASDI taxable private nonfarm wages including tips
 QWTSL = Quarterly OASDI taxable State and Local wages
 WTLQ = Quarterly OASDI taxable wage liabilities

2.5.5 OASDI quarterly taxable wage liability collections (WTLQC)

OASDI taxable private nonfarm wages by sub-quarterly periods

PTR = WTP/WCP
 MR = MAR(I) - .04346*(1.-PTR) + .08497*(1.-PTR)**2
 JR = JUN(I) - .02627*(1.-PTR) - .26844*(1.-PTR)**2
 SR = SEP(I) - .12321*(1.-PTR) - .02344*(1.-PTR)**2
 DR = DEC(I) - .12468*(1.-PTR) - .20710*(1.-PTR)**2
 MWTP(1) = QWTP(I,1)*MR
 MWTP(2) = QWTP(I,1) - MWTP(1)
 MWTP(3) = QWTP(I,2)*JR
 MWTP(4) = QWTP(I,2) - MWTP(3)
 MWTP(5) = QWTP(I,3)*SR
 MWTP(6) = QWTP(I,3) - MWTP(5)
 MWTP(7) = QWTP(I,4)*DR
 MWTP(8) = QWTP(I,4) - MWTP(7)

Where

DEC = Proportion of fourth quarter OASDI covered private nonfarm wages (excluding tips and household) paid in December
 DR = Proportion of fourth quarter OASDI taxable private nonfarm wages (excluding tips and household) paid in December

I = Calendar year
 JR = Proportion of second quarter OASDI taxable private nonfarm wages (excluding tips and household) paid in June
 JUN = Proportion of second quarter OASDI covered private nonfarm wages (excluding tips and household) paid in June
 MAR = Proportion of first quarter OASDI covered private nonfarm wages (excluding tips and household) paid in March
 MR = Proportion of first quarter OASDI taxable private nonfarm wages (excluding tips and household) paid in March
 MWTP = OASDI taxable private nonfarm wages (excluding tips and household) paid in last month and in first two months of quarter
 PTR = Ratio of annual OASDI taxable private nonfarm wages (excluding tips and household) to covered private nonfarm wages
 QWTP = Quarterly OASDI taxable private nonfarm wages (excluding tips and household)
 SEP = Proportion of third quarter OASDI covered private nonfarm wages (excluding tips and household) paid in September
 SR = Proportion of third quarter OASDI taxable private nonfarm wages (excluding tips and household) paid in September
 WCP = Annual OASDI covered private nonfarm wages
 WTP = Annual OASDI taxable private nonfarm wages (excluding tips and household)

OASDI taxable private nonfarm wages collected on in same quarter wages are paid

```

TRAT =RATEE(I,5)
CA =.95
MWCP(1)=QWSCPWF(I,1)*MAR(I)
MWCP(2)=QWSCPWF(I,1)-MWCP(1)
MWCP(3)=QWSCPWF(I,2)*JUN(I)
MWCP(4)=QWSCPWF(I,2)-MWCP(3)
MWCP(5)=QWSCPWF(I,3)*SEP(I)
MWCP(6)=QWSCPWF(I,3)-MWCP(5)
MWCP(7)=QWSCPWF(I,4)*DEC(I)
MWCP(8)=QWSCPWF(I,4)-MWCP(7)
RCSM =.80
QRMREQ=750.
QRWREQ=11250.
RMF=70786.*WSP(I)/1001400.
CALL ITERNU(QRMREQ,MWTP(2),MWCP(2),TRAT,RMF,PWCS(1))
CALL ITERNU(QRWREQ,MWTP(1),MWCP(1),TRAT,RMF,PWCE(1))
CALL ITERNU(QRMREQ,MWTP(4),MWCP(4),TRAT,RMF,PWCS(2))
CALL ITERNU(QRWREQ,MWTP(3),MWCP(3),TRAT,RMF,PWCE(2))
CALL ITERNU(QRMREQ,MWTP(6),MWCP(6),TRAT,RMF,PWCS(3))
CALL ITERNU(QRWREQ,MWTP(5),MWCP(5),TRAT,RMF,PWCE(3))
CALL ITERNU(QRMREQ,MWTP(8),MWCP(8),TRAT,RMF,PWCS(4))
CALL ITERNU(QRWREQ,MWTP(7),MWCP(7),TRAT,RMF,PWCE(4))
DO J=1,4
  QWTPC(I,J)=PWCS(J)+PWCE(J)*RCSM*CA
  QWTPF(I,J)=QWSTXPHH(I,J)-QWTPC(I,J)
END DO
  
```

Where

AWSCODXSRT = Annual average OASDI covered private nonfarm wages (excluding household)
 CA = Compliance allowance
 DEC = Proportion of fourth quarter OASDI covered private nonfarm wages (excluding tips and household) paid in December
 I = Calendar year
 J = Quarter
 JUN = Proportion of second quarter OASDI covered private nonfarm wages (excluding tips and household) paid in June
 MAR = Proportion of first quarter OASDI covered private nonfarm wages (excluding tips and household) paid in March
 MWCP = OASDI covered private nonfarm wages paid in third month and in first two months of each quarter
 MWTP = OASDI taxable private nonfarm wages paid in third month and in first two months of each quarter
 PWCE = OASDI taxable private nonfarm wages paid in the third month of each quarter
 PWCS = OASDI taxable private nonfarm wages paid in the first two months of each quarter on which taxes are collected in that quarter
 QWSCPWF = Quarterly OASDI covered private nonfarm wages
 QRMREQ = Monthly deposit requirement
 QRWREQ = Quarterly deposit requirement
 QWSTXPHH = Quarterly OASDI taxable private nonfarm wages (excluding household)
 QWTPC = Quarterly OASDI taxable private nonfarm wages on which employers deposit taxes in the quarter the wages were paid
 QWTPF = Quarterly OASDI taxable private nonfarm wages on which employers deposit taxes in the quarter after the wages were paid
 RATEE(I,5) = OASDI employee tax rate
 RCSM = Proportion of OASDI taxable private nonfarm wages wages paid in same quarter in which taxes are collected
 RMF = Current year average wage size of firm
 SEP = Proportion of third quarter OASDI covered private nonfarm wages (excluding tips and household) paid in September

TRAT = OASDHI employee tax rate
WSP = Economy-wide (NIPA) private wages

SUBROUTINE ITERNU(A11,QPAR,QTOT,T,RMF,AMTOUT)

```

R=QPAR/QTOT
X=A11/(T*2.+10)
DO
IWH=X*(.16011+.01998*LOG(X/RMF)-.01)
FWH=T*2.*X*((-1.4402*LOG(1.+X/RMF)+1.)*(1.-R)+R)
A1=IWH+FWH
D=A11/A1
N1=D*1000.
IF(N1.EQ.999.OR.N1.EQ.1000)THEN
RTAX=R+(1.-R)*(-1.07115*X/RMF+38633*(X/RMF)**2+1)
TOD=177.16+1142.7*DEXP(-(X/RMF))+1181.26*DEXP(-3.*(X/RMF))-907.88*DEXP(-4.*(X/RMF))+646.49*DEXP(-5.*(X/RMF))-
165.09*DEXP(-6.*(X/RMF))-20.92*X/RMF-2906.07/(X/RMF+1.)**2+831.44/(X/RMF+1.)**3
AMTOUT=QPAR-RTAX*TOD*QTOT
RETURN
END IF
X=X*D
END DO
END SUBROUTINE ITERNU

```

Where

A1 = Total (income plus FICA) taxes withheld
A11 = Deposit requirement
AMTOUT = OASDI taxable private nonfarm wages paid in sub-quarterly period and collected on in same quarter
D = Ratio of deposit requirement to total taxes withheld
FWH = FICA taxes withheld
IWH = Income taxes withheld
N1 = Ratio of deposit requirement to total taxes withheld times 1000 (used to see how close we are to target)
QPAR = OASDI taxable private nonfarm wages paid in sub-quarterly period
QTOT = OASDI covered private nonfarm wages paid in sub-quarterly period
R = Initial ratio of OASDI taxable to covered private nonfarm wages paid in sub-quarterly period
RMF = Current year average wage size of firm
RTAX = Ratio of OASDI taxable to covered private nonfarm wages paid in sub-quarterly period
T = OASDHI employee tax rate
TOD = Proportion of liabilities to be deposited in quarter after that in which wages paid
X = Taxable wage amount needed to meet deposit requirement

OASDI taxable private wages collected on in same quarter wages paid and in following quarter

QWTPCQ(L,J)=QWTPC(L,J)+QWTPHHCQ(L,J)+QWTFM(L,J)
QWTPFQ(L,J)=QWTPF(L,J)+QWTPHHFQ(L,J)

OASDI taxable State and Local wages collected on in same quarter wages paid and in following quarter

```

SLTR=WTSL/WCSL
LMPW(1)=MARSL(I)-.00329*(1.-SLTR**2)
LMPW(2)=JUNSL(I)-.68187*(1.-SLTR**3)+.52206*(1.-SLTR**4)
LMPW(3)=SEPSL(I)-1.33596*(1.-SLTR)+1.51187*(1.-SLTR**2)-.63523*(1.-SLTR**3)
LMPW(4)=DECSL(I)-2.03892*(1.-SLTR)+1.90430*(1.-SLTR**2)-.6633*(1.-SLTR**3)
DO J=1,4
SLCR(J)=(1.-LMPW(J))+LMPW(J)*LMCRPR(I-16,J)
QWTSCL(I,J)=SLCR(J)*QWTSL(I,J)
QWTSLF(I,J)=QWTSL(I,J)-QWTSCL(I,J)
END DO

```

Where

DECSL = Proportion of OASDI taxable State and Local wages paid in fourth quarter which are paid in December
I = Calendar year
J = Quarter
JUNSL = Proportion of OASDI taxable State and Local wages paid in second quarter which are paid in June
LMCRPR = Proportion of OASDI taxable State and Local wages paid in final month of quarter on which employers are to deposit taxes in the same quarter
LMPW = Proportion of quarterly OASDI taxable State and Local wages paid in final month of quarter
MARSL = Proportion of OASDI taxable State and Local wages paid in first quarter which are paid in March
QWTSL = Quarterly OASDI taxable State and Local wages paid in quarter
QWTSCL = Quarterly OASDI taxable State and Local wages paid in quarter on which taxes are deposited by the employer in the same quarter

QWTSLF	=	Quarterly OASDI taxable State and Local wages paid in quarter on which taxes are deposited by the employer in the following quarter
SEPSL	=	Proportion of OASDI taxable State and Local wages paid in third quarter which are paid in September
SLCR	=	Proportion of OASDI taxable State and Local wages paid in quarter on which taxes are deposited by the employer in the same quarter
SLTR	=	Ratio of OASDI taxable to covered State and Local wages
WCSL	=	Annual OASDI covered State and Local wages
WTSL	=	Annual OASDI taxable State and Local wages

OASDI taxable wages collected on in same quarter wages paid and in following quarter

WTQCQ(I,J)= QWTPCQ(I,J)+ QWTSLC(I,J)+QWTF(I,J)
WTQFQ(I,J)= QWTPFQ(I,J)+ QWTSLF(I,J)

Where

I	=	Calendar year
J	=	Quarter
QWTF	=	Quarterly OASDI taxable Federal wages
QWTPCQ	=	Quarterly OASDI taxable private wages collected on in same quarter wages paid
QWTPFQ	=	Quarterly OASDI taxable private wages collected on quarter following that in which wages paid
QWTSLCQ	=	Quarterly OASDI taxable State and Local wages collected on in same quarter wages paid
QWTSLFQ	=	Quarterly OASDI taxable State and Local wages collected on in quarter following that in which wages paid
WTQCQ	=	Quarterly OASDI taxable wages collected on in same quarter wages paid
WTQFQ	=	Quarterly OASDI taxable wages collected on in quarter following that in which wages paid

Quarterly OASDI wage tax collections

WTLQC(I,1) = TRW(I-1) * WTQFQ(I-1,4) + TRW(I) * WTQCQ(I,J)
DO J = 2, 4
WTLQC(I,J) = TRW(I) * (WTQFQ(I,J-1) + WTQCQ(I,J))
END DO

Where

I	=	Calendar year
J	=	Quarter
TRW	=	OASDI combined employee-employer tax rate
WTLQC	=	Quarterly OASDI wage tax collections
WTQCQ	=	Quarterly OASDI taxable wages collected on in same quarter wages paid
WTQFQ	=	Quarterly OASDI taxable wages collected on in quarter following that in which wages paid

2.5.6 Quarterly Self-Employed Net Income Tax Collections (SELQC)

DO J = 1, 4
SELQC(I,J) = SECRPY(I,J) * SEL(I) + SECRPY(I,J) * SEL(I-1)
END DO

Where

I	=	Calendar year
J	=	Quarter
SECRPY	=	Proportion of OASDI taxable self-employment earnings collected on in same year earned
SECRPY	=	Proportion of OASDI taxable self-employment earnings collected on in year following that in which earned
SEL	=	OASDI taxable self-employment earnings liabilities
SELQC	=	Quarterly OASDI self-employed net income tax collections

Appendix 2-2
Economic Acronyms

AA	Appropriation adjustments
ACE	Average OASDI covered earnings
ACSE	Average OASDI covered self-employed income
ACW	Average OASDI covered wage
ACWC	Average economy-wide wage
ADJ_FSA_FC	Adjustment to lower federal civilian covered wages relative to NIPA wages due to a presumed increase in the relative amount placed into an FSA
AWEFC_N	Average wage for Federal civilian employees not covered under OASDI
AWI	Average wage index calculated by SSA; based on the average wage of all workers with wages from Forms W-2
AWSE	Economy-wide average wage
AWSGEFC	Average wage for the Federal government enterprises
AWSGFC	Average wage for the Federal civilian government
AWSGFM	Average wage for the military
AWSGGEFC	Average wage for the Federal government & government enterprises
AWSGGESL	Average wage for State and local government and government enterprises
AWSP	Average wages, private sector
AWSPH	Average wage in private household sector
AWSPL	Average wages, private sector, 2-year moving average
AWSSP	Average compensation, private sector
AWSSPBNFXGE	Average compensation, private nonfarm business, excluding government enterprises
AWSSPES	Average compensation, private sector, educational services
AWSSPF	Average compensation, private farm, wage workers
AWSSPHS	Average compensation, private sector, health services
AWSSPL	Lagged average compensation for private sector workers
AWSSPSS	Average compensation, private sector, social services
AWSUI	Average wage of workers under UI
AWS_MEF	Average wage for employees with any wages (covered and noncovered) posted to the MEF
AYF	Average proprietor income, private farm
AYF_K	Ratio of average self-employment income to average wage-worker compensation for the agriculture sector
AYNF	Average proprietor income, private nonfarm business
AYNF_K	Ratio of average self-employment income to average wage-worker compensation for the nonagriculture sector
BEA	The Bureau of Economic Analysis
BLS	The Bureau of Labor Statistics
CFCGEFC	Compensation of fixed capital, Federal government enterprises
CFCGESL	Government consumption of fixed capital, Government enterprises, State & local
CFCGFC	Compensation of fixed capital, Federal civilian

CFCGFM	Federal Government Consumption Expenditures, Defense Consumption Expenditures
CFCGSL	State & Local Government consumption expenditures, Gross output of general government, Value added, consumption of general government fixed capital
CMB_TOT	Workers that have a combination of both OASDI covered wages and self-employed income.
CML	Ratio of Federal military OASDI covered wages to NIPA wages
COV	Economic Sub-Process: Covered Employment and Earnings
CP	Ratio of Private OASDI Covered to NIPA wages; OASDI private coverage ratio
CPI	The Consumer Price Index for Urban Wage Earners and Clerical Workers (CPI-W) is an official measure of inflation in consumer prices, published by the BLS.
CPS	Current Population Survey, conducted monthly by the Bureau of Census for the Bureau of Labor Statistics. It is the source of historical monthly economic data (such as labor force, civilian noninstitutional population, and unemployment) used to project US employment.
CR_UI	
CRAZI	Civilian pay raise
CSE_TOT	Total OASDI covered self-employed income
CSLA	Ratio of State and Local OASDI Covered to NIPA wages
CSRS	Civil Service Retirement System
CSW	Self-employed only workers (SEO) plus combination workers (CMB_TOT)
DNEDMIL	Decreases in the military population (as shown by the difference over four quarters)
DRTP, DRTPP, DRTPN, DRTP1Q, DNRTTP1Q, DPRTP1Q	Dummy variables for positive and negative economic cycle trends
E	Total employment, CPS concept (i.e., average of monthly estimates of total wage and salary workers, plus self-employed, plus unpaid family workers)
E_FE	Civilian employment level at full employment (i.e., at potential GDP)
EA	Total agricultural employment
EAS	Civilian Employment Level, Self-employed workers: agriculture, SA
EAW	Employment by class of worker, agricultural wage workers
EDMIL	Total number serving in the US Armed Forces estimated by the Department of Defense and published by the Census Bureau
EGFC	Federal civilian government employment
EGEFCPS	Employment, Establishment Data, All Employees: Government, Federal Government Enterprises, U.S. Postal Service
EGGEFC	Employment, Establishment Data, All Employees: Government, Federal Government, SA
EGGESL	Employment, State & Local government enterprises

EMPTRDI	DI employer tax rate
EMPTRHI	HI employer tax rate
EMPTROASI	OASI employer tax rate
ENA	Civilian Employment Level, Nonagricultural industries, 16 years and over, SA
ENAS	Employment by class of worker, nonagricultural self-employed
ENAU	Employment by class of worker, nonagricultural unpaid family workers
ENAW	Employment by class of worker, nonagricultural wage workers
ENAWPBXGE	Employment for private nonfarm business
ENAWPH	Employment by class of worker, nonagricultural wage workers, private household workers
ENAWSPBXGE	Employment for private nonfarm business and nonagricultural self-employed
EO	Total employment in the other-than-LPR population
EO_A	Total employment in the other-than-LPR population who are temporarily authorized to reside or work in the US
EO_ESF	Total employment in the other-than-LPR population whose reported earnings are posted to the Earnings Suspense File
EO_MEF	Total employment in the other-than-LPR population whose earnings are reported and posted to the Master Earnings File
EO_MEFC	Total employment in the other-than-LPR population whose earnings are reported and posted to the Master Earnings File and are OASDI-covered
EO_NA	Total employment in the other-than-LPR population who have overstayed their authorization
EO_NO	Total employment in the other-than-LPR population who were never authorized to reside or work in the US
EO_UND	Total employment in the other-than-LPR population that is strictly in the underground economy (i.e., with no earnings reported)
EP	Employees in Private industries
EPES_EST	Employees by industry, Private industries, Educational services
EPHS_EST	Employment for private health services
EPSS_EST	Employees by industry, Private industries, Social Assistance
ES	Self-employed workers
ETP	Effective annual taxable payroll, equal to total employer taxable OASDI wages plus total self-employed taxable income minus one half of the multi-employer refund wages
EU	Unpaid family workers
EW	Wage and salaried workers
FERS	Federal Employee Retirement System
GDP	Gross domestic product
GDP05	GDP, 2005\$
GDPG	GDP, General Government
GDPGE	GDP, Federal and State & local government enterprises
GDPGEFC	GDP, Federal civilian government enterprises
GDPGESL	GDP, State & local government enterprises
GDPGF	GDP, General Government, Federal

GDPGFC	GDP, Federal civilian
GDPGFM	GDP, military
GDPGGE	GDP, Federal and State & local government enterprises
GDPGGEFC	GDP, Federal civilian government and government enterprises
GDPGGESL	GDP, State & local government and government enterprises
GDPGSL	GDP, General Government, State & Local
GDPPBNFXGE	GDP, private nonfarm business, excluding government enterprises
GDPPF	GDP, private business sector, farm
GDPPH	GDP, Private Households
GDPPNI	GDP, Nonprofit institutions serving households
HI	Hospital insurance
HIFC_L	HI Employer Liability, Federal Civilian
HIFM_L	HI Employer Liability, Federal Military
HIP_L	HI Employer Liability, Private
HISL_L	HI Employer Liability, State & Local
KGDP05	Potential real GDP, 2005\$
LC	US labor force, equal to the sum of number of persons employed and number of persons seeking employment
LFPR	Labor force participation rate, defined as the ratio of the number of persons in the US labor force to the number of persons in the US noninstitutional population.
M	Military population
MER	Multi-employer refund wages
MRAZ	Military pay raise
N	Civilian noninstitutional population
NCE	Total noncovered employment
NIPA	The National Income and Product Accounts, published by the BEA, providing historical estimates of quarterly earnings and output measures
NRA	Normal retirement age
OASDI	Old-Age, Survivors, and Disability Insurance
OASDIFC_L	OASDI Employer Liability, Federal Civilian
OASDIFM_L	OASDI Employer Liability, Federal Military
OASDIP_L	OASDI Employer Liability, Private
OASDISL_L	OASDI Employer Liability, State & Local
OASDHI	Old-Age, Survivors, Disability, and Health Insurance
OLI	Employer contributions for employee pension and insurance funds
OLI_CSRS1	Contributions for CSRS employees' pay
OLI_FC	Other labor income, Federal civilian
OLI_FERS1	Contributions for FERS employees' pay
OLI_FERSFC	Employer contributions to Thrift Savings Plan for FERS employees
OLI_GGE	Other labor income, government and government enterprises
OLI_GHI	Other labor income by type, Employer contributions to pension and welfare funds, private welfare funds, Group health insurance
OLI_GHI_FC	Employer contributions for employee pension & insurance funds, group health insurance, Federal civilian government sector
OLI_GHI_P	Employer contributions for employee pension & insurance funds, group

	health insurance, private sector
OLI_GHI_SL	Employer contributions for employee pension & insurance funds, group health insurance, State & local government sector
OLI_GLI	Employer contributions for employee pension and insurance funds, Group life insurance
OLI_GLI_FC	Employer contributions for employee pension & insurance funds, group life insurance, Federal civilian government sector
OLI_GLI_P	Employer contributions for employee pension & insurance funds, group life insurance, private sector
OLI_GLI_SL	Employer contributions for employee pension & insurance funds, group life insurance, State & local government sector
OLI_P	Employer contributions for employee pension and insurance funds, private industries
OLI_PPPS	Other Labor Income, Private Sector Pension and Profit Sharing
OLI_PPS	Employer contributions for employee pension and insurance funds, Pension & profit-sharing
OLI_RETFC	Employer contributions for employee pension and insurance funds, Publicly administered government employee retirement plans, Federal civilian
OLI_RETFM	Employer contributions for employee pension and insurance funds, Publicly administered government employee retirement plans, Federal military
OLI_RETSL	Employer contributions for employee pension and insurance funds, Publicly administered government employee retirement plans, State and local
OLI_SL	Other labor income, State and local
OLI_SU	Employer contributions for employee pension and insurance funds, Supplemental unemployment
OLI_WC	Employer contributions for employee pension and insurance funds, Workers' compensation
OLI_WCP	Private employer contribution to other labor income, total for workers' compensation
OLI_WCSL	Employer contributions to workers' compensation, State and local
OLIF_RETFCO	Other government contributions to Federal civilian retirement
OOH	Owner-occupied housing
OP	Other-than-LPR population
OP_A	Other-than-LPR population who are temporarily authorized to reside or work in the US
OP_NA	Other-than-LPR population who have overstayed their authorization
OP_NO	Other-than-LPR population who were never authorized to reside or work in the US
PGDP	Gross Domestic Product Price Index, Units: 2005=100
PGDPAF	Deflator for farm output
PIA	Primary insurance amount
PIARR	PIA replacement rate, defined as the ratio of a hypothetical medium scale worker's PIA to his/her career average indexed earnings.

PGDP	GDP price deflator
PBNFXGE	Private nonfarm business excluding government enterprises
RCMB	Proportion of wage workers who are also self-employed (CMB_TOT/WSW)
RCSE	Covered self-employed ratio, defined as the ratio of total covered self-employment income to total proprietor income (CSE_TOT/Y).
RCWSF	Ratio of compensation to wages in the Federal government
RCWSM	Ratio of compensation to wages in the military
RCWSP	Ratio of compensation to wages in the private sector
RCWSSL	Ratio of compensation to wages in the State and local sector
RD	Disability prevalence ratio, defined as the ratio of disabled worker beneficiaries to the disability-insured population.
RELMAX	Ratio of the TAXMAX to averaged covered earnings
RELMAX_UI	Ratio of the aggregate weighted average of the UI taxable maximum to the average UI wage
RET	Earnings test ratio, defined the ratio of the maximum amount of earnings before an OASDI benefit is reduced to the average wage index.
RFS	Family size ratio, defined as the ratio of the number of children under 6 to mothers of a certain age.
RGR_GHI	
RHIP_L	Product of HI tax rate, private coverage ratio, and the taxable ratio
RM	Military ratio, the ratio of the US armed forces to the noninstitutionalized population.
RMER	Multi-employer refund wage ratio, defined as the ratio of multi-employer refund wages to total OASDI wages.
ROASDIP_L	Product of OASDI tax rate, private coverage ratio, and the taxable ratio
ROLI_PPPS	Ratio of employer contributions to private pension and profit-sharing to private wages
ROLI_SU	Ratio of private employer contributions for employee pension and insurance funds, Supplemental unemployment to private wages
ROLI_WCP	Ratio of private employer contribution to other labor income, total for workers' compensation to private wages
RSET	Self-employed net income taxable ratio, defined as the ratio of total self-employed taxable income to total OASDI wages.
RSOC_UIP	Ratio of private employer contributions to social insurance, total for unemployment insurance, to private wages
RSOC_WCP	Ratio of private employer contributions to social insurance, total for workers' compensation to private wages
RSOCF_PBG	Ratio of private employer insurance contribution to the Pension Benefit Guaranty Trust Corporation to private wages
RSOCSL_WC	Ratio of combined Private and State & local sector employer contributions to social insurance for workers' compensation to the combined Private and State and local sector employer contributions to workers' compensation
RTE	Ratio of total employment to the sum of wage & salary, self-employed workers, and the military (TE/(EW + ES + military))

RTP	A summary measure of the economic cycle equal to the ratio of real GDP to potential GDP
RU	Civilian unemployment rate defined as the ratio of the unemployed US labor force to the total US labor force
RUIWS	Effective tax rate for employer contributions to unemployment insurance
RWCWS	Effective tax rate for employer contributions to workers' compensation
RWSC	Covered wage ratio, defined as the ratio of OASDI covered wages to total wage and salary disbursements (WSC/WSD)
RWSD	Earnings ratio, defined as the ratio of total wage and salary disbursements to total wage and worker compensation (WSD/WSS)
RWSSPBNFXGE	Ratio of compensation to GDP in private business nonfarm excluding government enterprises
RWSSY	Total compensation ratio, defined as the ratio of total compensation to gross domestic product (WSSY/GDP)
RWTEE	Employee taxable ratio, defined as the ratio total employee OASDI taxable wages to total OASDI covered wages.
RY	Income ratio, defined as the percentage of total compensation attributable to proprietor income (Y/WSSY).
S&L	State and Local government
SEL	Self-employment tax liabilities
SELQC	Quarterly self-employed net income tax collections
SEO	Workers that report only OASDI covered self-employed earnings
SEPR	Self-employed participation rate (the proportion of employed persons that are self-employed)
SET	Total self-employed taxable income
SOC	Employer contributions for government social insurance
SOC_FC	Employer contributions for social insurance, Federal civilian sector
SOC_FM	Employer contributions for government social insurance, Federal government, military sector
SOC_GGE	Employer contributions for social insurance, government and government enterprises
SOC_P	Employer contributions for social insurance, private industries
SOC_SL	Employer contributions for social insurance, State and local sector
SOC_UIP	Private employer contributions to social insurance, total for unemployment insurance
SOC_UISL	State and local government employer contributions to social insurance, total for unemployment insurance
SOC_WCP	Private employer contributions to social insurance, total for workers' compensation
SOC_WCSL	State and Local government employer contributions to social insurance, total for workers' compensation
SOCF_HI	Contributions for Government Social Insurance, Employer Contributions, Federal Social Insurance Funds, Hospital Insurance
SOCF_MIFM	Contributions for Government Social Insurance, Employer Contributions, Federal Social Insurance Funds, Military Medical

	Insurance
SOCF_OASDI	Contributions for Government Social Insurance, Employer Contributions, Federal Social Insurance Funds, Old-age, Survivors, And Disability Insurance
SOCF_PBG	Contributions for Government Social Insurance, Employer Contributions, Federal Social Insurance Funds, Pension Benefit Guaranty
SOCF_RETRR	Contributions for Government Social Insurance, Employer Contributions, Federal Social Insurance Funds, Railroad Retirement
SOCF_UIF	Contributions for Government Social Insurance, Employer Contributions, Federal Social Insurance Funds, Federal Unemployment Tax
SOCF_UIFC	Total federal civilian government employer contributions to unemployment insurance
SOCF_UIFED	Contributions for Government Social Insurance, Employer Contributions, Federal Social Insurance Funds, Federal Employees' Unemployment Insurance
SOCF_UIFM	Total federal government employer contributions to unemployment insurance, military
SOCF_UIS	Contributions for Government Social Insurance, Employer Contributions, Federal Social Insurance Funds, State Unemployment Insurance
SOCF_WC	Contributions for Government Social Insurance, Employer Contributions, Federal Social Insurance Funds, Worker's Compensation
SOC SL_WC	Contributions for Government Social Insurance, Employer Contributions, State and Local Social Insurance Funds, Workers' Compensation
SSA	Social Security Administration
TAXMAX	OASDI contribution and benefit base
TAXPAY	Economic Sub-Process: Taxable Payroll
TCE	Total OASDI covered employment
TE	Total "at any time" employment
TEFC_N	Total "at any time" employment, Federal civilian, without Federal civilian OASDI
TEO	Total "at any time" employment in the other-than-LPR population
TEO_ESF	Total "at any time" employment in the other-than-LPR population whose reported earnings are posted to the Earnings Suspense File
TEO_MEF	Total "at any time" employment in the other-than-LPR population whose earnings are reported and posted to the Master Earnings File
TEO_MEFC	Total "at any time" employment in the other-than-LPR population whose earnings are reported and posted to the Master Earnings File and are OASDI-covered
TEO_UND	Total "at any time" employment in the other-than-LPR population that is strictly in the underground economy (i.e., with no earnings reported)
TMAXUI_SL	Taxable maximum for State & local unemployment insurance

TRSE	OASDI self-employed tax rate
TRW	Combined OASDI employee-employer tax rate
TXRP	OASDI private taxable ratio
U	The number of persons in the labor force who are unemployed
MODSOL2	Economic Sub-Process: U.S. Earnings
USEMP	Economic Sub-Process: U.S. Employment
WEFC_N	Wages for Federal civilian employees not covered under OASDI
WS	Compensation of Employees, Wage and Salary Accruals
WSC	Total OASDI covered wages
WSD	Total wage and salary disbursements
WSDP	Private wage and salary disbursements
WSGEFC	Government Wages and Salaries, Federal civilian, Government Enterprises
WSGFC	Wage and salary accruals by industry, Government, Federal civilian
WSGFM	Wage and salary accruals by industry, Government, Federal, Military
WSGGFC	Wages for the Federal government & government enterprises
WSGGESL	Wages for State and local government and government enterprises
WSP	Compensation of Employees, Wage and Salary Accruals
WSPF	Wage and salary accruals by industry, Private industries, Farms
WSPH	Wage and salary accruals by industry, Private industries, Households
WSPNI	Wage and salary accruals by industry, Private industries, Nonprofit institutions serving households
WSPRRB	Wages covered by Railroad Retirement Act
WS_MEF	Total wages posted to the MEF
WSS	Total wage worker compensation
WSSG	Compensation for Federal and State & local government
WSSGE	Compensation for Federal and State & local government enterprises
WSSGEFC	Compensation of employees by industry, Government, Federal
WSSGESL	Compensation of employees by industry, Government, State and local government enterprises
WSSGF	Federal Government Consumption Expenditures, Compensation of General Government Employees
WSSGFC	Compensation of employees by industry, Government, Federal civilian
WSSGFM	Compensation of employees by industry, Government, Military
WSSGGE	National Income w/o Capital Consumption Adjustment, Government
WSSGGESL	Compensation for the State & local government and government enterprises
WSSGSL	State & Local Government Consumption Expenditures, Compensation of General Government Employees
WSSP	Compensation of employees by industry, Private industries
WSSPBNFXGE	Compensation in private business nonfarm excluding government enterprises
WSSPES	Compensation of employees by industry, Private industries, Educational services
WSSPF	Compensation of employees by industry, Private industries, Farms

WSSPH	Compensation of employees by industry, Private industries, Households
WSSPHS	Compensation of employees by industry, Private industries, Health services
WSSPNI	Compensation of employees by industry, Private industries, Nonprofit institutions serving households
WSSPSS	Compensation of employees by industry, Private industries, Social assistance
WSSY	Total compensation for wage and salary workers and proprietors
WSW	Wage and salary workers that report some OASDI covered earnings
WSW_MEF	Total number of employees with any wages posted to the MEF
WTEE	Total employee OASDI taxable wages
WTER	Total employer OASDI taxable wages
WTL	Annual OASDI wage tax liabilities
WTLQ	Quarterly OASDI wage tax liabilities
WTLQC	Quarterly OASDI wage tax collections
Y	Total proprietor income
YF	National Income, Proprietors' income with Inventory Valuation (IVA) and Capital Consumption Adjustment (CCAdj): farm sector
YNF	National Income, Proprietors' income with IVA and CCAdj: nonfarm

Process 3:

Beneficiaries

3. Beneficiaries

OACT uses the Beneficiaries process to project the fully insured and disability insured population, the number of disabled workers and their dependent beneficiaries, the number of retired workers and their dependent beneficiaries, and the number of dependent beneficiaries of deceased workers. The Beneficiaries process receives input data from the Demography and Economics processes along with data received from the Social Security Administration and other government agencies. Output data is provided to the Economics and Trust Fund Operations and Actuarial Status processes.

The Beneficiaries Process is composed of three subprocesses: INSURED, DISABILITY, and OLD-AGE AND SURVIVORS. As a rough overview, INSURED projects the number of people in the Social Security area population that have sufficient work histories for disability and retirement benefit eligibility. DISABILITY projects the number of disabled workers and their dependent beneficiaries. OLD-AGE AND SURVIVORS projects the number of retired workers, their dependent beneficiaries, and the dependent beneficiaries of deceased workers.

All programs output data on an annual basis.

3.1. INSURED

3.1.a. Overview

Insured status is a critical requirement for a worker, who has participated in the covered economy, to receive Social Security benefits upon retirement or disability. The requirement for insured status depends on the age of a worker and his or her accumulation of quarters of coverage (QC).

INSURED is a simulation model that estimates the percentage of the population that is fully insured (FPRO) and disability insured (DPRO) throughout the projection period. These estimates are used in conjunction with estimates of the Social Security area population (SSAPOP) to estimate the number of people that are fully insured (FINPOP) and disability insured (DINPOP). FINPOP is then used by the OLD-AGE AND SURVIVORS INSURANCE subprocess, and both FINPOP and DINPOP are used by the DISABILITY subprocess. FINPOP and DINPOP are projected by age, sex, and cohort.

For each sex and birth cohort, INSURED simulates 30,000 work histories that represent the population with permanent legal work authorization. These histories are constructed from past and projected cover worker rates, median earnings, and amounts required for crediting QC.

The equations for this subprocess are given below:

$$\text{FPRO} = \text{FPRO}(\cdot) \quad (3.1.1)$$

$$\text{DPRO} = \text{DPRO}(\cdot) \quad (3.1.2)$$

$$\text{FINPOP} = \text{FPRO} * \text{SSAPOP} \quad (3.1.3)$$

$$\text{DINPOP} = \text{DPRO} * \text{SSAPOP} \quad (3.1.4)$$

3.1.b. *Input Data*

All data are updated annually, except those that are noted.

Long-Range OCACT Data

Demography

1. Social Security area population as end of year (1940-2100) by age (0-100, age 100 including age 100 and older), marital status (single, married, widowed, divorced) and sex (M, F).
2. Number of new “net lawful permanent resident (LPR) immigrants” (LPR immigrants – estimated legal emigrants) entering the Social Security area each year (1940-2100) by age (14-100, age 100 including age 100 and older) and sex (M, F).
3. “Other-than-LPR immigrant” population as end of year (1963-2100) by age (0-100, age 100 including age 100 and older) and sex (M, F).
4. The population granted deferred action for childhood arrivals (DACAs) as end of year (2012-2100) by age (0-100, age 100 including age 100 and older) and sex (M, F).
5. The population granted deferred action for parental arrivals (DAPAs) as end of year (2019-2100) by age (0-100, age 100 including age 100 and older) and sex (M, F).
6. The population that attain deferred action for childhood arrivals status (DACAATT) each year (2013-2100) by age (-1-100, age 100 including age 100 and older) and sex (M, F).
7. The population that attain deferred action for parental arrivals status (DAPAATT) each year (2019-2100) by age (-1-100, age 100 including age 100 and older) and sex (M, F).

Economics

8. Annual estimates of covered workers posted to the Master Earnings File (MEF) by sex (M, F) and age (0-100) for years (1937-2100).
9. Annual projection (2017-2100) of average wage index and median covered earnings.
10. “Other-than-LPR immigrant” workers with earnings posted to the (MEF) by sex (M, F), age (16-100), and for years (1964-2100).

Beneficiaries

11. Disabled-worker beneficiaries at year end (2019-2100) by age (15-66), sex (M, F) and duration (0-10, duration 10 including duration 10 and above) from the previous year’s Trustees Report. These data are read in from files that are generated annually from

the Beneficiaries/DISABILITY (#3.2) area.

Short-Range OCACT data

12. FINPOP by age (14-95, age 95 including age 95 and older) and sex (M, F) from the end of year 1969 to the end of Short-Range projection period (2030) (EOY 1969-2035 is provided).
13. DINPOP by age (15-66) and sex (M, F) from the end of year 1969 to the end of Short-Range projection period (2030) (EOY 1969-2035 is provided).

Other input data

14. Historical series of annual median earnings of covered workers by age group (<20, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, 70-74, 75-79, 80-84) and sex (M, F) for years 1937-2017. Data are updated using the CWHS file from the mainframe.
15. Number of disabled workers by age (20-69) and sex (M, F) for years 1958-2019. Ages 66-69 are zeros. Data are updated using the data from the historical disability file “wkrben”.
16. The amount required for crediting one quarter of coverage for years 1937-2017 from an OCACT web site.
17. Historical series of annual median earnings of all covered workers for years 1937-2017. Data are updated using the data in the most recent Social Security Annual Statistical Supplement Table 4.B6.
18. The number of all covered workers (wage/salary workers, self-employed workers) by sex and amount of earnings for 2017 in the most recent Social Security Annual Statistical Supplement Table 4.B7 & 4.B9. These are used to produce the input data for the distribution of earnings (FRACMOD.f90). **(No Change for the 2021 TR)**
19. ANNUAL factor (comparability factor between quarterly and annual reporting of earnings) by age (13-84) and sex (M, F) for years prior to 1978. **(No Change for the 2021 TR)**
20. SLCT factor (adjustment factor to bring simulated fully insured rate in line with historical fully insured rate) by age (13-84) and sex (M, F). These data were updated for the 2021 TR.
21. SRCH factor (adjustment factor to bring simulated fully insured rate in line with historical fully insured rate) by age (13-84) and sex (M, F). These data were updated for the 2021 TR.
22. DIADJ factor (adjustment factor to bring simulated disability insured rate in line with historical disability insured rate) by age (13-69) and sex (M, F). These data were updated for the 2021 TR.

3.1.c. Development of Output

Equation 3.1.1 & 3.1.2 -

Determining the QC distribution

There are three variables playing important roles in the simulation process starting from age 13 through 84 of a birth cohort by sex. They include historical and projected covered worker rates of the population with permanent legal work authorization (LEGWK) by age and sex, the amounts required for crediting QC, and a cumulative worker distribution by earnings level.

Covered worker rates of the LEGWK population (CPRO) are the ratio of the LEGWK covered workers to the LEGWK Social Security area population. A LEGWK population is its total population minus its other-than-LPR immigrant population¹. Historical and projected (total and other-than-LPR immigrant) numbers of covered workers and the Social Security area population, which are provided by the Economics and Demography sections respectively, are used to calculate the rates for ages 13 through 84.

The law specifies the amount of earnings needed to earn one QC for each year of the historical period. Its projection assumes the same growth rate as the Social Security average wage index.

The cumulative worker distribution by earnings level is ‘FRAC’. It is a function of covered earnings relative to median earnings. For a given ratio of covered earnings relative to median earnings, FRAC returns the percentage of covered workers whose earnings relative to median earnings are less than the given ratio. It is constructed based on the latest historical data. It is used for each age and sex and is assumed to remain constant throughout the projection period. The program uses FRAC to estimate the percentage of covered workers that earn 0, 1, 2, 3 or 4 QC in a given year. Thus, for a particular age and sex, the percentage of covered workers earning at least n QC is defined as:

$$QCDist = 1 - FRAC \left[\frac{n * QC \text{ amount}}{\text{median earnings}} \right], \quad \text{for } n = 1, 2, 3, 4$$

where *median earnings* is for that age and sex.

Simulation process – assigning QC to records

Once the QCDist is known, the simulation process begins with 30,000 records for each sex and birth cohort. Starting with the QC distribution at age 13, INSURED randomly assigns a number of QC (1, 2, 3 or 4) to these records based on QCDist.

For ages 14 to 84, INSURED begins the simulation process by randomly selecting records to represent new net LPR immigrants and the other-than-LPR immigrant population that attain DACA status from the covered worker portion of 30,000 records. For each record, a number of QC (0, 1, 2, 3 or 4) is assigned on a uniform basis. Once a record is assigned a number of QC, INSURED nullifies the previous earnings of the record. In 2013, the initial year of eligibility for DACAs, we assume that 10 percent of DACAs will retain covered earnings made prior to attaining LPR status. We apply this same assumption for newly eligible DACAs in 2021-2022 when the 2012 DACA program is restored.

¹ The insured model treats those given legal work authorization through the 2012 Deferred Action for Childhood Arrivals (DACA) program like LPR immigrants.

After the records for new immigrants are selected, the rest of the records for ages 14 to 84 are either non-covered workers or covered workers. The total number of records assigned as non-covered workers is set equal to $(1 - \text{covered worker rate}) * 30,000$. These records receive no QC. To identify records as non-covered workers, INSURED uses two parameters (SRCH, SLCT), which vary by age and sex.

SRCH sets a limit on the number of consecutive records to be searched for a non-covered worker. In general, the younger age groups have lower SRCH values. SLCT is the number of consecutive prior years in which no QC were earned that is required in order for a simulated record to be assigned as a non-covered worker. Lower SLCT values are set for the very young age groups. Sensitivity analyses show that insured percentages are negatively correlated with these two parameters. When the female covered worker rates approach the male rates, the female SRCH and SLCT values are graded toward the male values².

For each sex and birth cohort, the simulation process of assigning records as non-covered workers uses the following approach. This approach is repeated until the targeted number of non-covered workers is achieved.

1. One of the records, which is designated as one for LPR immigrants, is randomly selected as the starting record.
2. Beginning with the starting record, each record is examined until a record that matches the SLCT criterion is found.
3. However, if the number of records examined equals the value of SRCH and no record matches the SLCT criterion, then the record closest to the SLCT criterion is assigned no QC as a non-covered worker.

Initially, values for SRCH and SLCT are the same as those used in the prior Trustees Report. Adjustments to these values are only made when the results are not consistent with historical data.

The final step of the simulation process is to use QCDis to randomly assign QC of 0, 1, 2, 3 or 4 to the remaining covered worker records, which are not new net LPR immigrants, for ages 14-84.

Determining Insured Status

Once the simulation process is complete, the insured status for each record at any age can be determined based on the total QC assigned up to that age. The simulated LEGWK fully insured percentage (FSIM_LEG) is calculated as the percentage of the 30,000 simulated records meeting the QC requirements for insured status. The same calculation is applied to the disability-insured percentage (DSIM_LEG).

² This occurs when the female covered worker rate is greater than 90 percent of the male rate. When the female rate is equal to or greater than the male rate, the female SLCT and SRCH parameters are set equal to the male parameters. The parameters are linearly interpolated when the female covered worker rate is between 90 and 100 percent of the male rate.

For each sex and cohort, FSIM_LEG is determined at ages 13 to 84. DSIM_LEG is determined at ages 13 to 69.

An adjustment, DINADD, is made to DSIM_LEG. This additive adjustment accounts for workers who fail to meet the requirement for disability-insured status solely because of having no earnings while receiving disability benefits. INSURED assumes that workers who have been on the disability rolls for more than 3 years would be in this situation³. Thus, DINADD is

$$\frac{\# \text{ of workers on the disability rolls more than 3 years}}{\text{Social Security Area population}} \quad \text{by age, sex, and cohort.}$$

A small proportion of the other-than-LPR immigrant population, OTLPOP, is added to calculate the simulated fully insured rate of the Social Security Area population, FSIM. We assume that other-than-LPR immigrants who have their earnings posted to the MEF (CW_Other) are three-fourths as likely to be insured as the LEGWK population. We project FSIM as

$$\frac{FSIM_LEG * (LEGWK + ALPHA * (CW_OTHER/CPRO))}{SSAPOP}$$

by age, sex, and cohort, where ALPHA is equal to 0.75.

Hence, the simulated fully insured rate of the other-than-LPR immigrant population FSIM_OTL is

$$\frac{FSIM_LEG * ALPHA * CW_OTHER/CPRO}{OTLPOP}$$

by age, sex, and cohort.

FSIM_LEG is assumed to remain the same beyond age 84. FSIM_OTL is assumed to remain the same beyond age 69. For ages 70 and older, FSIM is projected by multiplying the LEGWK and other-than-LPR fully insured rates to their respective populations and dividing the sum by the Social Security Area population.

DSIM and DSIM_OTL are projected in a similar manner by using DSIM_LEG. If the simulated results for DSIM are not consistent with historical data, an additional age-sex-specific additive adjustment (DIADJ) is used to bring the simulated results in line with the historical estimates.

Finally, incorporation of Short-Range projections produces FPRO and DPRO. For the first 10 years, FPRO and DPRO are calculated by dividing the Short-Range estimates by the Social

³ Those who are on the rolls for less than 4 years are assumed to meet the requirement for disability-insured status based on their earnings histories.

Security area population. The difference in terms of the percentage between the Long-Range (FSIM and DSIM) and Short-Range projections at the end of 10th year is linearly phased out during the next ten years by cohort and sex. The Long-Range projections are assumed thereafter.

Number of Fully Insured and Disability Insured Workers

The numbers of Fully Insured and Disability Insured workers are obtained by applying FPRO and DPRO, respectively, to the Social Security area population. The result is an estimate of the number of people that are fully insured (FINPOP) and disability insured (DINPOP) by single year of age and sex, respectively. For a given age and sex, the proportion of the Social Security area population that is insured (FPRO) is assumed to be the same for each marital status.

3.2. DISABILITY

3.2.a. Overview

The Social Security Administration pays monthly disability benefits to disability-insured workers who meet the Disability Insurance program’s definition of “disability”. If they meet certain requirements, spouses and children of disabled-worker beneficiaries may also receive monthly benefits.

DISABILITY projects the number of disabled-worker beneficiaries in current-payment status (DIB) at the end of each year by age at entitlement, sex, and duration from entitlement. We base the number of DIB at the end of each year on the number of disabled-worker beneficiaries who are currently entitled to benefits (CE). We calculate the number of CE at the end of year by adding the number of newly entitled CE (New Entitlements) during the year and subtracting the number of CE who leave the disability rolls (Exits) during the year to the number of CE at the end of the prior year. Disabled-worker beneficiaries who leave the disability rolls (Exits) do so by recovering from disabilities (Recoveries), by dying (Deaths), or by converting to retired worker status (Conversions). A disabled-worker beneficiary converts to retired worker status upon reaching Normal Retirement Age (NRA), the age at which a person first becomes entitled to an unreduced retirement benefit.

DISABILITY also projects the number of future dependent beneficiaries of DIB by category, age, and sex. The six categories are minor child, student child, disabled adult child, young spouse, married aged spouse and divorced aged spouse. We generate the numbers of dependent beneficiaries of DIB by multiplying the relevant subset of the SSA area population (Exposures) by a series of probabilities that relate to the regulations and requirements for obtaining benefits (Linkages).

$$\text{New Entitlements}(\text{year}) = \text{Exposure}_{\text{BOY}} \times \text{Incidence Rate}(\text{year}) \quad (3.2.1)$$

where BOY is beginning of year.

$$\text{Exits}(\text{year}) = \text{Recoveries}(\text{year}) + \text{Deaths}(\text{year}) + \text{Conversions}(\text{year}) \quad (3.2.2)$$

where Recoveries(year) = $\text{CE}_{\text{BOY}} \times \text{Recovery Rate}(\text{year})$
where Deaths(year) = $\text{CE}_{\text{BOY}} \times \text{Death Rate}(\text{year})$.

$$\text{CE}_{\text{EOY}} = \text{CE}_{\text{EOY-1}} + \text{New Entitlements}(\text{year}) - \text{Exits}(\text{year}), \quad (3.2.3)$$

where EOY is end of year, EOY-1 is end of prior year.

$$\text{Dependent Beneficiaries of DIB}_{\text{EOY}} = \text{Exposures}_{\text{EOY}} \times \text{Linkages}_{\text{EOY}} \quad (3.2.4)$$

3.2.b. Input Data

Trustees Assumptions

Each year, the Trustees set the assumption for the ultimate incidence rates and the ultimate recovery rates (on an award rate basis) for the twentieth year of the projection period. For the

2021 Trustees Report, the age-sex-adjusted incidence rate (on an award rate basis) is 5.0 per 1,000 and the age-sex-adjusted recovery rate (on an award rate basis) is 10.4 per thousand. The following chart shows our targeted ultimate incidence rates (on an entitlement rate basis) by age group and sex. We further adjust the rates below with a 0.25 percent discount to reflect the regulation removing the inability to communicate in English as an education category in the disability determination process.

	Age Group										
	<20	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69
Men	1.34	2.05	1.53	2.23	2.83	3.96	5.26	9.01	15.29	17.40	9.14
Women	1.04	1.43	1.52	2.29	3.32	4.64	6.09	9.69	14.52	14.27	7.69

Using a standard population of disability insured who are not in current pay as of December 1999, the age-sex-adjusted incidence rate (on an entitlement rate basis) for the 2021 Trustees Report is 5.2 per 1,000. Using a standard population of DIBs as of December 1999, the age-sex-adjusted recovery rate (on an entitlement rate basis) for the 2021 Trustees Report is 9.3 per 1,000.

Long-Range OCACT Data

All data is updated annually except those noted otherwise. Population data are as of December 31. We assume data as of December 31 of year *z-1* is equal to data as of January 1 of year *z*.

Demography

1. Social Security area population by age, sex, and marital status¹ (dimensioned (0:100,1:2,1:4)) for years 1970-2100.
2. Probabilities of death by sex, age and year (1:2,-1:148,2019:2100).
3. Total children by sex of parent, age of parent and age of child (dimensioned (1:2,19:71,0:18)) for years 1970-2100.
4. Total married lives by age of husband crossed with age of wife (dimensioned (14:100,14:100)) for years 1970-2100.
5. Average number of children under 18 per couple with children by age group (<25, 25-29,...,60-64) of head of household (dimensioned (1:9)) for years 1970-2100.

Economics

6. Unemployment rates by age group (16-19, 20-24,...,60-64), sex and year (1990:2032).

Beneficiaries

INSURED subprocess #3.1

7. Disability-insured population by age, sex and year (15:69,1:2,1969:2100) from the 2021 Trustees Report.
8. Disability-insured population by age, sex and year (15:69,1:2,1969:2100) from the 2020 Trustees Report.
9. Fully insured population by age, sex and marital status (14:95,1:2,1:4) for years 1970-2100 from the 2021 Trustees Report.

¹ Single, married, widowed, divorced.

10. Lawful Permanent Resident (LPR) fully insured percentage by age, sex and year (14:95,1:2,1969:2100) from the 2021 Trustees Report

2020 Trustees Report DISABILITY subprocess #3.2

11. Death rate projection factors from the 2020 Trustees Report by age group (15-19,20-24,...,60-64), sex and year (1:2,1:10,2020:2100).
12. Recovery rate projection factors and recovery rates in 2021 Trustees Report format after method change using data from the 2020 Trustees Report by age group (15-19,20-24,...,65-69), sex and year (1:2,1:11,2020:2100) and (1:2,1:10,1970:2100), respectively.

Other input data

We update only the most recent year data annually for this category except as noted otherwise below:

13. The December 2020 data from the Master Beneficiary Record (MBR) containing the number of DIB by duration of entitlement, age of entitlement, sex and time of year (BOY or EOY) (0:55,15:66,1:2,1:2).
14. December 2020 and December 2019 data from the MBR containing the number of disabled workers in current-payment status by age, sex and year (15:66,1:2,1969:2020).
15. December 2020 data from the MBR containing the number of disabled workers in withheld or suspended status by age, sex and year (15:66,1:2,1969:2020).
16. December data from the MBR containing the number of dependent beneficiaries by age, sex of the account holder, and year for the following beneficiary categories.
 - Minor child (0:17,1:2,1970:2020)
 - Student child (18:21,1:2,1970:2020)
 - Disabled adult child (age group 1:9²,1:2,1970:2020)
 - Young spouse (19:64,1:2,1970:2020)
 - Married aged spouse (62:100,1:2,1970:2020)
 - Divorced aged spouse (62:100,1:2,1970:2020)

We also read totals for each category.

17. December data from the MBR containing the number of DIB awards by age, sex and year (15:67,1:2,1970:2020).
18. December data from the MBR containing (1) the number of DIB total terminations (recoveries and deaths) and (2) the number of conversions³. These data are by sex and year (1:2,1970:2020).
19. December data from the MBR containing the number of DIB deaths by age, sex and year (15:67,1:2,1975:2020).
20. December data from the MBR containing the number of estimated DIB recoveries by age, sex and year (15:67,1:2,1975:2020).
21. December data from the MBR containing the number of old-age beneficiaries who at some point in time were converted to retired worker status. This data is by age, sex and year: (62:95+,1:2,1970:2020)

² Age groups 1 through 9 are 18-19, 20-24, 25-29, ..., 55-59.

³ Conversions are DIB beneficiaries who become eligible for old-age benefits due to reaching the normal retirement age.

22. December data from the MBR containing the number of DIB entitled to the Hospital Insurance portion of Medicare by age group (<25,25-29,...,60-64,65+), sex and year (2:11,1:2,1973:2020). This file is not used for the SOSI.
23. December data from the MBR containing the number of DIB awards by duration, age, sex and year (0:5,0:65,1:2,1993:2019). 2019 Awards were retrieved for this year's TR, 1993-2018 Awards have not been updated.
24. Retroactive factors⁴ by year (1969:2019). These values are estimated using OCACT beneficiary data. This file is not used for the SOSI.

All numbers in the following categories are updated annually unless otherwise noted.

25. Average incidence rates by age and sex (15:65,1:2) for the base period 2005-2014 based on awards data from 2005-2019 (also known as the base incidence rates). We update these values when time and data are available. Note that rates for ages 60 through NRA are from the 2011 TR.
26. Probability of death for DIB's – in a multiple-decrement environment by duration, age and sex (0:10,15:65,1:2) for the base period 2011-2015. These numbers (also known as the base probabilities of death) are from Actuarial Study No. 125. We update these values when time and data are available.
27. Probability of recovery for DIB's – in a multiple-decrement environment by duration, age and sex (0:10,15:65,1:2) for the base period 2011-2015. These numbers (also known as the base probabilities of recovery) are from Actuarial Study No. 125. We update these values when time and data are available.
28. IBNR⁵ (incurred but not reported) factors by duration, age and sex (0:10, 15:69, 1:2) based on 2005-2014 entitlements (awards data from 2005-2019). We update these values when time and data are available.
29. For each year 2000-2110, (1) the Normal Retirement Age (NRA), (2) the proportion of DIBs who stay on the DI roll for that age, and (3) the proportion of DIBs who convert to an old-age benefit during that year for that age. We update these values only when there is a change in the NRA or in the present law.
30. The following linkages for the calculations of auxiliary beneficiaries; the probability that student is in an eligible school, the probability that adult child is disabled, the probability that beneficiary is not subject to the earnings test, and the probability that beneficiary was married 10 or more years are estimated and are updated when time and data are available.
31. Short-Range/Long-Range adjustment (APROJ) factors by auxiliary beneficiary category (1:7) for years 2021-2100. These seven categories are; minor child, student child, disabled adult child, young wife, young husband, age wife and aged husband. We calculate these values by comparing Short-Range and Long- Range numbers for auxiliary beneficiaries.
32. IPROJG, DPROJG and RPROJG adjustment factors used to adjust incidence, death and recovery rates to reconcile between the long-range model and the short-range model. Adjfprojg(1:11, 1:2, 2021-2100), Adjfacprojg(1:2, 1:10, 2021-2100) and

⁴ Retroactive factors for each calendar year are the ratio of the total monthly payments to DIBs to the monthly DIBs in current payment status times the average DIB monthly benefit.

⁵ IBNR factors reflect the proportion of DIBs entitled to benefits who have been awarded since the year of their entitlement.

- SRAdjustment(1:2,1:11, 2021-2030) adjustment factors are by age group, sex and year.
33. Ultimate RPROJG values by sex and age group $UltValueRprojg(1:2,1:10)$ calculated to reach a target value. We update these values when the probabilities of recovery for DIB's are updated.

3.2.c. *Development of Output*

Equation 3.2.1 – New Entitlements

We calculate new entitlements by multiplying age-sex-specific incidence rates to the exposed population at the beginning of the year. The exposed population is the disability-insured population less the currently entitled population. We calculate future age-sex-specific incidence rates by multiplying the base incidence rates by the incidence rate projection factors (IPROJGs). For the first ten years of the projection (short-range period), IPROJGs by 5-year age group and sex are obtained by using regression equations with the change in unemployment in the two prior years as the independent variables. Then, we run the IPROJGs through the main model and analyze the resulting incidence rates by age group and sex. We adjust the IPROJGs by age and sex to reach “target” incidence rates in the twentieth year of the projection period. These “target” ultimate incidence rates are calculated by age group and sex using a no-lag unemployment rate regression model for the years 1995-2018. For ages 60-64, rates are increased from the regression results to reflect the planned increase in the Social Security Normal Retirement Age from 66 to 67. We calculated rates for ages 65 and older using a weighted average of our base incidence rates and projected exposure. In 2030, at the end of the short-range period, age-sex-specific incidence rates approximate the ultimate rates assumed for the long-range period. For projection periods between the tenth and twentieth years, we linearly interpolate the IPROJGs between the ultimate IPROJGs values and the IPROJGs values at the end of short-range period. Additional adjustments to the IPROJGs during the short-range period may be necessary for reconciliation between the long-range model and the short-range model. For the 2021 Trustees Report, we made IPROJG adjustments in the short-range period to reflect that disability applications are expected to increase to pre-pandemic levels as the COVID-19 pandemic recedes, the economy will continue to recover, and there will be a temporary increase in applications partially offsetting reduced levels experienced during the pandemic.

Equation 3.2.2 – Exits

The long-range model projects three types of exits from the disability rolls; death, recovery and conversion to an old-age beneficiary upon reaching normal retirement age (NRA). Deaths and recoveries are projected by multiplying the beginning currently entitled population by the probabilities of death only and recovery only, $(q_x^{(d)})$ and $(q_x^{(r)})$, respectively. Projected $(q_x^{(d)})$ and $(q_x^{(r)})$ by age, sex, and duration are calculated by multiplying the base probabilities by the respective projection factors by age group and sex for that year.

For the first ten years, we derive the recovery projection factors (RPROJGs) by age group and sex from linear interpolation between an estimated starting level for the RPROJGs and an estimated tenth-year projection target level for the RPROJGs. For each age group and sex, we

calculate the starting RPROJGs the following way:

$$\text{RPROJG}^{\text{TR21}}(2020) = \text{RPROJG}^{\text{TR20}}(2020) \times \frac{\text{actual recovery rate}(2020)}{\text{recovery rate}^{\text{TR20}}(2020)}$$

Because there is no apparent upward or downward trend, we use the average recovery rates for the last ten historical years as the target values for the 10th year (2030). Then, for each age group and sex, we calculate the tenth year's RPROJGs as follows:

$$\text{RPROJG}^{\text{TR21}}(2030) = \text{RPROJG}^{\text{TR21}}(2020) \times \frac{\text{target value recovery rate}(2030)}{\text{actual recovery rate}(2020)}$$

For the second 10 years of the projection period, we linearly interpolate between the ultimate RPROJG value and the RPROJG value at the end of short-range period (2030). Ultimate recovery rates by age group and sex are determined by analyzing historical recovery rates. We may make additional adjustments to the RPROJGs to reconcile with the short-range model.

For the first year of the projection period, the death projection factors (DPROJGs) by age group and sex are determined so that they achieve a targeted death rate. The targeted death rate is determined by fitting an exponential curve to historical death rates for DIBs by age group and sex. For the 2021 Trustees Report, we made DPROJG adjustments in 2021 and 2022 to better reflect the extra mortality expected from the pandemic. For the rest of the projection period, we assume the DPROJGs improve at the same rate as the general population for that age group and sex. We calculate the DPROJGs for each year by 5-year age group and sex the following way:

$$\text{DPROJG}(\text{year}) = \text{DPROJG}(\text{year}-1) \times \left(\frac{\sum_{\text{age}=x}^{x+4} \frac{\text{year}}{\text{age}} \times \frac{1999}{\text{age}}}{\sum_{\text{age}=x}^{x+4} \frac{\text{year}-1}{\text{age}} \times \frac{1999}{\text{age}}} \right)$$

Equation 3.2.3 – Disabled-Worker Beneficiaries

The projection begins with the latest data available from the mainframe of disabled-worker beneficiaries in current-payment status. This data is from a 100 percent sample of the Master Beneficiary Record (MBR) at the end of the year. We split up disabled-worker beneficiaries by age at entitlement, sex and duration of entitlement. We convert this population to a currently entitled population by dividing each age, sex and duration cell by the appropriate duration-age-sex-year-specific IBNR factor. An iterative process begins with new entitlements added to and exits subtracted from the previous year's currently entitled population to get the following year's currently entitled population with advancement of duration within the age of entitlement. We reduce this currently entitled population by multiplying by the appropriate duration-age-sex-year-specific IBNR factor. The result is the following year's disabled-worker beneficiaries in current-payment status. The process repeats over each sex, age of entitlement and duration of entitlement throughout the projection period.

Equation 3.2.4 – Dependent Beneficiary of Disabled Workers

There are six dependent-beneficiary categories; minor child, student child, disabled adult child,

young spouse, married aged spouse and divorced aged spouse. We disaggregate projections by age of the beneficiary and sex of the account holder. We detail below the linkages and exposures used in each category of dependent beneficiaries.

Minor Child

Exposure: Single SSA population by single ages 0-17

Linkages: pMCAGA = Probability that parent is under NRA
 pMCDIA = Probability that parent is disability insured given that the parent is under NRA
 pMCDPA = Probability that disability insured parent under NRA is disabled
 MCRES = Residual Factor

Student Child

Exposure: Single SSA population by single ages 18-19

Linkages: pSCAGA = Probability that parent is under NRA
 pSCDIA = Probability that parent is disability insured given that the parent is under NRA
 pSCDPA = Probability that disability insured parent under NRA is disabled
 pSCDPC = Probability that student is in an eligible school
 SCRES = Residual Factor

Disabled Adult Child

Exposure: Total SSA population by age groups 18-19, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59

Linkages: pDCAGA = Probability that parent is under NRA
 pDCDIA = Probability that parent is disability insured given that the parent is under NRA
 pDCDPA = Probability that disability insured parent under NRA is disabled
 pDCDPC = Probability that adult child is disabled
 DCRES = Residual Factor

Young Spouse

Exposure: Married SSA population by sex and by single ages 20-64

Linkages: pYSAGA = Probability that account holder is under NRA
 pYSDIA = Probability that account holder is disability insured given that the account holder is under NRA
 pYSDPA = Probability that disability insured account holder under NRA is disabled
 pYSETB = Probability that young spouse is not subject to earnings test
 pYSMCB+ pYSDCB= Probability that young spouse has a minor child or a disabled child beneficiary in his/her care
 YSRES = Residual Factor

Married Aged Spouse

Exposure: Married SSA population by sex and by single ages 62-100

Linkages: pMSAGA = Probability that account holder is under NRA
pMSDIA = Probability that account holder is disability insured given that the account holder is under NRA
pMSDPA = Probability that disability insured account holder under NRA is disabled
pMSFIB = Probability that beneficiary is not insured
MSRES = Residual Factor

Divorced Aged Spouse

Exposure: Divorced SSA population by sex and by single ages 62-100

Linkages: pDSDEA = Probability that account holder is living
pDSAGA = Probability that account holder is under NRA
pDSDIA = Probability that account holder is disability insured given that the account holder is under NRA
pDSDPA = Probability that disability insured account holder under NRA is disabled
pDSFIB = Probability that beneficiary is not insured
pDSDMB = Probability that beneficiary was married 10 or more years

We estimate the residual factors for each of the dependent categories using a 10-year Least Squares regression formula. We then hold these residual factor values constant for the duration of the long-range period. If the 10-year Least Squares method results in a negative residual factor, we hold the last historical residual factor instead. For the 2021 Trustees Report, so that the 2020 experience during the pandemic does not skew our regression results, we retain the regression period of 2010-2019 from the 2020 Trustees Report.

We develop factors for a dependent beneficiary category to match short-range results during the first 10 years of the projection period. We phase these factors out linearly over the second ten years of the projection period.

Details about the regression equations used in determining incidence rates and death rates by age group and sex are available upon request.

3.3. Old-Age and Survivors Insurance

3.3.a. Overview

Every month, the Social Security program pays benefits to retired workers and their dependents. It also provides benefits to eligible dependents of deceased workers. The OLD-AGE AND SURVIVORS subprocess projects the number of people expected to receive benefits over the next 75 years. The projection method is very similar to the method used for dependent beneficiaries of disabled workers in the DISABILITY subprocess. We compute the projection of beneficiaries by multiplying a subset of the Social Security area population by a series of probabilities of the conditions that a person must meet to receive benefits. The main program receives all necessary input data and performs all preliminary calculations. It then calls each individual beneficiary type subroutine where it makes all beneficiary calculations.

We categorize retired workers and their dependent beneficiaries as follows:

- retired workers (*RWN*) by age (62-95+), sex, and marital status (single, married, widowed, divorced)
- aged spouses of retired workers (*ASRWN*), by age (62-95+), sex of the account holder, and marital status of the beneficiary (married, divorced)
- young spouses of retired workers (*YSRWN*) by age-group (under 25, 25-29, ..., 65-69) and sex of the account holder
- minor, student, and disabled adult children of retired workers (*MCRWN*, *SCRWN*, and *DCRWN*, respectively) by age of the child (0-17 for minor, 18-21 for student, age groups 18-19, 20-24, ..., 55-59, 60+ for disabled adult) and sex of the account holder

Dependent beneficiaries of deceased workers include:

- aged spouses of deceased workers, *ASDWN*, by age (60-95+), sex of the account holder, marital status (widowed, divorced) and insured status (insured, uninsured)
- disabled spouses of deceased workers (*DSDWN*) by age (50-69), sex of the account holder and marital status (widowed, divorced)
- young spouses of deceased workers (*YSDWN*) by age-group (under 25, 25-29, ..., 65-69), sex of the account holder and marital status of the beneficiary (widowed, divorced)
- minor, student, and disabled adult children of deceased workers (*MCDWN*, *SCDWN*, and *DCDWN*, respectively) by age of the child (0-17 for minor, 18-21 for student, age groups 18-19, 20-24, ..., 55-59, 60+ for disabled adult) and sex of the account holder

Lastly, we estimate the number of deaths of insured workers (*LUMSUM*) by 5-year age group (20-24, 25-29, ..., 80-84, 85+) and sex.

Equations 3.3.1-13 indicates the flow of calculations of beneficiaries.

$$ASDWN = ASDWN(\cdot) \quad (3.3.1)$$

$$RWN = RWN(\cdot) \quad (3.3.2)$$

$$ASRWN = ASRWN(\cdot) \quad (3.3.3)$$

$$DSDWN = DSDWN(\cdot) \quad (3.3.4)$$

$$MCRWN = MCRWN(\cdot) \quad (3.3.5)$$

$$MCDWN = MCDWN(\cdot) \quad (3.3.6)$$

$$SCRWN = SCRWN(\cdot) \quad (3.3.7)$$

$$SCDWN = SCDWN(\cdot) \quad (3.3.8)$$

$$DCRWN = DCRWN(\cdot) \quad (3.3.9)$$

$$DCDWN = DCDWN(\cdot) \quad (3.3.10)$$

$$YSRWN = YSRWN(\cdot) \quad (3.3.11)$$

$$YSDWN = YSDWN(\cdot) \quad (3.3.12)$$

$$LUMSUM = LUMSUM(\cdot) \quad (3.3.13)$$

The appendix 3.3-1 at the end of this section provides a listing with explanation of the acronyms used in this documentation.

3.3.b. Input Data

We update all data annually unless otherwise noted. Timing of data received is denoted ‘BOY’ (beginning of year) or ‘EOY’ (end of year).

Long-Range OCACT Data

Demography

1. Social Security area population by year (EOY 1970-2100), single year of age (0-100+), sex, and marital status (single, married, widowed, divorced)
2. Deaths by year (during years 2020-2100), age group (20-24, ..., 80-84, 85+) and sex
3. Average number of children per family by year (EOY 1970-2100), and age group of the householder (20-24, ..., 60-64)
4. Children by year (EOY 1970-2100), single year of age (0-18), sex of primary account holder (parent), status of primary (62+ or deceased), and age of the other parent (15-19, 20-24, ..., 65-69, 70+, total ages)
5. Married couples by year (EOY 1970-2100), age of husband (62-95+) and age of wife (62-95+)
6. Persons with an aged spouse by year (EOY 1970-2100), age group (15-24, 25-29, ..., 65-69) and sex
7. Probabilities of death by year (EOY 1941-2100), single year of age (-1,100), and sex

Economics

8. Covered wages and employment in the Federal Civilian and State and Local Sectors (during years 1998-2100)
9. Employment rates for age 62 by year (during years 1977-2100) and sex

Beneficiaries

10. Fully insured persons by year (EOY 1969-2100), age (14-95+), sex, and marital status (single, married, widowed, divorced)
11. Lawful permanent resident (LPR) fully insured rate by year (EOY 1969-2100), age (14-95+), and sex
12. Disabled-worker beneficiaries in current pay by year (EOY 1970-2100), age (62-66) and sex
13. Converted DI to OAI beneficiaries by year (EOY 1970-2100), age (62-95+) and sex
14. Disability prevalence rates by year (EOY 1970-2100), age (50-69) and sex

Short-Range OCACT Data

15. For EOY 1975-2005:
 - a. Aged spouses of deceased workers by age (60-95+), sex, and insured status
 - b. Retired workers by age (62-95+) and sex
 - c. Aged spouses of retired workers by age (62-95+) and sex
 - d. Disabled widow(er)s by age (50-64) and sex and marital status
 - e. Minor children by age (0-17) and status of parent (retired, deceased)
 - f. Student children by age (18-21) and status of parent (retired, deceased)
 - g. Disabled adult children by age (20-95+) and status of parent (retired, deceased)
 - h. Young spouses of retired workers by age group (under 25, 25-29,....,60-64,65-65) and sex
 - i. Young spouses of deceased workers by age group (under 25, 25-29,....,60-64,65-65) and sex

Note: We will not update this data.

16. Insured aged spouses of deceased workers by year (EOY 1974-2020), age (60-95+) and sex
17. Retired worker beneficiaries in-current-pay status by age (62-70, 70+) and sex for EOY 2019-2030
18. We receive the following for EOY 2020:
 - a. Aged spouses of deceased workers by age (60-95+), sex and marital status (widowed, divorced)
 - b. Retired workers by age (62-95+) and sex
 - c. Aged spouses of retired workers by age (62-95+), sex and marital status (married, divorced)
 - d. Disabled widow(er)s by age (50-66), sex and marital status (widowed, divorced)
 - e. Minor children by age (0-17), sex of parent and status of parent (retired, deceased)
 - f. Student children by age (18-21), sex of parent and status of parent (retired, deceased)

- g. Disabled adult children by age group (18-19, 20-24, ..., 55-59, 60+), sex of parent and status of parent (retired, deceased)
 - h. Young spouses of retired workers by age group (under 25, 25-29, ..., 65-66) and sex
 - i. Young spouses of deceased workers by age group (under 25, 25-29, ..., 65-66), sex and marital status (widowed, divorced)
 - j. Total parent beneficiaries
19. We receive the following for EOY 2030:
- a. Retired workers by age group (62-64, 65-69) and sex
 - b. Insured widows by age group (60-64, ..., 80-84, 85+)
 - c. Uninsured widows by age group (60-64, 65+)
 - d. Total disabled widows
 - e. Female young spouses of deceased workers
 - f. Female aged spouses of retired workers by age group (62-64, 65-67, 68-70, 71+)
 - g. Female young spouses of retired workers
 - h. Minor children by status of parent
 - i. Student children by status of parent
 - j. Disabled adult children by status of parent
20. Total amount of lump-sum death payments during 2019
21. Insured aged spouses in current pay by single year of age as of the end of the year for years 1974-2020.

Other Input Data

22. For EOY 1970-1974, obtained from the MBR10PER dataset on the mainframe:
- j. Aged spouses of deceased workers by age (60-95+), sex and marital status (widowed, divorced)
 - k. Retired workers by age (62-95+) and sex
 - l. Aged spouses of retired workers by age (62-95+), sex and marital status (married, divorced)
 - m. Disabled widow(er)s by age (50-64), sex and marital status (widowed, divorced)
 - n. Minor children by age (0-17), sex of parent and status of parent (retired, deceased)
 - o. Student children by age (18-21), sex of parent and status of parent (retired, deceased)
 - p. Disabled adult children by age (20-95+), sex of parent and status of parent (retired, deceased)
 - q. Young spouses of retired workers by age group (under 25, 25-29, ..., 60-64) and sex
 - r. Young spouses of deceased workers by age group (under 25, 25-29, ..., 60-64), sex and marital status (widowed, divorced)
 - s. Total parent beneficiaries
- Note: We will not update this data.
23. Number of beneficiaries with benefits withheld due to receipt of a significant government pension by sex and marital status (married, widowed) for EOY 2019 from the 2020 Annual Statistical Supplement

24. Age distribution of beneficiaries with benefits withheld due to receipt of a significant government pension by age (60-95+) and sex, computed as an average from the 2016 through 2020 WEP 100 percent sample
25. Proportions of disabled adult children of retired and deceased workers (proportioned by age and sex of the child) from the 2003 MBR ten-percent sample. (Note: The RSB program calculates disabled adult children by sex of the primary account holder, not by sex of the child. The RSB program outputs a file, which we use for Annual Update #9, which calculates beneficiaries by sex. Therefore, we apply the 2003 proportions to estimate the breakdown of disabled adult children by sex of the child. We will not update this input.). Not used for SOSI.
26. Schedule of normal retirement age (*NRA*), delayed retirement credit, and actuarial reduction factors for ages more than 3 years below *NRA* and less than 3 years below *NRA* for years 1970-2100 from the Social Security website (Note: these values are only updated when there is a Social Security law change regarding the *NRA*)
27. Prevalence rate regression coefficients (slopes and y-intercept value by sex)
28. Regressed prevalence rate by sex for the most recent historical year
29. Adjustment factors which account for the difference between estimated and actual historical retired worker prevalence rates by year (EOY 2021-2100), age (63-69) and sex
30. Adjustment factors which account for the difference between projected beneficiary values for the 10th year of the projection period made by the Long-Range and Short-Range offices. Factors are computed for:
 - a. Retired workers by age group (62-64, 65-69) and sex
 - b. Insured widows by age group (60-64, ..., 80-84, 85+)
 - c. Uninsured widows by age group (60-64, 65+)
 - d. Total disabled widows
 - e. Female young spouses of deceased workers
 - f. Female aged spouses of retired workers by age group (62-64, 65-67, 68-70, 71+)
 - g. Female young spouses of retired workers
 - h. Minor children by status of parent
 - i. Student children by status of parent
 - j. Disabled adult children by status of parent
31. Adjustment factors to auxiliary beneficiary categories for demographic assumption changes increasing the other-than-LPR population by sex for years 2021-2100. Factors are computed for:
 - a. Aged spouses of deceased workers (widow(er)s)
 - b. Aged spouses of retired workers
 - c. Disabled adult children of retired workers
 - d. Minor children of deceased workers

- e. Disabled adult children of deceased workers
- f. Young spouses of deceased workers

Note: Starting with the 2017 TR, these factors are all set to 1 due to the change to using LPR fully insured rates versus total fully insured for calculating FIB factors.

32. Number of one-earner couples by year of birth of the primary account holder where the primary account holder files for and suspends their retirement worker benefit and their spouse receives a spousal benefit for years 2008-2015 from a 1 percent MBR sample.

Note: We will not update this data.

33. EOY 2020 male and female retired worker counts by attained age and entitlement age from a 100 percent MBR sample.

34. Adjustments to the estimated historical retired worker prevalence rates to account for the increase in retired workers due to the elimination of the deemed filing claiming strategy.

3.3.c. *Development of Output*

We use several acronyms to describe the equations presented below. Acronyms not preceded by a subscript generally refer to the number of beneficiaries. For example, RWN refers to the number of retired workers. Acronyms preceded with a ‘*p*’ refer to probabilities. For example, pRW_{FIA} refers to the probability that a person is fully insured.

Equation 3.3.1 – Aged Spouses of Deceased Workers (ASDWN)

Aged Spouses of Deceased Workers

Exposures: SSA population by age (60-95+), sex and marital status (widowed and divorced)

- Linkages:
- $pASDW_{DEA}$ = probability that the primary account holder (PAH) is deceased
 - $pASDW_{FIA}$ = probability that the PAH was fully insured at death
 - $pASDW_{MBB}$ = probability that the widow(er) is not receiving a young-spouse benefit for the care of a child
 - $pASDW_{FIB}$ = probability that the aged-widow(er) is or is not fully insured
 - $pASDW_{GPB}$ = probability that the aged-widow(er)’s benefits are not withheld or offset totally because of receipt of a significant government pension based on earnings in non-covered employment
 - $pASDW_{RES}$ = probability that a widow(er) eligible to receive his/her own retired-worker benefits would instead apply for and receive widow(er) benefits

We project the number of aged spouses of deceased workers (widow(er)s), along with all

linkage factors, by age, sex of the account holder ($sa = 1$ for male, $sa = 2$ for female), marital status and insured status. Age ranges from 60 to 95+, marital status includes widowed ($mb = 1$) and divorced ($mb = 2$), and insured status includes insured ($fin = 1$) and uninsured ($fin = 2$). Note that all variables preceded by the letter p refer to calculated probabilities. We calculate the projected number of insured aged spouses of deceased workers age 60 to 70, and uninsured aged spouses of deceased workers age 60 to 95+ as follows:

$$ASDWN = ASDW_{POP} \times pASDW_{DEA} \times pASDW_{FIA} \times pASDW_{MBB} \times pASDW_{FIB} \times pASDW_{GPB} \times pASDW_{RES} \quad (3.3.1)$$

For each sex we calculate the projected number of insured aged spouses of deceased workers over age 70 by applying mortality rates to the population already receiving such benefits:

$$ASDWN_{N,YEAR} = ASDWN_{N-1,YEAR-1} \times (1 - qx_{N-1,YEAR})$$

Where N is the age, and $qx_{N-1,YEAR}$ is the death rate for age $N-1$ in the given year.

$ASDW_{POP}$ represents the subset of the population from which we draw these beneficiaries. We set this equal to the Social Security area population ($SSAPOP_{mb}$) for each possible marital status.

$$ASDW_{POP} = SSAPOP_{mb}$$

$pASDW_{DEA}$ represents the probability that the primary account holder (PAH) is deceased. For the widowed population, we set this factor equal to one. For the divorced population, we set this factor equal to the portion of the total widowed ($SSAPOP_{wid}$) and married ($SSAPOP_{mar}$) population who are widowed.

$$pASDW_{DEA} = \begin{cases} 1, & mb = 1 \text{ (widowed)} \\ \frac{SSAPOP_{wid}}{SSAPOP_{mar} + SSAPOP_{wid}}, & mb = 2 \text{ (divorced)} \end{cases}$$

$pASDW_{FIA}$ represents the probability that the PAH was fully insured at death. For a given age of widow, AW , we assume that the age of her deceased husband, AH , ranges from $AW-6$ to $AW+12$ with a lower and upper bound of 60 and 95+. Further, we assume that the more likely age of the husband is $AW+3$. For each age, we calculate $pASDW_{FIA}$ as a weighted average of the portion of the Social Security area population who are fully insured at each possible age of the husband (INS_{AH}). For example, for a widow age 70, we assume that the age of her husband is between 64 and 82, therefore we calculate the weighted average of the portion of the population who are fully insured men, applying the highest weight of 10 to age 73, and a linearly reduced weight to zero for each age above and below 73. We use the same concept for widow(er)s with the assumption that the age of his deceased wife ranges from $AH-12$ to $AH+6$, with a greater likelihood of her age being $AH-3$. Let $WEIGHT$ represent the specific weight applied to each potential age of the spouse.

$$WEIGHT_{AH} = 10 - |AW + 3 - AH|$$

$$WEIGHT_{AW} = 10 - |AH - 3 - AW|$$

$$pASDW_{FIA} = \begin{cases} \frac{\sum_{AH=AW-6}^{AW+12} WEIGHT_{AH} \times INS_{AH}}{\sum_{AH=AW-6}^{AW+12} WEIGHT_{AH}}, & sa = 1 \\ \frac{\sum_{AW=AH-12}^{AH+6} WEIGHT_{AW} \times INS_{AW}}{\sum_{AW=AH-12}^{AH+6} WEIGHT_{AW}}, & sa = 2 \end{cases}$$

$pASDW_{MBB}$ represents the probability that the widow(er) is not receiving a young-spouse benefit for the care of a child. A widow(er) can receive a young-spouse benefit up to age 69 if he/she meets all other eligibility requirements. Since the minimum age requirement to receive a widow(er) benefit is 60, it is necessary to remove those receiving a young-spouse benefit ($YSDWN^{ab}$), where ab represents the 5-year age bracket¹. We assume a uniform breakdown to divide the age groups into single-age estimates.

For $fin = 1$ (insured):

$$pASDW_{MBB} = 1$$

For $fin = 2$ (uninsured):

$$FACTOR_{AGE} = \begin{cases} 1, & 65 \leq age \text{ and } NRA \geq age + 1 \\ NRA - age, & age < NRA < age + 1 \\ 0, & elsewhere \end{cases}$$

$$pASDW_{MBB} = \begin{cases} 1 - \frac{0.2 \times YSDWN^{60-64}}{ASDW_{POP} \times pASDW_{DEA} \times pASDW_{FIA}}, & age = 60 - 64 \\ 1 - \frac{YSDWN^{65-69}}{ASDW_{POP} \times pASDW_{DEA} \times pASDW_{FIA}} \times \frac{FACTOR_{age}}{\sum_{65}^{69} FACTOR_{age}}, & 65 \leq age \leq NRA \\ 1, & age > NRA \end{cases}$$

$pASDW_{FIB}$ represents the probability that the aged widow(er) is fully insured. For insured widow(er)s, $pASDW_{FIB}$ is equal to the LPR fully insured rate ($FILEG$) at each age and sex. For uninsured widow(er)s, $pASDW_{FIB}$ is simply one minus the probability for insured widow(er)s.

$$pASDW_{FIB} = \begin{cases} FILEG, & fin = 1 \\ 1 - FILEG, & fin = 2 \end{cases}$$

Where fin represents the insured status of the account holder.

¹ There are no young spouses at NRA or above.

$pASDW_{GPB}$ represents the probability that the aged-widow(er)'s benefits are not withheld or completely offset because of receipt of a significant government pension based on earnings in non-covered employment. According to the 1977 amendments, Social Security benefits are subject to reduction by up to two-thirds of non-covered government pension. $GPWHL D$ represents the total number of widow(er) beneficiaries (for all ages) expected to receive a significant government pension. $rGPOAGE$ represents the ratio of the total for each given age. If a person is insured, this implies that he/she is eligible to receive Social Security benefits based on his/her own earnings regardless of a government pension. Therefore, we do not apply a factor.

For $fin = 1$ (insured):

$$pASDW_{GPB} = 1$$

For $fin = 2$ (uninsured):

$$pASDW_{GPB} = \begin{cases} 1, & year \leq 1978 \\ 1 - \frac{rGPOAGE \times GPWHL D}{ASDW_{POP} \times pASDW_{DEA} \times pASDW_{FIA} \times pASDW_{MBB} \times pASDW_{FIB}}, & year > 1978 \end{cases}$$

$pASDW_{RES}$ represents the probability that a widow(er), who is eligible to receive widow(er)'s benefits actually receives the benefits. In particular, for $fin = 1$, this factor is equivalent to the probability that a widow(er) eligible to receive his/her own retired-worker benefits would instead apply for and receive widow(er) benefits. This factor accounts for other eligibility requirements not previously mentioned. For example, in the case of an aged widow(er), that the widow(er) did not remarry before age 60. In the case of a divorced widow(er), that the marriage to the PAH lasted at least 10 years. For all historical years, we calculate $pASDW_{RES}^{year}$ as the ratio of $ASDWN$, the actual number of widow(er)s, to the number of persons meeting all previously mentioned requirements by age, sex, insured status, and marital status.

$$pASDW_{RES}^{year} = \frac{ASDWN}{ASDW_{POP} \times pASDW_{DEA} \times pASDW_{FIA} \times pASDW_{MBB} \times pASDW_{FIB} \times pASDW_{GPB}}, \quad year < TRYR$$

Where TRYR is the Trustees Report year.

For each age, sex, insured status, and marital status, we use a least squares regression over the last ten years of historical data to determine a starting value in TRYR-1 for $pASDW_{RES}^{year}$ from which we project future values. In addition, for each sex, insured status, and marital status, we graduate the regressed values of $pASDW_{RES}^{TRYR-1}$ over age using a weighted minimized third-difference formula to produce $ESTRES^{ASDW}$. $ESTRES^{ASDW}$ are the preliminary estimates of $pASDW_{RES}^{TRYR+9}$, the values in the 10th year of the projection period². In addition, we apply adjustments to the widows ($sa = 1$) by age group (60-64, 65-69 for insured; 60-64, 65+ for uninsured), $SRADJ^{ASDW}$, to the 10th year of the projection

² For the 2021 Trustees Report, the ten-year regression is over the period 2010-2019, and we graduate the TRYR-2 regressed values of $pASDW_{RES}$, excluding the 2020 data due to the Covid-19 pandemic.

period in order to match the projections made by the Short-Range office. We linearly interpolate the values of $pASDW_{RES}^{year}$ for intermediate years between $pASDW_{RES}^{TRYR-1}$ and $pASDW_{RES}^{TRYR+9}$ (equal to $ESTRES^{ASDW} \times SRADJ^{ASDW}$). After the 10th year, we linearly grade these adjustment factors to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to long-range projections.

We also apply adjustments to the 70+ insured widows by age group (70-74, 75-79, 80-84, 85+). The adjustments for the intermediate years between the TRYR and TRYR + 9 are applied on a cohort basis to the number of aged spouses of deceased workers (ASDWN) using the following formula:

$$ASDWN_{N,YEAR} = ASDWN_{N,YEAR} \times SRADJ\left(\frac{1}{h}\right)$$

Where h is the number of years the adjustment is applied for the cohort. For age 70, we linearly grade these adjustment factors to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to long-range projections.

Equation 3.3.2 – Retired Workers (RWN)

Retired Workers

Exposure: SSA population by age (62-95+), sex and marital status (single, married, widowed and divorced)

Linkages: pRW_{FIA} = probability that the primary account holder (PAH) is insured

pRW_{DBB} = probability that the PAH is not receiving a disabled-worker benefit

pRW_{WBB} = probability that the PAH is not receiving a widow(er) benefit

pRW_{RES} = retirement prevalence rate; probability that a fully insured worker not receiving disability or widow(er)'s benefits would receive a retired-worker benefit

We project the numbers of retired-worker beneficiaries, along with all linkage factors, by age, sex, and marital status. Age ranges from 62 to 95+, and marital status includes single, married, widowed, and divorced ($ms = 1-4$). We calculate the projected number of retired-worker beneficiaries as follows:

$$RWN = RW_{POP} \times pRW_{FIA} \times pRW_{DBB} \times pRW_{WBB} \times pRW_{RES} \quad (3.3.2)$$

RW_{POP} represents the subset of the population from which we draw these beneficiaries. We set this equal to the Social Security area population ($SSAPOP_{ms}$) for $ms = 1-4$.

$$RW_{POP} = SSAPOP_{ms}$$

pRW_{FIA} represents the probability that the primary account holder (PAH) is insured. We set

this factor equal to the portion of the Social Security area population that is fully insured (*INS*) for $ms = 1-4$.

$$pRW_{FIA} = \frac{INS}{RW_{POP}}$$

pRW_{DBB} represents the probability that the PAH is not receiving a disabled-worker or disability-conversion benefit. We set this factor equal to the portion of fully insured workers who are neither disabled-worker beneficiaries nor converted from disabled-worker beneficiaries (*DIBCON*). *ASDWN* represents the number of aged spouses of deceased workers.

$$pRW_{DBB} = \begin{cases} 1 - \frac{DIBCON}{RW_{POP} \times pRW_{FIA}}, & ms = 1, 2 \\ \left(1 - \frac{DIBCON + ASDWN}{RW_{POP} \times pRW_{FIA}}\right) \left(\frac{DIBCON}{DIBCON + ASDWN}\right), & ms = 3, 4 \end{cases}$$

pRW_{WBB} represents the probability that the PAH is not receiving a widow(er) benefit. We set this factor equal to the portion of fully insured workers that is not aged spouses of deceased workers.

$$pRW_{WBB} = \begin{cases} 1, & ms = 1, 2 \\ \left(1 - \frac{DIBCON + ASDWN}{RW_{POP} \times pRW_{FIA}}\right) \left(\frac{ASDWN}{DIBCON + ASDWN}\right), & ms = 3, 4 \end{cases}$$

$pRW_{RES}^{N,year}$ represents the retirement prevalence rate, which is the probability that a fully insured worker not receiving disability or widow(er)'s benefits would receive retired-worker benefits as of the given age, N , for the given year. In order to estimate the future prevalence rate, the program first calculates the historical values of $pRW_{RES}^{N,year}$.

For each historical year and sex, we calculate $pRW_{RES}^{N,year}$ as the ratio of *RWN*, the actual number of retired workers, to the number of persons meeting all previously mentioned requirements by age, sex, and marital status.

$$pRW_{RES}^{N,year} = \frac{RWN}{RW_{POP} \times pRW_{FIA} \times pRW_{DBB} \times pRW_{WBB}}, \quad N = 62-95+ \text{ and } year < \text{TRYR}$$

Historical prevalence rates at age 62 follow an inverse relationship with (1) employment rates ($EMPRATE^{year}$) at age 62, by sex, and (2) increases in the normal retirement age over the historical period. We assume this relationship holds in the projection period, and therefore we used it to calculate $REGPR^{year}$, the regressed prevalence rate based on the projected $EMPRATE^{year}$ at age 62 for each year and sex, and the number of months from age 62 to the normal retirement age ($monthNRA^{year}$). Note that we calculate prevalence rates on a cohort

basis³. The regression equation used to estimate the prevalence rates is:

$$REGPR^{year} = -1.35852 \times EMPRATE^{year} + -0.00417 \times monthNRA^{year} + 1.25532 \text{ for male}$$

with an R² value of 0.88795

and

$$REGPR^{year} = -0.46351 \times EMPRATE^{year} + -0.00955 \times monthNRA^{year} + 1.00156 \text{ for female}$$

with an R² value of 0.89175

We then set the future prevalence rate at age 62, $pRW_{RES}^{62,year}$, equal to the sum of the regressed prevalence rate ($REGPR^{year}$) and $ERROR$, the difference between the actual prevalence rate and the regressed prevalence rate in the most recent historical year, which we phase out linearly over 10 years.

$$pRW_{RES}^{62,year} = REGPR^{year} + (ERROR) \times \max\left(0, \frac{TRYR+9-year}{10}\right), \quad N = 62 \text{ and } year \geq TRYR-1$$

To compute $pRW_{RES}^{N,year}$ for ages 63 through 69 in the projection period, we must calculate several preliminary variables. These include:

- $MBAPIA_N$, for $N = 62-70$ (same for both sexes),
- $ESTPR_N^{year}$, for $N = 63-69$ and by sex, and
- $DIFFADJ_N$, for $N = 63-69$ and by sex.

$MBAPIA_N$ is the ratio of the monthly benefit amount (MBA) to the primary insurance amount (PIA) at age N and is calculated on a cohort basis for $N = 62-70$. We base the calculation of $MBAPIA_N$ on the normal retirement age (NRA), delayed retirement credits (DRC), and actuarial reduction factors, $ARFLE3$ when the difference between NRA and age at retirement is less than 3, and $ARFGT3$ when the difference is greater than 3 within each cohort. If a person retires after NRA , his/her benefits are increased by DRC for each year the age exceeds NRA . If a person retires before NRA , his/her benefits are decreased by $ARFLE3$ for each of the first three years that NRA exceeds the age, and further decreased by $ARFGT3$ for any remaining years.

$$MBAPIA_N = \begin{cases} 1 + (N - NRA) \times DRC, & N \geq NRA \\ 1 - (NRA - N) \times ARFLE3, & NRA - 3 \leq N \leq NRA \\ 1 - 3 \times ARFLE3 - (NRA - 3 - N) \times ARFGT3, & N < NRA - 3 \end{cases}$$

$ESTPR_N^{year}$, the estimated prevalence rate at age N , is then calculated as the prevalence rate at age 62 ($pRW_{RES}^{62,year-(N-62)}$) plus an estimate on the expected portion of the remaining probability ($1 - pRW_{RES}^{62,year-(N-62)}$), that a potential retired worker will actually retire by that given age. We base this estimate on $MBAPIA_N$, assuming that the retirement decision by a worker is totally and completely influenced by the expected change in the portion of

³For example, to calculate the projected number of 65 year olds in a given year, the prevalence rate at age 62 is needed. This is actually the prevalence rate that occurred three years ago at age 62.

PIA that is payable at each age relative to the potential change after the initial eligibility.

$$ESTPR_N^{year} = pRW_{RES}^{62,year-(N-62)} + (1 - pRW_{RES}^{62,year-(N-62)}) \times \frac{MBAPIA_N - MBAPIA_{62}}{MBAPIA_{70} - MBAPIA_{62}}, \quad N = 63-69$$

In the first year of the projection period, an adjustment ($DIFFADJ_N$) is made which accounts for the difference between the actual and estimated prevalence rate at each age in the most recent historical years. For ages 63 to 69, the value used beginning in TRYR is a regressed value based on the last 5 years' differences between the actual and estimated prevalence rate. With the exception of age 66, we hold this value constant throughout the projection period.

$DIFFADJ_N$ is greater during NRA transition periods. This affects $DIFFADJ_{66}$ due to the scheduled increase in NRA from age 66 to 67 over the years 2017-2022. For the years preceding the NRA transition, we use the average from the last five historical years to calculate $DIFFADJ_{66}$. We assume that the effects on age 66 prevalence rates after the NRA change from 66 to 67 will be similar to the effects on age 65 prevalence rates after the NRA change from 65 to 66. Thus we set $DIFFADJ_{66}$ to the average difference between actual and estimated prevalence rates for age 65 during the five years after the NRA changed from 65 to 66. Then after the NRA increases to age 67, we use $DIFFADJ_{66}$ to decrease age 66 prevalence rates to the level that age 65 prevalence rates dropped to after the NRA increase to 66. In the years during which the NRA transitions to age 67 we use a linear interpolation based on the number of months that the NRA has increased in order to phase out $DIFFADJ_{66}$ and phase in $DIFFADJ_{67}$. It will remain at $DIFFADJ_{66}$ for the rest of the projection period.

$$pRW_{RES}^{N,year} = ESTPR_N + DIFFADJ_N$$

We apply additional adjustments to lower age 66 prevalence rates for the 1955 through 1960 birth cohorts to account for the NRA transition to age 67. The target age 66 prevalence for the 1960 cohort (the first cohort with an NRA of 67) is set by matching the increase in prevalence between ages 65 and 66 to the increase between ages 64 and 65 of the 1943 cohort (the first with an NRA of 66). The adjustment factor needed to reduce the 1960 cohort age 66 prevalence to the target value is graded in over the age 66 prevalence rates of the 1955 through 1959 cohorts.

For age 70 men, we assume that the values of the latest actual $pRW_{RES}^{N,year}$, change linearly to the ultimate level of 0.995 over the first 20 years of the projection period. For age 70 women, we assume an ultimate level of 0.990 through 2023 and an ultimate level of 0.995 starting in 2024. This is due to the Bipartisan Budget Act of 2015 (BBA), which expands deemed filing to those at NRA or above, thus eliminating the deemed filing claiming strategy for those turning 62 in 2016 or later.

For $year \geq TRYR$:

$$pRW_{RES}^{70,year} = \begin{cases} 0.995 - [0.995 - pRW_{RES}^{70,TRYR-1}] \times \max\left(0, \frac{TRYR + 19 - year}{20}\right), & sa = 1 \\ 0.990 - [0.990 - pRW_{RES}^{70,TRYR-1}] \times \max\left(0, \frac{TRYR + 19 - year}{20}\right), & sa = 2 \text{ and } year \leq 2023 \\ 0.995 - [0.995 - pRW_{RES}^{70,TRYR-1}] \times \max\left(0, \frac{TRYR + 19 - year}{20}\right), & sa = 2 \text{ and } year > 2023 \end{cases}$$

For ages 71 and older, we assume $pRW_{RES}^{N,year}$ stays constant at the level when the age was 70 because there is no incentive to delay applying for benefits beyond age 70.

$$pRW_{RES}^{N,year} = pRW_{RES}^{70,year-(N-70)}, \text{ for } N = 71-95+ \text{ and } year \geq TRYR$$

In addition, we apply adjustments by age group (62-64, 65-69) and sex, $SRADJ^{RW}$, to the 10th year of the projection period in order to match the projections made by the Short-Range office. We also apply these adjustments to $pRW_{RES}^{N,year}$ for all years after TRYR+9. The values of $pRW_{RES}^{N,year}$ for intermediate years are linearly interpolated between $pRW_{RES}^{N,TRYR-1}$ and $pRW_{RES}^{N,TRYR+9}$.

In response to the elimination of the file and suspend and deemed filing claiming strategies for aged spouses by the BBA, we expect that the married and divorced retired workers who would have delayed receiving their worker benefits will instead file for benefits earlier. As such, we use the $unsuspRW(69)$ factor to increase the projected number of age 69 married workers for those who would have suspended their benefits but will now begin receiving them at age 69.

$$unsuspRW(69) = \frac{suspRW(69) + RWN_{69,M,Mar} + RWN_{69,F,Mar}}{RWN_{69,M,Mar} + RWN_{69,F,Mar}} - 1$$

Where $suspRW(69)$ is the number of age 69 retired workers who would have suspended their benefits for the file and suspend strategy. Since the utilization of this filing strategy had been expected to continue to grow in popularity had it not been eliminated, we apply a behavioral increase factor, $RWBehavioralInc(69)$, to $unsuspRW(69)$ to reflect the increasing number of retired workers who will no longer delay their benefits now that the filing strategy is no longer available.

$$RWBehavioralInc(69) = \begin{cases} 1.5, & \text{for those born in 1947} \\ 1.03, & \text{for those born in 1948} \\ 1.0, & \text{for those born after 1948} \end{cases}$$

We use the $DFRetWrkr(ag,sx)$ factor to increase the number of married and divorced retired workers for those affected by the expansion of deemed filing. This includes both the insured aged spouses who will no longer be able to delay receiving their worker benefit while

receiving their aged spouse benefit, and the insured aged spouses who would never claim their retired worker benefit because their aged spouse benefit is higher. We calculate these factors as:

$$DFRetWrkr(ag, sx) = \begin{cases} \frac{DeemdFiler(ag, sx) + RWN_{ag, sx, Mar} + RWN_{ag, sx, Div}}{RWN_{ag, sx, Mar} + RWN_{ag, sx, Div}} - 1, & year - ag \geq 1955 \\ \frac{RWN_{ag, sx, Mar} + RWN_{ag, sx, Div}}{RWN_{ag, sx, Mar} + RWN_{ag, sx, Div} - DeemdFiler(ag, sx)} - 1, & year - ag < 1955 \end{cases}$$

Where $DeemdFiler(ag, sx)$ is the number of insured aged spouses who will now have to claim their retired worker benefit due to the expansion of deemed filing. We assume that all of the insured aged spouses who would never have claimed their retired worker benefit will file at NRA. We identify this group as the insured aged spouses still receiving a spouse benefit at age 71. For those who are delaying claiming their worker benefit, we assume that 60 percent will file for worker benefits earlier now that the strategy is no longer available, with 1/2 claiming at age 66, and 1/6 at each age 67 through 69. As the NRA transitions from 66 to 67 we reduce $DFRetWrkr(66, sx)$ by one minus the increase in NRA expressed as a fraction of a year with each factor being set to zero once the NRA is 67.

For each age, retired workers are further broken down by age at entitlement, AE , by multiplying the number of retired workers at age N by the ratio of the incidence rate ${}_{AE}^N INCRATE^{TRYR-1}$ to the prevalence rate at age N ,

$${}_{AE}^N RWN^{year} = {}^N RWN^{year} \times \frac{{}_{AE}^N INCRATE^{TRYR-1}}{pRW_{RES}^{N, year}}, \quad AE \leq N$$

where we calculate the incidence rate for a given $AE = N$ and year as the change in the prevalence rate at age N to the prevalence rate at age $N-1$ in the previous year

$${}_{AE} INCRATE^{year} = \begin{cases} pRW_{RES}^{N, year}, & N = AE = 62 \\ pRW_{RES}^{N, year} - pRW_{RES}^{N-1, year-1}, & 63 \leq N = AE \leq 69 \\ 1 - pRW_{RES}^{N-1, year-1}, & N = AE = 70 \end{cases}$$

and we set the incidence rate for a given $AE < N$ and year to the incidence rate for AE and age $N-1$ in the previous year.

$${}_{AE}^N INCRATE^{year} = {}_{AE}^{N-1} INCRATE^{year-1}, \quad AE < N$$

For the last historical year, we use the TRYR-1 male and female retired worker counts by age and age of entitlement from a 100 percent sample to recalculate the TRYR-1 incidence rates

for all attained ages, N , and all entitlement ages, AE .⁴

Additional adjustments are applied to the incidence rates in order to account for the increase in retired worker beneficiaries due to the elimination of the deemed filing and file and suspend strategies. Since many of the beneficiaries utilizing these strategies delayed receiving their worker benefit until age 70, we assume that the retired workers added through the $unsuspRW(ag)$ and $DFRetWrkr(ag,sx)$ factors will survive until age 70. As such, our incidence rate calculations are adjusted to reflect the age at which the new retired workers claim benefits and to ensure that they remained on the rolls until age 70.

Equation 3.3.3 – Aged Spouses of Retired Workers (ASRWN)

Aged Spouses of Retired Workers

Exposures: SSA population by age (62-95+, sex, and marital status (married and divorced))

Linkages: $pASRW_{DEA}$ = probability that the primary account holder (PAH) is not deceased

$pASRW_{AGA}$ = probability that the PAH is of the required age

$pASRW_{FLA}$ = probability that the PAH is fully insured

$pASRW_{CPA}$ = probability that the PAH is receiving benefits

$pASRW_{MBB}$ = probability that the beneficiary is not receiving a young-spouse benefit

$pASRW_{FIB}$ = probability that the aged-spouse is not fully insured

$pASRW_{GPB}$ = probability that the aged-spouse's benefits are not withheld because of receipt of a significant government pension based on earnings in non-covered employment

$pASRW_{RES}$ = probability that a person who is eligible to receive aged-spouse benefits actually receives the benefits

$pASRW_{FS2}$ = probability that a couple will engage in a filing strategy in which the PAH files for and may or may not suspend their benefit and the fully insured spouse files only for their aged-spouse benefit.

We project the number of aged spouses of retired workers, along with all linkage factors, by age, sex of the account holder ($sa = 1, 2$), and marital status of the beneficiary. Age ranges from 62 to 95+, and marital status includes married ($mb = 1$) and divorced ($mb = 2$). We calculate the projected number of aged spouses of retired workers as follows:

$$ASRWN = ASRW_{POP} \times pASRW_{DEA} \times pASRW_{AGA} \times pASRW_{FLA} \times pASRW_{CPA} \times pASRW_{MBB} \times pASRW_{FIB} \times pASRW_{GPB} \times pASRW_{RES} \times pASRW_{FS2} \quad (3.3.3)$$

$ASRW_{POP}$ represents the subset of the population from which these beneficiaries are drawn,

⁴ For the oaicp.1992+ file produced for the Trust Fund Operations and Actuarial Status/Awards area, TRYR-1 incidence rates are not recalculated from the 100 percent sample and instead are calculated as described in the previous paragraph.

and we set it equal to the Social Security area population ($SSAPOP_{mb}$) for $mb = 1, 2$.

$$ASRW_{POP} = SSAPOP_{mb}$$

$pASRW_{DEA}$ represents the probability that the PAH is not deceased. For the married population, we do not apply a factor. For the divorced population, we set the factor equal to the portion of the total married and widowed population who are married.

$$pASRW_{DEA} = \begin{cases} 1, & mb = 1 \text{ (married)} \\ \frac{SSAPOP_{mar}}{SSAPOP_{mar} + SSAPOP_{wid}}, & mb = 2 \text{ (divorced)} \end{cases}$$

$pASRW_{AGA}$ represents the probability that the PAH is of the required age, and we set it equal to the portion of the married population with a spouse (PAH) at least age 62.

$$pASRW_{AGA} = \frac{\sum_{AA=62}^{95} mar(AA, AS)}{SSAPOP_{AS, mar}}$$

Where $mar(AA, AS)$ is the number of married couples where the age of the account holder is AA and the spouse of the account holder is age AS.

$pASRW_{FIA}$ represents the probability that the PAH is fully insured, and we set it equal to the portion of married couples of the required age where the PAH is fully insured (FI_PAH). For example, when the program estimates the number of female aged spouse of retired workers, this factor will find the portion where their spouse, the male PAH, is fully insured.

$$pASRW_{FIA} = \frac{\sum_{AA=62}^{95} [mar(AA, AS) \times FI_PAH_{AA, mar}]}{\sum_{AA=62}^{95} mar(AA, AS)}$$

$pASRW_{CPA}$ represents the probability that the PAH is receiving benefits. We set this factor equal to the portion of eligible married couples where the PAH is receiving benefits ($RETIRED$). If the beneficiary is divorced, we do not apply a factor, since it is not required for the retired worker to be receiving benefits for the divorced aged spouse to receive benefits.

$$pASRW_{CPA} = \begin{cases} 1, & \text{year} \geq 1985 \text{ and} \\ & mb = 2 \\ \frac{\sum_{AA=62}^{95} [mar(AA, AS) \times FI_PAH_{AA, mar} \times RETIRED]}{\sum_{AA=62}^{95} mar(AA, AS) \times FI_PAH_{AA, mar}}, & \text{elsewhere} \end{cases}$$

$pASRW_{MBS}$ represents the probability that the beneficiary is not receiving a young-spouse benefit. If the beneficiary is age 70 or older or if the beneficiary is divorced, we do not apply a factor. Otherwise, we set this factor equal to the portion of potentially eligible widow(er)s

where the spouse of the PAH is not receiving a young-spouse benefit ($YSRWN^{ab}$), where ab represents the 5-year age group.⁵

For $mb = 1$ (married):

$$FACTOR_{AGE} = \begin{cases} 1, & 65 \leq age \text{ and } NRA \geq age + 1 \\ NRA - age, & age < NRA < age + 1 \\ 0, & elsewhere \end{cases}$$

$$pASRW_{MBB} = \begin{cases} 1 - \frac{0.2 \times YSRWN^{60-64}}{ASRW_{POP} \times pASRW_{DEA} \times pASRW_{AGA} \times pASRW_{FIA} \times pASRW_{CPA}}, & age = 62 - 64 \\ 1 - \frac{YSRWN^{65-69}}{ASRW_{POP} \times pASRW_{DEA} \times pASRW_{AGA} \times pASRW_{FIA} \times pASRW_{CPA}} \times \frac{FACTOR_{ag}}{\sum_{65}^{69} FACTOR_{ag}}, & 65 \leq age \leq NRA \\ 1, & elsewhere \end{cases}$$

For $mb = 2$ (divorced):

$$pASRW_{MBB} = 1$$

$pASRW_{FIB}$ represents the probability that the aged spouse is not fully insured, and is therefore not receiving a retired-worker benefit based on his/her own earnings. We set this factor equal to one minus the LPR fully insured rate ($FILEG$) at each age and sex.

$$pASRW_{FIB} = 1 - FILEG$$

$pASRW_{GPB}$ represents the probability that the aged-spouse's benefits are not withheld because of receipt of a significant government pension based on earnings in non-covered employment. $GPWHLD$ represents the total number of aged spouse of retired-worker beneficiaries (for all ages) expected to receive a significant government pension. $rGPOAGE$ represents the ratio of the total for each given age.

$$pASRW_{GPB} = \begin{cases} 1, & year \leq 1978 \\ 1 - \frac{rGPOAGE \times GPWHLD}{ASRW_{POP} \times pASRW_{DEA} \times pASRW_{AGA} \times pASRW_{FIA} \times pASRW_{CPA} \times pASRW_{MBB} \times pASRW_{FIB}}, & elsewhere \end{cases}$$

$pASRW_{RES}$ represents the probability that a person who is eligible to receive aged-spouse benefits actually receives the benefits. This factor accounts for other eligibility requirements not previously mentioned. For example, that the aged spouse has been married to the PAH for at least one year and in the case of a divorced aged spouse that the spouse was married to

⁵ There are no young spouses at NRA or above.

the PAH for at least 10 years. For all historical years, we calculate $pASRW_{RES}^{year}$ as the ratio of uninsured aged-spouse beneficiaries receiving benefits to the number of persons meeting all previously mentioned requirements by age, sex, and marital status. We calculate the number of uninsured aged-spouse beneficiaries as the actual number of aged spouses receiving benefits ($ASRWN$), both insured and uninsured, less couples ages NRA and above where the PAH has filed for benefits and their fully insured spouse is receiving only a spousal benefit ($FSCouple2(ag, sa, mb)$).

For $year < TRYR$:

$$pASRW_{RES}^{year} = \frac{ASRWN_{ag,sa,mb} - FSCouple2(ag, sa, mb)}{ASRW_{POP} \times pASRW_{DEA} \times pASRW_{AGA} \times pASRW_{FIA} \times pASRW_{CPA} \times pASRW_{MBB} \times pASRW_{FIB} \times pASRW_{GPB}}$$

For each age, sex, and marital status, we use a least squares regression over the last ten years of historical data to determine a starting value in $TRYR-1$ for $pASRW_{RES}^{year}$ from which we project future values. In addition, for each sex and marital status, we graduate the regressed values of $pASRW_{RES}^{TRYR-1}$ over age using a weighted minimized third-difference formula to compute $ESTRES^{ASRW}$. $ESTRES^{ASRW}$ are the preliminary estimates of $pASRW_{RES}^{TRYR+9}$, the values in the 10th year of the projection period⁶. For female spouses, we apply additional adjustments by age group (62-64, 65+), $SRADJ^{ASRW}$, to the 10th year of the projection period in order to match the projections made by the Short-Range office. We linearly interpolate the values of $pASRW_{RES}^{year}$ for intermediate years interpolated between $pASRW_{RES}^{TRYR-1}$ and $pASRW_{RES}^{TRYR+9}$ (equal to $ESTRES^{ASRW} \times SRADJ^{ASRW}$). After the 10th year of the projection period, we linearly grade these adjustment factors to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to the long-range projections.

$pASRW_{FS2}$ represents the probability that a couple engages in the deemed filing claiming strategy or that the insured spouse has a spouse benefit that is higher than their retired worker benefit. The deemed filing claiming strategy is a strategy in which the PAH files for and may or may not suspend their worker benefit and their fully insured spouse receives only a spousal benefit thus delaying receiving their worker benefit. For historical years 2006 to $TRYR-1$, we set $pASRW_{FS2}$ equal to the ratio of $ASRWN$ to the number of uninsured aged spouses. Due to the BBA, those turning 62 in 2016 or later will no longer be able to use the filing strategy. For the projection period prior to the elimination of the filing strategy, we set $pASRW_{FS2}^{year}$ equal to $pASRW_{FS2}^{year-1}$ multiplied by a behavioral increase factor ($BehaviorInc(ag, sa)$). This factor accounts for the increase in the number of couples engaging in the filing strategy due to knowledge of the strategy becoming more widespread as well as the growth in the age 62 LPR fully insured married and divorced population ($filpop62$). We set $pASRW_{FS2}$ to 1 in the projection period as those age 62 in 2016 reach NRA and above.

⁶ For the 2021 Trustees Report, the ten-year regression is over the period 2010-2019, and we graduate the $TRYR-2$ regressed values of $pASRW_{RES}$, excluding the 2020 data due to the Covid-19 pandemic.

For $year < TRYR$:

$$pASRW_{FS2}^{year} = \begin{cases} \frac{ASRWN}{ASRW_{POP} \times ASRW_{DEA} \times ASRW_{AGA} \times ASRW_{FIA} \times ASRW_{CPA} \times ASRW_{MBB} \times ASRW_{FIB} \times ASRW_{GPB} \times ASRW_{RES}}, & 2006 \leq year \text{ and } NRA \leq ag \\ 1, & \text{elsewhere} \end{cases}$$

For $year \geq TRYR$:

$$BehaviorInc(ag, sa) = \begin{cases} 1.00 \times filpop62, & \text{for those born in 1950 – 1951} \\ 1.05 \times filpop62, & \text{for those born in 1952} \\ 1.15 \times filpop62, & \text{for those born in 1953} \\ 1.00 \times filpop62, & \text{for those born after 1953} \end{cases}$$

$$pASRW_{FS2}^{year} = \begin{cases} 1 + (pASRW_{FS2}^{year-1} - 1) \times BehaviorInc(ag, sa), & NRA \leq ag \text{ and } \\ & year - 2016 + 62 < ag \\ 1, & \text{elsewhere} \end{cases}$$

Equation 3.3.4 – Disabled Spouses of Deceased Workers (DSDWN)

Disabled Spouses of Deceased Workers

Exposure: SSA population by age (50-69), sex and marital status (widowed and divorced)

- Linkages: $pDSDW_{DEA}$ = probability that the primary account holder (PAH) is deceased
 $pDADW_{FIA}$ = probability that the PAH was fully insured at death
 $pDSDW_{SSB}$ = probability that the spouse is indeed disabled
 $pDSDW_{DEB}$ = probability that the disabled spouse is not receiving another type of benefit
 $pDSDW_{RES}$ = probability that a person who is eligible to receive disabled-spouse benefits actually receives the benefits

We project the number of disabled spouses of deceased workers, along with all linkage factors, by age, sex of the account holder ($sa = 1$ for male, $sa = 2$ for female), and marital status. Age ranges from 50 to 69, and marital status includes widowed ($mb = 1$) and divorced ($mb = 2$). We calculate the projected number of disabled spouses of deceased workers as follows:

$$DSDWN = DSDW_{POP} \times pDSDW_{DEA} \times pDSDW_{FIA} \times pDSDW_{SSB} \times pDSDW_{DEB} \times pDSDW_{RES} \quad (3.3.4)$$

$DSDW_{POP}$ represents the subset of the population from which we draw these beneficiaries, and we set it equal to the Social Security area population ($SSAPOP_{mb}$) for $mb = 1, 2$.

$$DSDW_{POP} = SSAPOP_{mb}$$

$pDSDW_{DEA}$ represents the probability that the primary account holder is deceased. For the widowed population, we do not apply a factor. For the divorced population, we set this factor equal to the portion of the total widowed and married population that is widowed.

$$pDSDW_{DEA} = \begin{cases} 1, & mb = 1 \text{ (widowed)} \\ \frac{SSAPOP_{wid}}{SSAPOP_{wid} + SSAPOP_{mar}}, & mb = 2 \text{ (divorced)} \end{cases}$$

$pDSDW_{FIA}$ represents the probability that the PAH was fully insured at death. Given the age of the widow, AW , we assume that the age of her deceased husband, AH , ranges from $AW-6$ to $AW+12$ with a lower and upper bound of 50 and 95+. Further, we assume that the more likely age of the husband is $AW+3$. For each age, we calculate $pDSDW_{FIA}$ as a weighted average of the portion of the Social Security area population that is fully insured (INS), at each possible age of the husband. For example, if the widow is age 65, we assume that the age of the husband is between 59 and 77. Therefore, when we calculate the weighted average of the portion of the population who are fully insured men, we apply the highest weight of ten to age 68 and linearly reduce the weight to zero for each age above and below 68. We use the same concept for widow(er)s with the assumption that the age of his deceased wife ranges from $AH-12$ to $AH+6$, with a greater likelihood of her age being $AH-3$. Let $WEIGHT$ represent the specific weight applied to each age.

$$WEIGHT_{AH} = 10 - |AW + 3 - AH|$$

$$WEIGHT_{AW} = 10 - |AH - 3 - AW|$$

$$pDSDW_{FIA} = \begin{cases} \frac{\sum_{AH=AW-6}^{AW+12} WEIGHT_{AH} \times INS_{AH}}{\sum_{AH=AW-6}^{AW+12} WEIGHT_{AH}}, & sa = 1 \\ \frac{\sum_{AW=AH-12}^{AH+6} WEIGHT_{AW} \times INS_{AW}}{\sum_{AW=AH-12}^{AH+6} WEIGHT_{AW}}, & sa = 2 \end{cases}$$

$pDSDW_{SSB}$ represents the probability that the spouse is indeed disabled. We set this factor equal to the disability prevalence rates ($DISPREV$) by age and sex received from the DISABILITY subprocess.

$$pDSDW_{SSB} = DISPREV$$

$pDSDW_{DEB}$ represents the probability that the disabled spouse is not dually eligible for another type of benefit. We assume this factor remains at a constant level by sex.

$$pDSDW_{DEB} = \begin{cases} 0.85, & sa = 1 \\ 0.06, & sa = 2 \end{cases}$$

$pDSDW_{RES}$ represents the probability that a person who is eligible to receive disabled-spouse benefits actually receives the benefits. This factor accounts for other eligibility requirements not previously mentioned. For example, that the disabled widow met the seven-year deadline

for surviving spouses to qualify for benefits on the basis of disability. For all historical years, we calculate $pDSDW_{RES}^{year}$ as the ratio of $DSDWN$, the actual number of disabled spouses of deceased workers receiving benefits, to the number of persons meeting all previously mentioned requirements by age, sex, and marital status.

$$pDSDW_{RES}^{year} = \frac{DSDWN}{DSDW_{POP} \times pDSDW_{DEA} \times pDSDW_{FIA} \times pDSDW_{SSB} \times pDSDW_{DEB}}, \quad year < TRYR$$

For ages 50 to 64, and each sex, and marital status, we use the average $pDSDW_{RES}^{year}$ over the last five years of historical data to determine starting values for $pDSDW_{RES}^{year}$ from which we project future values. We phase in these averages over five years using a linear interpolation between the five-year averages and $pDSDW_{RES}^{TRYR-17}$. For female disabled spouses, we apply an adjustment, $SRADJ^{DSDW}$, to the 10th year of the projection period in order to match the projections made by the Short-Range office. We exponentially interpolate the values of $pDSDW_{RES}^{year}$ for intermediate years between $pDSDW_{RES}^{TRYR-1}$ and $pDSDW_{RES}^{TRYR+9}$ (equal to $pDSDW_{RES}^{year} \times SRADJ^{DSDW}$). After the 10th year of the projection period, we linearly grade the adjustment factors to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to the long-range projections.

For the projection period, for ages 65 to 69 where age is less than the NRA when the beneficiary was age 60, $pDSDW_{RES}^{year}$ is equal to $pDSDW_{RES}^{year}$ at age 64 times an adjustment that accounts for the additional ages as NRA changes.

$$FACTOR_{age} = \begin{cases} 1, & NRA \text{ at age } 60 \geq age + 1 \\ NRA \text{ at age } 60 - age, & age < NRA \text{ at age } 60 < age + 1 \end{cases}$$

$$pDSDW_{RES}^{year} = pDSDW_{RES}^{year,64} \times \left(\frac{pDSDW_{RES}^{year,64}}{pDSDW_{RES}^{year,63}} \right)^{(age-64)} \times FACTOR_{age}, \quad \begin{matrix} 65 \leq age \leq 69 \text{ and} \\ age < NRA \text{ at age } 60 \end{matrix}$$

Equation 3.3.5 – Minor Children of Retired Workers (MCRWN)

We project the number of minor children of retired workers, $MCRWN$, by age of the minor ($am = 0-17$) and sex of the account holder ($sa = 1$ for male, $sa = 2$ for female).

For children of male retired workers:

$$MCRWN_{M,sa}^{year} = mcrwChi_{M,am}^{year} \times \frac{rw_sum_{year}}{pop_sum_{year}} \times pMCRW_{RES}^{year} \quad (3.3.5.1)$$

For the number of minor children of male retired workers, we multiply the number of

⁷ For the 2021 Trustees Report, the five-year average is over the period 2015-2019, and we phase between the five-year averages and $pDSDW_{RES}^{TRYR-2}$, excluding the 2020 data due to the Covid-19 pandemic.

children under the age of 18 with a father who is at least 62 years old and who's other parent is not deceased ($mcrwChi_{M,am}^{year}$) by the total number of male retired workers ages 62 to 71 (rw_sum_{year}) divided by the total number of men in the population ages 62 to 71 (pop_sum_{year}).

$pMCRW_{RES}$ represents the probability that a child who is eligible to receive minor-child benefits actually receives the benefits. This factor accounts for other eligibility requirements not previously mentioned. For example, the marital status of the child or the dependency status of the child. In the historical period, we calculate this as the ratio of the total number of minor children of retired workers actually receiving benefits to the total eligible to receive benefits.

$$pMCRW_{RES}^{year} = \frac{MCRWN_M^{year}}{mcrwChi_{M,am}^{year} \times \frac{rw_sum_{year}}{pop_sum_{year}}}$$

For the projection period, we take the average of these ratios over the last ten historical years and phase it in using a linear interpolation between the average and $pMCRW_{RES}^{TRYR-18}$. We apply an adjustment, $SRADJ^{MCRW}$, to the 10th year of the projection period in order to match the projections made by the Short-Range office. The adjustment factor is phased in over the first 10 years of the projection period with the full factor applied in the 10th year. After the 10th year of the projection period, we linearly grade these adjustment factors to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to the long-range projections.

For children of female retired workers:

$$MCRWN_{F,sa}^{year} = \frac{MCRWN_{Total,M}^{year} \times mcrwPrctFem}{1 - mcrwPrctFem} \times \frac{MCRWN_{am,F}^{TRYR-1}}{MCRWN_{Total,F}^{TRYR-1}} \quad (3.3.5.2)$$

$mcrwPrctFem$ represents the average over the last five historical years⁹ of the percentage of the total number of minor children of female retired workers, ages 0 to 17, to the total number of minor children for both male and female retired workers.

$$mcrwPrctFem = \frac{MCRWN_{Total,F}^{year}}{MCRWN_{Total,F}^{year} + MCRWN_{Total,M}^{year}}$$

We maintain the total $MCRWN_{0-17,F}^{YR}$ at this percentage over the entire projection period by multiplying our estimate of $MCRWN_{0-17,M}^{YR}$ by $mcrwPrctFem$ divided by its complement.

⁸ For the 2021 Trustees Report, the ten-year average is over the period 2010-2019, and we phase between the ten-year average and $pMCRW_{RES}^{TRYR-2}$, excluding the 2020 data due to the Covid-19 pandemic.

⁹ For the 2021 Trustees Report, the five-year average is over the period 2015-2019, excluding the 2020 data due to the Covid-19 pandemic.

$$MCRWN_{0-17,F}^{YR} = \frac{MCRWN_{0-17,M}^{YR} \times mcrwPrctFem}{1 - mcrwPrctFem}$$

In order to distribute $MCRWN_{0-17,F}^{YR}$ among the ages 0 to 17, we multiply $MCRWN_{0-17,F}^{YR}$ by the proportion of beneficiaries at each age in the last historical year¹⁰.

Equation 3.3.6 – Minor Children of Deceased Workers (MCDWN)

Minor Children of Deceased Workers

Exposure: SSA population by age (0-17) and sex of the account holder

Linkages: $pMCDW_{DEA}$ = probability that the parent is deceased

$pMCDW_{FIA}$ = probability that the primary account holder (PAH) is fully insured

$pMCDW_{RES}$ = probability that a child who is eligible to receive minor-child benefits actually receives the benefits

We project the number of minor children of deceased workers, along with all linkage factors, by age of the minor ($am = 0-17$) and sex of the account holder ($sa = 1$ for male, $sa = 2$ for female). We calculate it as follows:

$$MCDWN = MCDW_{POP} \times pMCDW_{DEA} \times pMCDW_{FIA} \times pMCDW_{RES} \quad (3.3.6)$$

$MCDW_{POP}$ represents the subset of the population from which we draw these beneficiaries, and we set it equal to the Social Security area population ($SSAPOP$).

$$MCDW_{POP} = SSAPOP$$

$pMCDW_{DEA}$ represents the status of the parent (PAH). This is set equal to the portion of the minor population where at least one parent is deceased. CHI_DEA represents the number of children having at least one deceased parent.

$$pMCDW_{DEA} = \frac{CHI_DEA}{MCDW_{POP}}$$

$pMCDW_{FIA}$ represents the probability that the parent (PAH) is fully insured. We set this equal to the portion of the population aged $25+am$ to $35+am$ where the PAH is fully insured (FI_PAH).

$$pMCDW_{FIA} = \frac{\sum_{25+am}^{35+am} [SSAPOP \times FI_PAH]}{\sum_{25+am}^{35+am} SSAPOP}$$

$pMCDW_{RES}$ represents the probability that a child who is eligible to receive minor-child benefits actually receives the benefits. This factor accounts for other eligibility requirements not previously mentioned. For example, the marital status of the child or the dependency status of the child. For all historical years, we calculate $pMCDW_{RES}^{year}$ as the ratio of $MCDWN$,

¹⁰ For the 2021 Trustees Report, we use TRYR-2.

the actual number of minor children of deceased workers receiving benefits, to the number of persons meeting all previously mentioned requirements by age and sex of the parent.

$$pMCDW_{RES}^{year} = \frac{MCDWN}{MCDW_{POP} \times pMCDW_{DEA} \times pMCDW_{FIA}}, \quad year < TRYR$$

For each age and sex of parent, we use a least squares regression over the last ten years¹¹ of historical data to determine a starting value in TRYR-1 for $pMCDW_{RES}^{year}$ from which we project future values. We apply an adjustment, $SRADJ^{MCDW}$, to the 10th year of the projection period in order to match the projections made by the Short-Range office. We linearly interpolate the values of $pMCDW_{RES}^{year}$ for intermediate years between the regressed values for $pMCDW_{RES}^{TRYR-1}$ and $pMCDW_{RES}^{TRYR+9} \times SRADJ^{MCDW}$. After the 10th year of the projection period, we linearly grade the adjustment factors to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to the long-range projections.

Equations 3.3.7-8 – Student Children of Retired and Deceased Workers (SCRWN and SCDWN)

Student Children of Retired Workers

- Exposure: SSA population by age of the student (18-21) and sex of the account holder
- Linkages: $pSCRW_{DEA}$ = probability that at least one parent is retired
 $pSCRW_{AGA}$ = probability that the primary account holder (PAH) is age 62 or older
 $pSCRW_{FIA}$ = probability that the PAH is fully insured
 $pSCRW_{CPA}$ = probability that the PAH is receiving benefits
 $pSCRW_{SSB}$ = probability that the child is indeed attending school
 $pSCRW_{RES}$ = probability that a child who is eligible to receive student-child benefits actually receives the benefits

Student Children of Deceased Workers

- Exposure: SSA population by age of the student (18-21) and sex of the account holder
- Linkages: $pSCDW_{DEA}$ = probability that at least one parent is deceased
 $pSCDW_{AGA}$ = probability that the PAH is age 62 or older (set to 1)
 $pSCDW_{FIA}$ = probability that the PAH is fully insured
 $pSCDW_{CPA}$ = probability that the PAH is receiving benefits (set to 1)
 $pSCDW_{SSB}$ = probability that the child is indeed attending school
 $pSCDW_{RES}$ = probability that a child who is eligible to receive student-child benefits actually receives the benefits

We project the number of student children of retired and deceased workers, along with all linkage factors, by age of the student ($as = 18, 19$) and sex of the account holder ($sa = 1$ for male, $sa = 2$ for female). We calculate the projected number of student children of retired and deceased workers as follows:

¹¹ For the 2021 Trustees Report, the ten-year regression is over the period 2010-2019, excluding the 2020 data due to the Covid-19 pandemic.

$$SCRWN = SCRW_{POP} \times pSCRW_{DEA} \times pSCRW_{AGA} \times pSCRW_{FIA} \\ \times pSCRW_{CPA} \times pSCRW_{SSB} \times pSCRW_{RES} \quad (3.3.7)$$

$$SCDWN = SCDW_{POP} \times pSCDW_{DEA} \times pSCDW_{AGA} \times pSCDW_{FIA} \\ \times pSCDW_{CPA} \times pSCDW_{SSB} \times pSCDW_{RES} \quad (3.3.8)$$

$SCRW_{POP}$ and $SCDW_{POP}$ represent the subset of the population from which these beneficiaries are drawn, and we set them equal to the Social Security area population ($SSAPOP$).

$$SCRW_{POP} = SCDW_{POP} = SSAPOP$$

$pSCRW_{DEA}$ and $pSCDW_{DEA}$ represent the status of the parent (PAH). For student children of retired workers, we set this equal to the proportion of the subset of the population where neither parents are deceased. For student children of deceased workers, we set this equal to the proportion of the subset of the population where at least one parent is deceased. CHI_DEA represents the number of student children having at least one deceased parent.

$$pSCRW_{DEA} = 1 - \frac{CHI_DEA}{SCRW_{POP}}$$

$$pSCDW_{DEA} = \frac{CHI_DEA}{SCDW_{POP}}$$

$pSCRW_{AGA}$ and $pSCDW_{AGA}$ represent the probability that the PAH is age 62 or older. For student children of retired workers, we set this equal to the proportion of the student population that has one parent age 62 or older, CHI_62+ . For student children of deceased workers, we set the factor equal to one.

$$pSCRW_{AGA} = \frac{CHI_62+}{SCRW_{POP}}$$

$$pSCDW_{AGA} = 1$$

$pSCRW_{FIA}$ and $pSCDW_{FIA}$ represent the probability that the PAH is fully insured. For student children of retired workers, we set this equal to the portion of the population aged 62 to 64+ as where the PAH is fully insured (FI_PAH). For student children of deceased workers, we calculate the factor similarly with the population being aged 25+ as to 35+ as .

$$pSCRW_{FIA} = \frac{\sum_{62}^{64+as}[SSAPOP \times FI_PAH]}{\sum_{62}^{64+as} SSAPOP}$$

$$pSCDW_{FIA} = \frac{\sum_{25+as}^{35+as}[SSAPOP \times FI_PAH]}{\sum_{25+as}^{35+as} SSAPOP}$$

$pSCRW_{CPA}$ and $pSCDW_{CPA}$ represent the probability that the PAH is receiving benefits. For student children of retired workers, we set this factor equal to the portion of the population

aged 62 to 64+*as* where the PAH is receiving benefits (*RETIRED*). For student children of deceased workers, we set this factor equal to one.

$$pSCRW_{CPA} = \frac{\sum_{62}^{64+as}[SSAPOP \times FI_PAH \times RETIRED]}{\sum_{62}^{64+as}[SSAPOP \times FI_PAH]}$$

$$pSCDW_{CPA} = 1$$

$pSCRW_{SSB}$ and $pSCDW_{SSB}$ represent the probability that the child is indeed attending school (full-time elementary or secondary school). This factor is dependent upon the age of the child, and we calculate it as follows.

$$pSCRW_{SSB} = pSCDW_{SSB} = \begin{cases} \frac{1}{as - 16}, & year \leq 1981 \\ \frac{0.5}{as - 16}, & year > 1981 \end{cases}$$

$pSCRW_{RES}$ and $pSCDW_{RES}$ represent the probability that a child who is eligible to receive student-child benefits actually receives the benefits. This factor accounts for other eligibility requirements not previously mentioned. For example, the marital status of the child or the dependency status of the child. For all historical years, we calculate $pSCRW_{RES}^{year}$ and $pSCDW_{RES}^{year}$ as the ratio of $SCRWN$ and $SCDWN$, the actual number of student children receiving benefits, to the number of persons meeting all previously mentioned requirements by age and sex of the parent.

$$pSCRW_{RES}^{year} = \frac{SCRWN}{SCRW_{POP} \times pSCRW_{DEA} \times pSCRW_{AGA} \times pSCRW_{FIA} \times pSCRW_{CPA} \times pSCRW_{SSB}}, \quad year < TRYR$$

$$pSCDW_{RES}^{year} = \frac{SCDWN}{SCDW_{POP} \times pSCDW_{DEA} \times pSCDW_{AGA} \times pSCDW_{FIA} \times pSCDW_{CPA} \times pSCDW_{SSB}}, \quad year < TRYR$$

For each age and sex of parent, we use a least squares regression over the last ten years¹² of historical data to determine a starting value in TRYR-1 for $pSCRW_{RES}^{year}$ from which we project future values. We apply an adjustment, $SRADJ^{SCRW}$, to the 10th year of the projection period in order to match the projections made by the Short-Range office. We linearly interpolate the values of $pSCRW_{RES}^{year}$ for intermediate years between the regressed values for $pSCRW_{RES}^{TRYR-1}$ and $pSCRW_{RES}^{TRYR+9} \times SRADJ^{SCRW}$. After the 10th year of the projection period, we linearly grade the adjustment factors to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to the long-range projections. We calculate the values of $pSCDW_{RES}^{year}$ similarly.

Equations 3.3.9-10 – Disabled Adult Children of Retired and Deceased Workers (DCRWN and

¹² For the 2021 Trustees Report, the ten-year regression is over the period 2010-2019, excluding the 2020 data due to the Covid-19 pandemic.

DCDWN)

Disabled Adult Children of Retired Workers

Exposure: SSA population by age of the adult child (18-95)

- Linkages: $pDCRW_{AGA}$ = probability that the primary account holder (PAH) is age 62 or older
 $pDCRW_{DEA}$ = probability that the parent is retired
 $pDCRW_{FIA}$ = probability that the PAH is fully insured
 $pDCRW_{CPA}$ = probability that the PAH is receiving benefits
 $pDCRW_{SSB}$ = probability that the child is indeed disabled
 $pDCRW_{RES}$ = probability that a child who is eligible to receive disabled-child benefits actually receives benefits

Disabled Adult Children of Deceased Workers

Exposure: SSA population by age of the adult child (18-95)

- Linkages: $pDCDW_{AGA}$ = probability that the PAH is age 62 or older (set to 1)
 $pDCDW_{DEA}$ = probability that the parent is deceased
 $pDCDW_{FIA}$ = probability that the PAH is fully insured
 $pDCDW_{CPA}$ = probability that the PAH is receiving benefits (set to 1)
 $pDCDW_{SSB}$ = probability that the child is indeed disabled
 $pDCDW_{RES}$ = probability that a child who is eligible to receive disabled-child benefits actually receives the benefits

We project the number of disabled adult children of retired and deceased workers, along with all linkage factors, by age group of the disabled adult child ($ad = 1-10$) and sex of the account holder ($sa = 1$ for male, $sa = 2$ for female). The age groups are 18-19, 20-24, ..., 55-59, 60+. We calculate the projected number of disabled adult children of retired and deceased workers as follows:

$$DCRWN = DCRW_{POP} \times pDCRW_{AGA} \times pDCRW_{DEA} \times pDCRW_{FIA} \times pDCRW_{CPA} \times pDCRW_{SSB} \times pDCRW_{RES} \quad (3.3.9)$$

$$DCDWN = DCDW_{POP} \times pDCDW_{AGA} \times pDCDW_{DEA} \times pDCDW_{FIA} \times pDCDW_{CPA} \times pDCDW_{SSB} \times pDCDW_{RES} \quad (3.3.10)$$

We calculate all factors similarly to those for student children with the exception of the following.

$pDCRW_{DEA}$ is set equal to the proportion of the married and widowed population who are married (for ages of the parent that are reasonable based on the given age range of the disabled child). We calculate $pDCDW_{DEA}$ similarly for disabled children of deceased workers.

$$pDCRW_{DEA} = \begin{cases} \frac{SSAPOP_{mar}}{SSAPOP_{mar} + SSAPOP_{wid}}, & ad = 1 - 9 \\ \frac{0.25 \times SSAPOP_{mar}}{SSAPOP_{mar} + SSAPOP_{wid}}, & ad = 10 \end{cases}$$

$$pDCDW_{DEA} = \begin{cases} \frac{SSAPOP_{wid}}{SSAPOP_{mar} + SSAPOP_{wid}}, & ad = 1 - 9 \\ \frac{0.25 \times SSAPOP_{wid}}{SSAPOP_{mar} + SSAPOP_{wid}} + 0.75, & ad = 10 \end{cases}$$

$pDCRW_{SSB}$ and $pDCDW_{SSB}$ represent the probability that the adult child is indeed disabled. DCPREM is the preliminary calculation of this factor and we assume it to remain constant. For the projection period, for $ad = 6-10$, we set $pDCRW_{SSB}$ and $pDCDW_{SSB}$ equal to the preliminary factor, plus an adjustment which accounts for the year.

$$DCPREM = \begin{cases} 0.012, & ad = 1, 2 \\ 0.009, & ad = 3 \\ 0.007, & ad = 4 \\ 0.006, & ad = 5 \\ 0.005, & ad = 6 \\ 0.004, & ad = 7-10 \end{cases}$$

$$pDCRW_{SSB} = pDCDW_{SSB} = \begin{cases} \min[0.005, DCPREM + 0.0001 \times (year - TRYR)], & ad = 7 - 10 \text{ and } year > TRYR + 1 \\ DCPREM, & elsewhere \end{cases}$$

$pDCRW_{RES}$ and $pDCDW_{RES}$ represent the probability that a child who is eligible to receive disabled-child benefits actually receives the benefits. This factor accounts for other eligibility requirements not previously mentioned. For example, the marital status of the child or the dependency status of the child. For all historical years, we calculate $pDCRW_{RES}^{year}$ and $pDCDW_{RES}^{year}$ as the ratio of $DCRWN$ and $DCDWN$, the actual number of disabled children receiving benefits, to the number of persons meeting all previously mentioned requirements by age and sex of the parent.

For $year < TRYR$:

$$pDCRW_{RES}^{year} = \frac{DCRWN}{DCRW_{POP} \times pDCRW_{DEA} \times pDCRW_{AGA} \times pDCRW_{FIA} \times pDCRW_{CPA} \times pDCRW_{SSB}}$$

$$pDCDW_{RES}^{year} = \frac{DCDWN}{DCDW_{POP} \times pDCDW_{DEA} \times pDCDW_{AGA} \times pDCDW_{FIA} \times pDCDW_{CPA} \times pDCDW_{SSB}}$$

For each age and sex of parent, we use a least squares regression over the last ten years¹³ of historical data to determine a starting value in TRYR-1 for $pDCRW_{RES}^{year}$ from which we project future values. We apply an adjustment, $SRADJ^{DCRW}$, to the 10th year of the projection period in order to match the projections made by the Short-Range office. We linearly interpolate the values of $pDCRW_{RES}^{year}$ for intermediate years between $pDCRW_{RES}^{TRYR-1}$ and $pDCRW_{RES}^{TRYR+9} \times SRADJ^{DCRW}$. After the 10th year of the projection period, we linearly grade the adjustment factors to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to the long-range projections. We calculate the values of $pDCDW_{RES}^{year}$ similarly.

Equations 3.3.11-12 – Young Spouses of Retired and Deceased Workers (YSRWN and YSDWN)

Young Spouses of Retired Workers

Exposure: SSA population by age (15-69), sex of account holder, and marital status (married)

- Linkages: $pYSRW_{AGA}$ = probability that the primary account holder (PAH) is of the required age.
 $pYSRW_{ECB}$ = probability that the young spouse has an entitled child in their care
 $pYSRW_{FSB}$ = probability that the young spouse is not already receiving benefits based on another child in their care
 $pYSRW_{RES}$ = probability that a person who is eligible to receive young-spouse benefits actually receives the benefits

Young Spouses of Deceased Workers

Exposure: SSA population by age (15-69), sex of the account holder and marital status (widowed and divorced)

- Linkages: $pYSDW_{DEA}$ = probability that the PAH is deceased
 $pYSDW_{ECB}$ = probability that the young spouse has an entitled child in their care
 $pYSDW_{FSB}$ = probability that the young spouse is not already receiving benefits based on another child in their care
 $pYSDW_{RMB}$ = probability that the young spouse is not remarried
 $pYSDW_{RES}$ = probability that a person who is eligible to receive young-spouse benefits actually receives the benefits

We project the number of young spouses of retired and deceased-workers, along with all linkage factors, by age group ($ab = 1-10$) of the young spouse and sex of the account holder ($sa = 1$ for male, $sa = 2$ for female). We also project young spouses of deceased workers by marital status of the young spouse ($mb = 1$ for widowed and $mb = 2$ for divorced). The age groups are under 25, 25-29 ..., 65-69. We calculate the projected number of young spouses of retired and deceased-workers as follows:

¹³ For the 2021 Trustees Report, the ten-year regression is over the period 2010-2019, excluding the 2020 data due to the Covid-19 pandemic.

$$YSRWN = YSRW_{POP} \times pYSRW_{AGA} \times pYSRW_{ECB} \times pYSRW_{FSB} \times pYSRW_{RES} \quad (3.3.11)$$

$$\begin{aligned} YSDWN &= YSDW_{POP} \times pYSDW_{DEA} \times pYSDW_{ECB} \times pYSDW_{FSB} \\ &\quad \times pYSDW_{RMB} \times pYSDW_{RES} \end{aligned} \quad (3.3.12)$$

$YSRW_{POP}$ and $YSDW_{POP}$ represent the subset of the population from which we draw these beneficiaries. We set $YSRW_{POP}$ equal to the married Social Security area population ($SSAPOP_{mar}$) and we set $YSDW_{POP}$ equal to $SSAPOP_{mb}$ for $mb = 1, 2$.

$$YSRW_{POP} = SSAPOP_{mar}$$

$$YSDW_{POP} = SSAPOP_{mb}$$

$pYSRW_{AGA}$ represents the probability that the PAH is of the required age. We set $pYSRW_{AGA}$ equal to the portion of the married population who has an aged spouse ($AGSP$).

$$pYSRW_{AGA} = \frac{AGSP}{YSRW_{POP}}$$

$pYSDW_{DEA}$ represents the probability that the PAH is deceased. For $mb = 1$, we do not apply any factor. For $mb = 2$, we set this factor equal to the portion of young spouses that is widowed.

$$pYSDW_{DEA} = \begin{cases} 1, & mb = 1 \text{ (widowed)} \\ \frac{SSAPOP_{wid}}{SSAPOP_{wid} + SSAPOP_{mar}}, & mb = 2 \text{ (divorced)} \end{cases}$$

$pYSRW_{ECB}$ and $pYSDW_{ECB}$ represent the probability that the young spouse has an entitled child in their care. We set $YSRW_{ECB}$ equal to the portion of persons meeting the previously mentioned requirements who have a minor or disabled adult child in their care. We set $YSDW_{ECB}$, by marital status, equal to the portion of persons meeting the previously mentioned requirements who have a minor or disabled adult child in their care. $MCRWN^{ab}$ and $DCRWN^{ab}$ represent the total number of minor and disabled adult children of retired workers where the other parent (young spouse) is in the age bracket ab .

$$pYSRW_{ECB}^{ab} = \frac{MCRWN^{ab} + DCRWN^{ab}}{YSRW_{POT} \times pYSRW_{AGA}}$$

$$pYSDW_{ECB}^{mb} = \frac{(MCDWN^{ab} + DCDWN^{ab}) \times \left[\frac{YSDW_{POP}^{mb} \times pYSDW_{DEA}^{mb} \times pYSDW_{AGA}^{mb}}{YSDW_{POP}^{total} \times pYSDW_{DEA}^{total} \times pYSDW_{AGA}^{total}} \right]}{YSDW_{POP} \times pYSDW_{DEA}}$$

$pYSRW_{FSB}$ and $pYSDW_{FSB}$ represent the probability that the young spouse is not already receiving benefits based on another child in their care. We set this factor equal to one divided by the number of children in the average family ($ASOF_{ab}$) for the given age bracket

of the spouse. For young spouses of retired workers, we do not apply a factor for $sa = 2$.

$$pYSRW_{FSB} = \begin{cases} \frac{1}{ASOF_{ab}}, & sa = 1 \\ 1, & sa = 2 \end{cases}$$

$$pYSDW_{FSB} = \frac{1}{ASOF_{ab}}$$

$pYSDW_{RMB}$ represents the probability that the spouse is not remarried. We assume this factor remains constant at 0.600.

$$pYSDW_{RMB} = 0.600$$

$pYSRW_{RES}$ and $pYSDW_{RES}$ represent the probability that a person who is eligible to receive young-spouse benefits actually receives the benefits. This factor accounts for other eligibility requirements not previously mentioned. For example, that the young spouse is not entitled to a widow(er) benefit. For all historical years, we calculate $pYSRW_{RES}^{year}$ as the ratio of $YSRWN$, the actual number of young spouses of retired workers receiving benefits, to the number of persons meeting all previously mentioned requirements by age, sex, and marital status. We calculate $pYSDW_{RES}^{year}$ similarly, using the number of young spouses of deceased workers.

$$pYSRW_{RES} = \frac{YSRWN}{YSRW_{POP} \times pYSRW_{AGA} \times pYSRW_{ECB} \times pYSRW_{FSB}}, \quad year < TRYR$$

$$pYSDW_{RES} = \frac{YSDWN}{YSDW_{POP} \times pYSDW_{DEA} \times pYSDW_{ECB} \times pYSDW_{FSB} \times pYSDW_{RMB}}, \quad year < TRYR$$

For each age, sex, and marital status, we hold $pYSRW_{RES}^{year}$ constant based on the value for the last historical year. For female young spouses, we apply an adjustment, $SRADJ^{YSRW}$, to the 10th year of the projection period in order to match the projections made by the Short-Range office. We exponentially interpolate the values of $pYSRW_{RES}^{year}$ for intermediate years between $pYSRW_{RES}^{TRYR-1}$ and $pYSRW_{RES}^{TRYR+9}$ ¹⁴. After the 10th year of the projection period, we linearly grade the adjustment factors to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to the long-range projections. We calculate the values of $pYSDW_{RES}^{year}$ similarly.

Equation 3.3.13 – Number of Deaths of Insured Workers (LUMSUM_{ab})

¹⁴ For the 2021 Trustees Report, we hold $pYSRW_{RES}^{year}$ constant based on the value for TRYR-2 and interpolate between $pYSRW_{RES}^{TRYR-2}$ and $pYSRW_{RES}^{TRYR+9}$, excluding the 2020 data due to the Covid-19 pandemic.

We project the number of deaths of insured workers by sex and 5-year age group ($ab = 1-14$). Age groups include 20-24, 25-29, ..., 80-84, 85+. We calculate $EXPOSURE_{ab}$, the estimated number of lump-sum payments paid during the year for age group ab , as the number of total deaths during the year times the probability that the deceased was fully insured and has a surviving spouse or child. We calculate BASE as the ratio of the actual total amount of lump-sum death payments paid in TRYR-1 to the estimated total amount of lump-sum payments paid in TRYR-1. We then calculate $LUMSUM_{ab}$ for each year in the projection period.

$$LUMSUM_{ab} = EXPOSURE_{ab} \times BASE \quad (3.3.13)$$

Appendix 3.3-1: Glossary

AB: age group of the beneficiary

AD: age of the disabled child

AGSP: married population where at least one spouse is age 62 or older

AM: age of the minor child

ARFGT3: actuarial reduction factor for ages more than 3 years below normal retirement age

ARFLE3: actuarial reduction factor for ages less than 3 years below normal retirement age

AS: age of the student child

ASDW: aged spouse of deceased worker by linkage factor, age (60-95+), sex of the account holder, marital status (widowed, divorced) and insured status (insured, uninsured). Linkage factors are:

$ASDW_{POP}$:	population of potential aged spouse of retired workers
$pASDW_{DEA}$:	probability that the primary account holder (PAH) is deceased
$pASDW_{FIA}$:	probability that the PAH was fully insured at death
$pASDW_{MBB}$:	probability that the widow(er) is not receiving a young-spouse benefit for the care of a child
$pASDW_{FIB}$:	probability that the aged widow(er) is fully insured
$pASDW_{GPB}$:	probability that the aged-widow(er)'s benefits are not withheld or offset totally because of receipt of a significant government pension based on earnings in non-covered employment
$pASDW_{RES}$:	probability that a widow(er) eligible to receive his/her own retired-worker benefits would instead apply for and receive widow(er) benefits

ASDWN: final number of aged spouse of deceased workers (product of all linkage factors)

ASOF: average number of children in a family, by age group (under 25, 25-29 ..., 60-64)

ASRW: aged spouse of retired worker by linkage factor, age (62-95+), sex of the account holder, and marital status of the beneficiary (married, divorced). Linkage factors are:

$ASRW_{POP}$:	population of potential aged spouse of retired worker beneficiaries
$pASRW_{DEA}$:	probability that the primary account holder (PAH) is not deceased
$pASRW_{AGA}$:	probability that the PAH is of the required age
$pASRW_{FIA}$:	probability that the PAH is fully insured
$pASRW_{CPA}$:	probability that the PAH is receiving benefits

$pASRW_{MBB}$: probability that the beneficiary is not receiving a young-spouse benefit
 $pASRW_{FIB}$: probability that the aged spouse is not fully insured
 $pASRW_{GPB}$: probability that the aged-spouse's benefits are not withheld because of receipt of a significant government pension based on earnings in non-covered employment
 $pASRW_{FS2}$: probability that a couple will engage in a filing strategy in which the PAH files for and may or may not suspend their benefit and the fully insured for their aged spouse benefit.
 $pASRW_{RES}$: probability that a person who is eligible to receive aged-spouse benefits actually receives the benefits
ASRWN: final number of aged spouse of retired workers (product of all linkage factors)
AH: age of husband
AW: age of wife
BASE: ratio of actual to estimated total amount of lump-sum death payments paid in TRYR-1
BBA: Bipartisan Budget Act of 2015. A provision of this act closes unintended loopholes by eliminating (1) the ability to receive only a retired-worker benefit or an aged-spouse benefit when eligible for both, for those attaining age 62 in 2016 and later, and (2) the ability of a family member other than a divorced spouse to receive a benefit based on the earnings of a worker with a voluntarily suspended benefit, for voluntary suspensions requested after April 29, 2016.
CHI_62+: number of children having at least one parent aged 62 or older
CHI_DEA: number of children having at least one deceased parent
CON: number of persons converted from disabled-worker beneficiaries
DCDW: disabled child of deceased workers by linkage factor, age group of the child (18-19, 20-24, ..., 55-59, 60+) and sex of the account holder. Linkage factors are same as SCDW.
 $DCDW_{POP}$: population of potential disabled children
 $pDCDW_{AGA}$: probability that the PAH is age 62 or older
 $pDCDW_{DEA}$: probability that the parent is either retired or deceased
 $pDCDW_{FIA}$: probability that the PAH is fully insured
 $pDCDW_{CPA}$: probability that the PAH is receiving benefits
 $pDCDW_{SSB}$: probability that the child is indeed disabled
 $pDCDW_{RES}$: probability that a child who is eligible to receive disabled-child benefits actually receives the benefits
DCDWN: final number of disabled children of deceased workers (product of all linkage factors)
DCPREM: preliminary calculation of the probability that a child is disabled, by age
DCRW: disabled child of retired workers by linkage factor, age group of the child (18-19, 20-24, ..., 55-59, 60+) and sex of the account holder. Linkage factors are same as those for DCDW.
DCRWN: final number of disabled children of retired workers (product of all linkage factors)
DIB: number of disabled-worker beneficiaries
DIFFADJ: adjustment that accounts for the difference between the actual and estimated prevalence rate at each age in the most recent historical years
DISPREV: disability prevalence rate by age and sex
DRC: delayed retirement credit
DSDW: disabled spouse of deceased worker by linkage factor, age (50-69), sex of the account holder, and marital status (widowed, divorced). Linkage factors are:

$DSDW_{POP}$: population of potential beneficiaries
 $pDSDW_{DEA}$: probability that the primary account holder (PAH) is deceased
 $pDSDW_{FIA}$: probability that the PAH was fully insured at death
 $pDSDW_{SSB}$: probability that the spouse is indeed disabled
 $pDSDW_{DEB}$: probability that the disabled spouse is not receiving another type of benefit
 $pDSDW_{RES}$: probability that a person who is eligible to receive disabled-spouse benefits actually receives the benefits
DSDWN: final number of disabled spouse of deceased workers (product of all linkage factors)
EMPRATE: employment rates for age 62, by sex
ERROR: actual prevalence rate minus the regressed prevalence rate in the most recent historical year
ESTPR: preliminary estimate of the prevalence rate for retired workers
ESTRES: preliminary estimate of the RES factor for the 10th year of the projection period
EXPOSURE: estimated number of lump-sum payments by age group (20-24, 25-29, ..., 80-84, 85+)
FACTOR: adjustment for calculation of MBB factor of aged spouse of deceased worker
FI_PAH: portion of married population where one spouse is fully insured
FILEG: fraction of the LPR SSA population that is fully insured
FIN: insured status of the beneficiary
FP: status of the parent (retired, deceased)
GPOAGE: portion, by age, of the total beneficiaries expected to receive a significant government pension
GPWHLD: total number of beneficiaries (for all ages) expected to receive a significant government pension
INS: portion of the SSA population that is fully insured
LUMSUM: number of deaths of insured workers by sex and age group (20-24, ..., 80-84, 85+)
MAR62PLUS: number of couples where both husband and wife are age 62 and over
MS: marital status of the primary account holder
MB: marital status of the beneficiary
MBAPIA: ratio of the monthly benefit amount (MBA) to the primary insurance amount (PIA) by age (62-70) and sex
MCDW: minor children of deceased workers by linkage factor, age of the child (0-17) and sex of the account holder. Linkage factors are:
 $MCDW_{POP}$: population of potential minor children
 $pMCDW_{DEA}$: probability that the parent is either retired or deceased
 $pMCDW_{FIA}$: probability that the PAH is fully insured
 $pMCDW_{RES}$: probability that a child who is eligible to receive minor-child benefits actually receives the benefits
MCDWN: final number of minor children of deceased workers (product of all linkage factors)
MCRW: minor children of retired workers by linkage factor, age of the child (0-17) and sex of the account holder.
MCRWN: final number of minor children of retired workers (product of all linkage factors)
NRA: normal retirement age
PAH: primary account holder

REGPR: regressed prevalence rate for retired workers

RETIRED: number of retired workers receiving benefits

RW: retired workers by linkage factor, age (62-95+), sex, and marital status (single, married, widowed, divorced). Linkage factors are:

RW_{POP} :	population of potential retired-worker beneficiaries
pRW_{FIA} :	probability that the primary account holder (PAH) is insured
pRW_{DBB} :	probability that the PAH is not receiving a disabled-worker benefit
pRW_{WBB} :	probability that the PAH is not receiving a widow(er) benefit
pRW_{RES} :	retirement prevalence rate; probability that a fully insured worker (not receiving disability or widow(er)'s benefits) would receive a retired-worker benefit

RWN: final number of retired workers (product of all linkage factors)

SA: sex of the account holder

SCDW: student children of deceased workers by linkage factor, age of the student (18-21) and sex of the account holder. Linkage factors are:

$SCDW_{POP}$:	population of potential student children
$pSCDW_{DEA}$:	probability that the parent is either retired or deceased
$pSCDW_{AGA}$:	probability that the PAH is age 62 or older
$pSCDW_{FIA}$:	probability that the PAH is fully insured
$pSCDW_{CPA}$:	probability that the PAH is receiving benefits
$pSCDW_{SSB}$:	probability that the child is indeed attending school
$pSCDW_{RES}$:	probability that a child who is eligible to receive student-child benefits actually receives the benefits

SCDWN: final number of student children of deceased workers (product of all linkage factors)

SCRW: student children of retired workers by linkage factor, age of the student (18-21) and sex of the account holder. Linkage factors are same as SCDW.

SCRWN: final number of student children of retired workers (product of all linkage factors)

SRADJ: adjustment to match short-range projections in 10th year of projection period

SSAPOP: Social Security area population by age (0:100), sex, and marital status (single, married, widowed, divorced)

SX: sex of the beneficiary

TRYR: first year of the projection period

WEIGHT: estimated probability applied to each possible age of the spouse, given the age of the primary account holder

YSDW: young spouse of deceased worker by linkage factor, age group (under 25, 25-29, ..., 65-69), sex of the account holder and marital status (widowed, divorced). Linkage factors are:

$YSDW_{POP}$:	population of potential young spouse of deceased workers
$pYSDW_{DEA}$:	probability that the primary account holder (PAH) is of the required age
$pYSDW_{ECB}$:	probability that the young spouse has an entitled child in their care
$pYSDW_{FSB}$:	probability that the young spouse is not already receiving benefits based on another child in their care
$pYSDW_{RMB}$:	probability that the young spouse is not remarried

$pYSDW_{RES}$: probability that a person who is eligible to receive young-spouse benefits actually receives the benefits

YSDWN: final number of young spouse of deceased workers (product of all linkage factors)

YSRW: young spouse of retired worker by linkage factor, age group (under 25, 25-29,...,65-69) and sex of the account holder. Linkage factors are:

$YSRW_{POP}$: population of potential young spouse of retired workers

$pYSRW_{AGA}$: probability that the primary account holder (PAH) is of the required age

$pYSRW_{ECB}$: probability that the young spouse has an entitled child in their care

$pYSRW_{FSB}$: probability that the young spouse is not already receiving benefits based on another child in their care

$pYSRW_{RES}$: probability that a person who is eligible to receive young-spouse benefits actually receives the benefits

YSRWN: final number of young spouse of retired workers (product of all linkage factors)

Process 4:

**Trust Fund
Operations &
Actuarial Status**

4. Trust Fund Operations and Actuarial Status

OCACT uses the Trust Fund Operations and Actuarial Status Process to project (1) the annual flow of income from payroll taxes, taxation of benefits, and interest on assets in the trust fund and (2) the annual flow of cost from benefit payments, administration of the program, and railroad interchange. The annual flows are projected for each year of the 75-year projection period. In addition, this subprocess produces annual and summarized values to help assess the financial status of the Social Security program.

The Trust Fund Operations and Actuarial Status Process is composed of three subprocesses: TAXATION OF BENEFITS, AWARDS, and COST. As a rough overview, TAXATION OF BENEFITS projects, for each year during the 75-year projection period, the amount of income from taxation of benefits as a percent of benefits paid. AWARDS projects information needed to determine the benefit levels of newly awarded retired workers and disabled workers by age and sex. COST uses information from the AWARDS and TAXATION OF BENEFITS subprocesses, as well as information from other processes, to project the annual flow of income and cost to the trust funds. In addition, COST produces annual and summarized measures of the financial status of the Social Security program.

4.1. TAXATION OF BENEFITS

4.1.a. Overview

The Social Security Amendments of 1983 (P.L. 98-21) amended the Internal Revenue Code to establish taxation of Social Security benefits. The 1983 law specifies including up to 50 percent of the Social Security benefits in a tax return filer's adjusted gross income (AGI) for tax liability if a tax return filer's "*income*", defined as the sum of *modified adjusted gross income*¹ and one-half of Social Security and Tier 1 Railroad Retirement benefits, is above the specified income threshold amount of \$25,000 as a single filer (or \$32,000 as a joint filer). Subsequently, the 1993 OBRA (Omnibus Budget Reconciliation Act—P.L. 103-66) further amended the Internal Revenue Code to provide for taxation of up to 85 percent of Social Security benefits if a tax return filer's "*income*" is above the specified income threshold amount of \$34,000 as a single filer (or \$44,000 as a joint filer).

The proceeds from taxing up to 50 percent of the OASDI benefits, as a result of the 1983 law, are credited to the OASI and DI Trust Funds, while additional taxes on the OASDI benefits, as a result of the 1993 law, are credited to the HI Trust Fund.

Income to the Trust Funds from such taxation is estimated by using ratios of taxes on benefits to benefits for the OASI and DI programs separately. These ratios, called "RTBs", are applied to projected OASI and DI benefit amounts to estimate tax revenues to the OASI and DI Trust Funds.

Initially we rely on the Department of the Treasury Office of Tax Analysis's (OTA) estimates of the percent of benefits taxable for OASI and DI benefits separately and the average marginal tax rates applicable to those taxable OASI and DI benefits. These estimates are based on Internal Revenue Service (IRS) tax returns data.

For the short range period (first 10 years of the projection), the Cost sub-process (4.3) uses OTA's projected estimates for (1) the percent of benefits taxable and (2) the average marginal tax rates applicable to those taxable OASI and DI benefits. The multiplication of (1) and (2) produces projected RTBs under the 1983 law (up to 50 percent of benefits taxable) and the 1993 OBRA (additional up to 35 percent of benefits taxable).

For the long range period (11th through 75th year of the projection period), we compute the RTB ratios for OASI benefits and those for DI benefits under the 1983 law and the combined 1983 and 1993 laws with the following formula for each projection year.

$$\text{RTB}(\text{yr}) = \text{RTB}(\text{tryr}+9) * \{ \text{AWI}(\text{tryr}+9) / \text{AWI}(\text{yr}) \}^{\wedge} \text{P} + \text{RTB}(\text{ultimate}) * \{ 1 - \text{AWI}(\text{tryr}+9) / \text{AWI}(\text{yr}) \}^{\wedge} \text{P}, \quad (4.1.1)$$

where

¹ *Modified adjusted gross income* equals adjusted gross income (before Social Security and Railroad Retirement benefits are considered) plus nontaxable interest income.

tryr = first year of the projection period (year of the Trustees Report)

RTB(ultimate) = ratio of taxes on benefits to benefits assuming income
Threshold amounts equal zero.

AWI = SSA average wage index series

P = exponential parameter for a trend curve line.

After initial estimates of projected ratios of taxes on benefits to benefits by Trust Fund, based on the OTA's data, we include nonresident alien withholding tax revenue projections by our Short Range office since the OTA does not include nonresident withholding tax revenues. We add estimated nonresident tax revenues, as a ratio of taxes on benefits to benefits by Trust Fund, to the above projected ratios of taxes on benefits to benefits based on OTA data, to arrive at the final RTB ratios.

Finally, the Cost sub-process (4.3) applies the projected RTB ratios to our own estimates of the projected OASI and DI benefit payments to produce taxation of benefit revenues to the OASI and DI Trust Funds.

4.1.b. *Input Data*

OCACT Data

Economics—process 2

- 1a. Projected SSA wage index series by year, updated yearly
- 1b. Projected COLAs and average wage index levels under the intermediate assumptions of the prior Trustees Report

Beneficiaries—process 3

- 2a. Projected OASI beneficiaries by age and sex under the intermediate assumptions of the prior Trustees Report
- 2b. Projected DI beneficiaries by age and sex under the intermediate assumptions of the prior Trustees Report

Trust Fund Operations—process 4

3. Aggregate OASI and DI benefit ratios (as a ratio of total OASDI benefits) under the intermediate assumptions of the prior Trustees Report

OCACT Short-Range Office Data

- 4a. Nonresident alien withholding taxes for the OASI and DI benefits for the short range period

4b. Total OASI and DI benefits respectively for the short range period

Other input Data

5. OTA's projected percent of benefits taxable and average marginal tax rates by type of benefit (OASI and DI) for the short range period (updated yearly).
6. OTA's ultimate ratios of taxes on benefits to benefits (i.e., with income thresholds, assumed equal to 0). Such ultimate ratios are provided on a combined OASDI benefit basis, and are expected to be updated annually based on OTA's update.
7. Current Population Survey (CPS) data for year 2018
8. Marginal income tax bracket amounts for tax years 2018 - 2020
9. Marginal income tax rates for tax years 2018 - 2020
10. General filing requirement amounts (standard deduction amounts) for personal income tax purposes for tax years 2018 - 2020
11. Ratios of taxable income to adjusted gross income by income level (IRS data) for tax year 2018
12. Treasury's aggregate taxable benefit amount (IRS data) for tax year 2018
13. OTA's estimated taxes on benefits for the OASDI and HI Trust Funds for tax year 2018
14. Aggregate OASDI benefit payment for calendar year 2018

4.1.c. Development of Output

For the short range period, the Cost sub-process (4.3) uses OTA's projected RTBs for OASI and DI benefits under the 1983 law, to project taxation of benefit revenues to the OASI and DI Trust Funds.

For the long range period, formula 4.1.1 computes projected ratios of taxes on OASI benefits to OASI benefits and projected ratios of taxes on DI benefits to DI benefits under the 1983 law (up to 50 percent of benefits taxable). This formula essentially provides more weight to the ultimate RTB ratios as time progresses, using the ratio of AWI (10th year) to AWI (projection year) as the "weight." Additionally, an exponential parameter P

value to the AWI “weights” is set judgmentally such that the estimate continues the short range trend into the transitional 11th through 20th projection years before it approaches the ultimate RTB ratio. For the RTB ratios for up to 50 percent of benefits taxable, the P values were set at 0.99 and 0.99 to project smooth transitional RTB ratios for OASI and DI benefits, respectively.

The ultimate RTB ratios used in the projection are based on OTA’s ultimate ratios, reduced by about 2.6 percent. This reduction reflects estimates of the effect of the higher proportion of “old elderly” beneficiaries at about the 75th projection year OASDI beneficiary population distribution relative to the 2030 OASDI beneficiary population distribution, due to improved mortality.

For the 2021 Trustees Report, the ultimate RTB ratios for up to 50 percent of OASI and DI benefits taxable were set at 0.0604 and 0.0201, as compared to 0.0630 and 0.0210 for the 2020 Trustees Report. These decreases in the ultimate RTB ratios reflect OTA’s updated zero threshold estimates.

Lastly, the Cost sub-process (4.3) applies these projected RTB ratios to projected OASI and DI benefit payments to develop estimated taxation of benefit revenues to the OASI and DI Trust Funds.

4.2. AWARDS

Each year over 3 million workers begin receiving either retired-worker or disabled-worker benefits. The monthly benefits for these new awards are based on their primary insurance amount (PIA). The PIA is computed using the average indexed monthly earnings (AIME) and the PIA benefit formula as specified in the 1977 amendments. The AIME depends on the worker's number of computation years, Y , and the earnings in each year. For retired-worker beneficiaries who have attained or will attain age 62 in 1991 or later, $Y = 35$.

The AWARDS subprocess (AWARDS) selects records from a 10 percent sample of newly entitled worker beneficiaries obtained from the Master Beneficiary Record (MBR).¹ The selected sample, referred to as “sample”, contains 293,416 beneficiary records, and each record, r , includes a worker's history of taxable earnings under the OASDI program as well as additional information such as sex, birth date, month of initial entitlement, and type of benefit awarded. To estimate the benefit levels of future newly entitled worker beneficiaries, AWARDS modifies the earnings records in the sample to reflect the expected work histories and earnings levels of future beneficiaries (equation 4.2.1). After the modifications, AWARDS computes an AIME for each record in the future sample of beneficiaries (equation 4.2.2). AWARDS subdivides the AIME value of each record into *bend point subintervals*² (equation 4.2.3). As input to the Cost subprocess, the AIME values are used to calculate aggregate percentages of AIME in each *bend point subinterval* for each age at entitlement, sex and trust fund (equation 4.2.4). Equations 4.2.1 through 4.2.4 outline the overall structure and solution sequence. The subscript n refers to the *bend point subinterval* and r refers to the sample record.

$$\text{Projected Earnings} = \text{Projected Earnings}(\cdot) \quad (4.2.1)$$

$$\text{AIME}(r) = \frac{\sum \text{Highest } Y \text{ Indexed Earnings}(r)}{Y * 12} \quad (4.2.2)$$

$$\text{AIME}_n(r) = \text{AIME}_n(\cdot) \quad (4.2.3)$$

$$\text{PAP}_n = \frac{\sum_r \text{AIME}_n(r)}{\sum_r \text{bp}_n} \quad (4.2.4)$$

where bp_n is the length of the n th bend point subinterval,
 Y is the number of computation years, and
 $\text{AIME}_n(r)$ is the AIME amount contained within the n th interval for record r .

¹ A record is selected if the year of initial entitlement equals 2017 as of the December 2019 MBR extract file date, and the beneficiary is not in death status as of the December 2017 MBR extract file date. Retired beneficiaries over age 70 and disability beneficiaries under age 20 are excluded.

² The current law PIA formula has two bend points. For the purposes of PAP, the Awards subprocess instead uses 30 subintervals.

4.2.b. Input Data

Long-Range OCACT Projection Data

Demography–

1. Total Social Security area population (as of July) by sex and age.
 - From 1941 to 2100 (1951 to 2100 used in SOSI)
 - Updated annually
2. Other-than-lawful permanent resident (“other-than-LPR”) population (as of July) by sex and age
 - From 1964 to 2100 (1964 to 2100 used in SOSI)
 - Updated annually
3. Deferred Action for Childhood Arrivals (DACA) population (as of July) by sex and age.
 - From 2012 to 2100 (2012 to 2100 used in SOSI)
 - Updated annually

Economics –

4. Covered workers by sex and age—with earnings posted to the Master Earnings File (MEF) only - used with CWHS data to project future earnings levels
 - From 1937 to 2100 (1951 to 2100 used in SOSI)
 - Updated annually
5. Covered workers not in the other-than-LPR population by sex and age—with earnings posted to the Master Earnings File (MEF) only
 - From 1937 to 2100 (1951 to 2100 used in SOSI)
 - Updated annually
6. Average Wage Index (AWI), projected values.
 - From 2020 to 2100
 - Updated annually
7. Total taxable earnings and number of workers with taxable earnings by age, sex, and year from the Continuous Work History Sample (CWHS).
 - From 1951 to 2018
 - Updated annually
8. Historical Average Taxable Earnings (ATE) —with earnings posted to the Master Earnings File (MEF) only – used with CWHS data to project future earnings levels
 - From 1937 to 2019 (only 2014- 2019 data used in SOSI)
 - Updated annually
9. Projected Average Taxable Earnings (ATE) —with earnings posted to the Master Earnings File (MEF) only – used with CWHS data to project future earnings levels
 - From 2020 to 2100
 - Updated annually
10. Projected Covered Worker Rate based on total covered workers/total population (not used in SOSI)
 - From 2020 to 2100

- Updated annually
- 11. COLA (Cost of Living Adjustment) – not used in SOSI
 - From 2020 to 2100
 - Updated annually
- 12. Projected Wage Base (current law)
 - From 2022 to 2100
 - Updated annually

Fully Insured –

- 13. Historical and projected fully insured rates by sex and single year of age 14-95
 - From 1969 to 2100
 - Updated annually

Beneficiaries –

- 14. Distribution of number of in-current-pay retired beneficiaries by age at retirement from age 62 to 70 by year, sex and age
 - From 2020 to 2100
 - Updated annually

Other input data

- 15. 10% Awards Sample from the MBR and Master Earnings File
 - Newly entitled OASI / DI beneficiaries, whose initial entitlement year was 2017 as of the December 2019 MBR extract file date, and the beneficiary is not in death status as of the December 2017 MBR extract file date.
 - SSN
 - Type of benefit
 - Type of claim (retirement or disability)
 - Sex
 - Date of birth
 - Date of initial entitlement
 - Date of disability onset
 - PIA amount
 - Type of dual entitlement
 - Dual entitlement status code
 - PIFC
 - LAF
 - Eligibility year
 - Trust fund
 - Earnings histories for each worker from 1951 to 2016
 - Generally updated annually, pending validation of the sample
- 16. AWI, Average Wage Index, historical values
 - From 1951 to 2019
 - Data obtained from OCACT internet site.
 - Updated annually
- 17. Wage base, historical values
 - From 1951 to 2021
 - Data obtained from OCACT internet site.
 - Updated annually

18. COLA, cost of living adjustment (historical values) – not used in SOSI
 - From 1975 to 2020
 - Data obtained from OCACT internet site.
 - Updated annually
19. Amount of earnings needed to earn one quarter of coverage
 - From 1951 to 2021
 - 1978-2021 data obtained from OCACT internet site. 1951-1977 values estimated by applying projection methodology backwards from 1978.
 - Updated annually
20. Windfall Elimination Provision (WEP) factors, the percent of sample cases affected by the WEP which will no longer be affected by the WEP, by sex and projection year
 - From 2020 to 2100
 - Data obtained from OCACT internal calculations
 - Updated annually
21. PIA bend points – not used in SOSI (except for 1979 bend points)
 - From 1979 to 2021
 - Data obtained from OCACT internet site.
 - Updated annually
22. Hypothetical Wage Base (to reflect relative changes in relative taxable maximum levels over time)
 - From 1951 to 2021
 - Updated annually
 -
23. HI scaled factors, scaled factors computed using HI earnings instead of taxable earnings
 - From age 16 to 100.
 - Data obtained from OCACT internal calculations.
 - Updated periodically.
24. Bucket earnings file
 - Detail earnings for years 1978 and later for each worker in Awards Sample with at least one year of earnings at the taxable maximum during those years.
 - SSN
 - Tax year
 - Total Compensation (W-2 box #1)
 - Social Security taxable wages and tips (W-2 boxes #3 and #7)
 - Medicare taxable wages and tips (W-2 box #5)
 - Social Security taxable self-employment income
 - Medicare taxable self-employment income
 - Deferred Compensation Distributions (W-2 box 11)
 - Deferred Compensation Contributions (W-2 box 12)
 - Updated when changing the initial entitlement sample.
25. Taxable ratio goals for OASI and DI
 - Data obtained from OCACT internal calculations
 - Updated annually.

26. CWHS Median earnings / ratios of average earnings to average taxable earnings (MEF only) below and above median
 - Data obtained from OCACT internal calculations
 - Updated as needed.

27. Distribution of number of in-current-pay retired beneficiaries from 1992 to 2100 – only use the data from sample year 2017
 - Data obtained from Beneficiaries area
 - Updated annually

4.2.c. *Development of Output*

All equations described below are projected separately for the OASI and DI program.

Equation 4.2.1 – Projected Earnings

In order to estimate future benefit levels, the work histories and earnings levels in the current sample must be modified to represent those for a sample of worker beneficiaries who are newly entitled in future years. Three distinct modifications are made to the earnings records. For each future year, changes are made to the earnings records in order to reflect:

- *Changes in Wage Bases.*
For some years, the projected wage base (contribution and benefit base), on an AWI discounted basis, is higher than the historical wage base. Therefore, the taxable earnings of future beneficiaries need to include covered earnings above the reported historical wage base. Thus, for each record with reported taxable earnings at the wage base in a given year, AWARDS obtains or imputes his/her covered earnings. See below section “Change in Wage Bases” and appendix 4.2-1 for more information.

- *Changes in Covered Worker Rates.*
Adjustments are made to work histories to be consistent with the projected changes in the economy-wide covered worker rates. Economy-wide covered worker rates are defined as the ratio of “legal” covered workers (from Economics subprocess) to the Social Security area “legal” population (from Demography subprocess). See the below detailed section “Changes in Covered Worker Rates” for more information.

- *Earnings Experience in the CWHS³.*
Earning levels are modified to capture the changes to date that are reflected in the average taxable earnings reported in the CWHS by age and sex and the changes expected in the future.

Change in Wage Bases

³ This file is a 1% sample of individuals who had covered earnings at some point in their work histories.

The earnings posted in the sample are limited by the historical wage base (contribution and benefit base). Prior to 1975, the maximum annual amount of earnings on which OASDI taxes were paid was determined by ad hoc legislation. After 1974, however, the annual maximum level was legislated to be determined automatically, based on the increase in the Social Security Average Wage Index (AWI). Prior to these automatic wage base increases, a relatively large portion of workers earned amounts above the base. Additional legislation raising the annual maximum taxable amount occurred in 1979, 1980, and 1981 to improve the financial future of the OASDI Trust Funds. In addition, the AWI used in the automatic calculation of the annual taxable maximum was modified in the early 1990s to include deferred compensation amounts.

Therefore, for each record in the sample with earnings at the wage base, AWARDS uses an additional data source to obtain covered earnings for 1994 and later, and to impute as best as possible covered earnings for years before 1994. This is done in order to reflect higher maximum taxable amounts imposed on future newly entitled beneficiaries. Then, these projected covered earnings are capped at the wage base values that would be in effect for future samples of retired workers (using the “projected wage base” input file) to determine the taxable earnings to use in the benefit calculations. Please refer to appendix 4.2-1 for more details on this method.

Change in Covered Worker Rates

The *sample covered worker rate* by age group and sex is defined as the ratio of (1) the number of those beneficiaries with covered earnings in the sample to (2) the total number of beneficiaries in the sample. For both men and women, the work histories are modified to reflect changes in the covered worker rates that would apply to a future sample of beneficiaries. These changes in the covered worker rates are based on changes in the *economy-wide covered worker rates*. The *economy-wide covered worker rate* is defined for an age-sex group in a particular period which represents a future sample cohort as the ratio of (1) the number of “legal” workers in the economy in this group that have some earnings in this period, to (2) the total midyear “legal” population in this group in this period.⁴ *Economy-wide covered worker rates* are calculated separately for each age-sex group and each historical and projected calendar year based on input data from the Economics, Demography, and Fully Insured subprocesses.

In projecting *sample covered worker rates*, examination is done for the change in *adjusted economy-wide covered worker rates*, by age-sex group, between the “base period” (representing individuals retiring in the sample year) and the “projection period” (representing individuals retiring in a year later than the sample year). The adjusted economy-wide covered

⁴ For this purpose, we define the “legal” population as the total SSA area population *minus* the other-than-lawful permanent resident (“other-than-LPR”) population (those in the U.S. illegally and those in the U.S. on a temporary basis, e.g., individuals with specific non-immigrant worker or student visas) *plus* the DACA population (DACA individuals are included in the other-than-LPR data but had been effectively made legal, for Social Security purposes, by policy directive or executive action).

worker rates in the base year take into account 100% (for men) and 40% (for women) of the change in projected age-62 fully insured rates relative to the base year. These two percentages are based on analyses of different cohorts of historical data. For each projection year, the method estimates what the base year economy wide covered worker rates would have been if 100% (for men) and 40% (for women) of the increase in the covered worker rates were attributable to those individuals becoming newly fully insured (or losing insured status if the change in covered worker rates is negative). Details of how this change is used to estimate the change in a covered worker rate for retired workers, from a current period in the sample to a future period, are given below. The method used is the same for men and women in projecting sample covered worker rates. For additional explanation of this calculation, refer to example 1.1 in appendix 4.2-2 of this subprocess.

Projected sample covered worker rate equals (for increasing economy-wide rates):

- The ratio of (1) the potential difference in the *economy-wide male (or female) covered worker rate* in the projection year to (2) the potential difference in the *adjusted economy-wide male (or female) covered worker rate in the sample year (i.e., 1 – adjusted economy-wide male (or female) covered worker rate)*, multiplied by
- The corresponding potential difference in the *sample's male (or female) covered worker rates (i.e., 1 – sample male (or female) covered worker rate)*.
- The above result is subtracted from 1 to get the projected sample covered worker rate.

This presentation above presumes that economy-wide covered rates increase over time, which is very common for women but not always true for men. The calculation of the change in covered worker rate differs if there is a reduction in relevant economy-wide covered worker rates. Example 1.2 shows the calculations done for men and women if economy-wide covered worker rates decline.

A similar procedure exists for projecting sample covered worker rates for disabled workers, except that the calculations are further broken down by entitlement age group. For disabled workers, years used to determine disability insured status do not have earnings added or removed (the last 10 years for disabled workers initially entitled after age 31). See Example 1.3 for an illustration of the method.

Once the covered worker rates for the future sample of beneficiaries are determined, modifications to work histories of the sample to attain these rates are generally done by randomly removing or adding earnings.⁵ For men, the procedure is to select records randomly with no other restrictions or criteria. However, for women, an additional selection criterion is included in order to achieve a specified distribution of the number of years of earnings for retired female beneficiaries. Female records with 10 or fewer years of earnings are not modified. A distribution limit is set for those female workers with 11 to 25 total years of career earnings within the projection year. This distributional limit changes each projected year. In the first year after the sample year, the distribution limit for women is equal to the

⁵ Individuals in the sample affected by the Windfall Elimination Provision are less likely to have earnings removed or added by this process.

male distribution plus 97% of the difference between the initial male and female distributions within the sample. In each subsequent year, the percentage decreases by three percent until it reaches 0%. Thus, the women's years of earnings distribution for those with 11 to 25 years of earnings is adjusted to approach that of the men.

If a record is selected to remove the earnings in a particular year, the full amount of earnings will be taken away. If a record is selected for adding earnings in a particular year, the amount of earnings added is based on the career earnings pattern of the selected record. When earnings are added to a record, AWARDS calculates the ratio of (1) the record's Average Indexed Earnings, $AIE(r)^6$, to (2) the AIE of a hypothetical worker, w , whose year of birth and sex are the same as the record and whose annual earnings are set equal to *average taxable earnings*. For this purpose, *average taxable earnings* are determined by averaging the earnings over all records in the sample with the same sex and year of birth and earnings year. Then, the preliminary amount of earnings⁷ in year t that is added to the record is

$$\text{Earnings}(r, t) = {}^S\text{ATE}(tf, \text{sex}, t, a) * \frac{AIE(r)}{AIE(w)},$$

where ${}^S\text{ATE}(tf, \text{sex}, t, a)$ is the average taxable earnings in year t , for those in the sample with the same sex as that of the record for year of birth a by trust fund tf .

For additional explanation of this calculation, refer to example 2 in appendix 4.2-2 of this subprocess. Note that all earnings levels get further adjusted by earnings experience in the CWSHS for recent workers, as discussed in the below section.

Once the earnings are added or taken away for some records, the earnings are further adjusted to reflect the dispersion in historical taxable earnings observed from 1970 to 2010 in the model's earnings projections, using a 1% sample of workers likely to be eligible for retirement benefits. Over this period, increases in taxable earnings were generally higher for workers with taxable earnings over the median than for workers with taxable earnings under the median. A side model computes changes in average taxable earnings (ATE) levels above and below the median, as a ratio to the overall ATE level, for each earnings year, age, and sex combination from the 1% earnings sample. In the Awards program, the changes in these ratios (above and below median) in the projection year relative to the sample year were used to project taxable earnings from historical to projected years. The changes in these ratios (above and below median) are then applied to the earnings after coverage loads. See example 2.1 for an illustration.

The overall targeted projected ATE level for each projection year, earnings year, sex, and age combination, as explained in the next section, is unaffected by this dispersion adjustment.

⁶ AIE is the average indexed annual earnings, average over the highest Y years of earnings (similar to AIME, but an annual amount). Y is 35 for retired workers and is based on years of non-disability for disabled workers (resulting in a low of 2 and a high of 35).

⁷ In this subprocess, earnings histories of projected beneficiaries are all reflected as wage-indexed earnings histories in the 2017 sample.

Earnings Experience in the CWHS

For historical years beginning with 1951, AWARDS uses average taxable earnings by age and sex ($CWHSATE_{as}$) and numbers of covered workers by age and sex ($CWHS CW_{as}$) as tabulated from the most recent CWHS file⁸. To estimate ATE levels for the first projection year, AWARDS uses the average values of normalized average taxable earnings calculated from the last five historical years of the most recent CWHS file. These computed normalized values take into account changes in aggregate ATEs from the Economics subprocess between each of the five historical years and the first projection year; this allows the comparison of corresponding values from different years in a way that reduces time series effects.

Projections are made for each year after the base year through the end of the 75-year projection period using projected economy-wide number of covered workers by age and sex and aggregate annual average taxable earnings (ATE) from the Economics process⁹ attributable to ages 15-80. The first step is to determine preliminary $CWHSATE'_{as}$ (preliminary average taxable earnings by age and sex) by using the annual growth rate in the total economy-wide ATE and projected number of covered workers from the Economics process. A further multiplicative adjustment is made to each $CWHSATE'_{as}$ such that the resulting aggregate average taxable earnings, determined by combining the projected values of covered workers from the Economics process and $CWHSATE_{as}$ for the year, produces the same aggregate ATE level for that year as projected by the Economics process. A small constant adjustment is also made, based on historical data, to reflect the difference between aggregate ATE levels for ages 15-80 (as used here) and aggregate ATE levels for all ages produced by the Economics subprocess.

For additional explanation of this calculation, refer to example 3 in appendix 4.2-2 of this subprocess.

The historical and projected $CWHSATE_{as}$ are then used to change the earnings histories of the sample of newly entitled beneficiaries so that the earnings better represent newly entitled beneficiaries in future years. For a given sex, year of birth, trust fund, and earnings year, the expected annual average taxable earnings of a future sample is denoted as ATE'_f . ATE'_f equals the sample's average taxable earnings for a specific earnings year, multiplied by the comparable changes in the $CWHSATE_{as}$, that is, the wage-indexed changes in the $CWHSATE_{as}$ between the year of earnings in the sample of new beneficiaries and the year of earnings in the projected sample. Refer to example 3.1 in appendix 4.2-2 for additional explanation.

ATE'_f is then compared to the average taxable earnings of the sample (after adjustments to the records' earnings levels for changes in wage bases and covered worker rates), denoted as ATE_f and computed by sex, trust fund, earnings year, and year of birth. The difference between these values is the amount by which the average annual earnings levels are adjusted. Let

⁸ These historical values are tabulated by the Economic subprocess.

⁹ Aggregate ATE values for all ages are based on earnings posted to the Master Earnings File (MEF), excluding earnings posted to the suspense file. Historical experience from the CWHS sample is used to estimate the ATE for the large age 15-80 subgroup, and is used throughout this part of the model.

$$\delta(t) = ATE_f' - ATE_f ,$$

for a specific age (or year of birth in the sample) in the projection year t . Denote the total workers in the sample in year t as $TotalWorkers(t)$. Then, $(\delta(t) * TotalWorkers(t))$ is the total amount of earnings which the model distributes for a given sex and single age in a way so that the average taxable earnings after distribution is ATE_f' .

For additional explanation of the calculation $\delta(t)$, refer to example 4 in the appendix 4.2-2.

When $\delta(t)$ is negative, earnings for the year are decreased. To achieve ATE_f' for the given sex, age, trust fund, earnings year, and year of birth, AWARDS multiplies $CoveredEarnings(r, t)$ by a ratio,

$$ratio(t) = 1 + \frac{\delta(t)}{ATE_f(t)} + \alpha .$$

The term, α , is a necessary additional adjustment because covered earnings near or above the wage base, may have either a partial effect or no effect on modifying ATE_f to ATE_f' . These α values vary by sex, trust fund, and whether earnings increase or decrease. For OASI, the adjustment, α , is further broken down by year of birth to better match the targeted ATE levels. Otherwise, the determination process for OASI and DI is the same. These α values are set to best target ATE_f' while making adjustments throughout as many of the sample records as possible.

As AWARDS applies a $ratio(t)$ to $CoveredEarnings(r, t)$ by each record, it makes sure that the total earnings adjustment in a given earnings year and single age does not exceed $\delta(t)*TotalWorkers(t)$. For additional explanation of this calculation, refer to example 5 in appendix 4.2-1 of this subprocess.

Shuttling Method

The change in distribution of the retired worker beneficiaries in projection years, as compared to the sample year, is another element in projecting earnings. To account for the general projected shifting of retirement by workers to later ages relative to the initial entitlement sample, the “forward shuttling” method facilitates the shift and estimates the additional earnings for the corresponding years before later retirement. Records are randomly chosen for additional earnings in the later retirement years. The amount of earnings for these shuttled records with delayed retirement are assumed to be the projected sample average taxable earnings. The number of shuttled records with additional earnings depends on the projected sample covered worker rates by sex and age. In the case of “backwards shuttling”, which is also possible in projected years, workers could retire at younger ages than in the Awards initial entitlement sample. When this happens, the worker will have fewer years of earnings. The earnings are removed in the appropriate age(s) for backwards shuttling. Since backwards shuttling rarely occurs, this condition does not have a major effect.

Due to slightly different methodologies, the age distribution of newly entitled retired workers in the awards sample is slightly different than the corresponding age distribution from the OASI beneficiaries program for the sample year (2017). Therefore, an adjustment was made to align the OASI beneficiaries area distribution of newly retired workers by age to the distribution from the Awards area. This alignment process, applicable to beneficiary age distributions in projection years, occurs for the sample year used by the Awards area and effectively eliminates the differences in the two sample year distributions in projecting average retired worker benefit levels by age. This alignment restores the procedure in place before the 2010 Trustees Report.

For each age in a given projection year, the result of the shuttling method (forward or backward) is a weighted average of the PAPs that correspond to retirement at that age. As mentioned in the preceding paragraph, when the sample retirement age is earlier than the age in question (that is, individuals who delay retirement in the projection year), this calculation includes potential additional earnings in the AIME calculation for the additional years before retirement. Refer to example 7 in appendix 4.2-3 for additional explanation and illustration.

Equation 4.2.2 – Average Indexed Monthly Earnings (AIME)

Step 1: Index Earnings

To compute an individual’s AIME, all taxable earnings after 1950 are considered. First, the earnings are indexed up to the index year, i , which is defined as the year of attaining age 60 for retired-worker beneficiaries (eligible for benefits at age 62). For disabled-worker beneficiaries, i is set to be 2 years before the sample year. Thus,

$$\text{IndexedEarnings}(r, t) = \begin{cases} \text{Earnings}(r, t) * \frac{\text{AverageWage}(i)}{\text{AverageWage}(t)}, & \text{if } t < i \\ \text{Earnings}(r, t), & \text{if } t \geq i \end{cases}$$

Step 2: Determine Computation Years

For each record, the number of computation years, Y , is determined. For a retired-worker beneficiary in the sample, Y is 35.

For a disabled-worker beneficiary, Y is calculated as follows:

- Determine the number of elapsed years, which is equal to the year of disability onset (not later than the year the worker turned age 62) *minus* the greater of 1951 or the year the disabled worker turned age 22.

$$\text{Elapsed Years} = \min \{ \text{Year of disability onset, Year attained age 62} \} - \max \{ 1951, \text{Year attained age 22} \}$$
- Divide the elapsed years by five and truncate. Subtract this number (cannot exceed five)

from the number of elapsed years.

$$Y = \text{Elapsed Years} - \min \left\{ \text{int} \left[\frac{\text{Elapsed Years}}{5} \right], 5 \right\}$$

- Y must be at least 2.

Step 3: Determine AIME

Finally, an individual's AIME is computed by summing the highest Y indexed earnings and dividing by the number of months in those years. Hence, for each record,

$$\text{AIME}(r) = \frac{\sum \text{Highest Y Indexed Earnings}(r)}{Y * 12}.$$

Equation 4.2.3 – AIME_n(r)

The Possible AIME value is divided into 30 intervals (*bend point subintervals*). The length of each interval in 1979 dollars is given below:

$$\text{bp}_n = \begin{cases} \$45, & \text{if } 0 < n \leq 13 \\ \$100, & \text{if } 14 \leq n \leq 18 \\ \$200, & \text{if } 19 \leq n \leq 28 \\ \$1000, & \text{if } 29 \leq n \leq 30 \end{cases}$$

Thus, the interval points of AIME division given below in 1979 dollars, y_k , are equal to

$$\sum_{n=1}^k \text{bp}_n \text{ and}$$

$$y_k = \begin{cases} \$180, & \text{if } k = 4 \\ \$1085, & \text{if } k = 18 \\ \$5085, & \text{if } k = 30. \end{cases}$$

For each record (r), the values for bp_n are indexed from 1977 to his/her indexing year i using the Social Security average wage index (AWI). So for $n = 1$ to 30,

$$\text{bp}_n(r) = \text{bp}_n * \frac{\text{AWI}(i)}{\text{AWI}(1977)}$$

Next the record's AIME amount, AIME (r), is compared to the indexed intervals. If

$$\sum_{n=1}^{k-1} \text{bp}_n(r) < \text{AIME}(r) \leq \sum_{n=1}^k \text{bp}_n(r),$$

then AIME (r) falls within the *k*th interval. And for *n* = 1 to 30,

$$\text{AIME}_n(r) = \begin{cases} \text{bp}_n(r), & \text{if } n < k \\ \text{AIME}(r) - \sum_{n=1}^k \text{bp}_n(r), & \text{if } n = k \\ 0, & \text{if } n > k \end{cases}$$

Equation 4.2.4 – Potential AIME Percentages (PAPS)

Finally, for *n* = 1 to 30, AWARDS sums the values of AIME_{*n*} and bp_{*n*} across all the records for all projection years by sex, age (20-65 for disabled workers, and 62-70 for retired workers), and trust fund. The ratio of these values gives the average potential AIME percentages (PAPS):

$$\text{PAP}_n = \frac{\sum_r \text{AIME}_n(r)}{\sum_r \text{bp}_n(r)}.$$

For an example of this calculation, refer to example 6 in appendix 4.2-2 of this subprocess.

Appendix 4.2-1

This appendix provides additional details on how the AWARDS process imputes covered earnings above the historical wage base. The taxable earnings in the Awards sample are less than or equal to the historical wage base.

The Hospital Insurance (HI) wage base was phased out from 1991 to 1993 and eliminated in 1994. For years 1994 and later, HI earnings are a good proxy for Social Security covered earnings. For years 1991 to 1993, HI earnings are a good proxy for Social Security covered earnings if they are under the HI wage base. Otherwise they give a lower bound for Social Security covered earnings. Total compensation (TC) earnings, available starting in 1978, are earnings subject to federal income tax. TC can differ from Social Security covered earnings but, where other data is lacking, gives a reasonable estimate of Social Security covered earnings. The program uses Hospital Insurance (HI) covered earnings for years 1991 and later and Total Compensation (TC) earnings for years 1978 and later to estimate covered earnings for those years. For years in which no good HI or TC estimate is available, including all years before 1978, the program estimates covered earnings based on available data from later years or from the application of HI “scaled” earning factors by age. The program then adjusts covered earnings estimates for years 1993 and earlier in order to align taxable ratios to taxable ratio “targets” based on CWHS, Economics, and sample data.

To start with, the covered earnings methodology determines which workers in the Awards sample had taxable earnings at the wage base in years 1978 and later. For these individuals we obtained ORES “bucket” file records. These records contain individual HI earnings, total compensation (TC) earnings, and deferred compensation, and are only available for years 1978 and later. Program module HI_EarningsMod.f90 in the AwardsPart1 program reads the bucket file and makes the earnings available to module CoveredEarningsMod.90.

The program estimates covered earnings in three phases. In phase one, the program makes a best estimate of covered earnings in each year, for each record. In phase two, taxable ratio “targets” (the ratio of taxable to covered earnings) are computed by trust fund, sex, and year. These targets, for years 1993 and earlier, are externally computed based on economics data, Continuous Work History Sample data, and sample covered earnings after 1993. Finally, for phase 3, in an iterative process, the program adjusts covered earnings for years 1993 and earlier until the computed taxable ratios match the taxable ratio targets.

More details of each phase follow.

Phase 1

Phase 1 makes a best estimate of covered earnings in each year for each worker using sample taxable earnings, bucket earnings, and scaled factors based on HI earnings.¹⁰ For years with

¹⁰ HI scaled factors are computed using the same methodology as regular scaled factors [see actuarial note 2021.3 at <http://www.ssa.gov/OACT/NOTES/ran3/an2021-3.pdf> except that HI taxable earnings are used in the calculations instead of Social Security taxable earnings. We discovered that the method worked better when HI scaled factors for ages under 22 were set equal to the age 22 factor so the factors hold steady under age 22.

sample taxable earnings under the wage base, covered earnings are simply set equal to taxable earnings. If taxable earnings are equal to the wage base then the program estimates covered earnings using the following data sources, in order of perceived accuracy: 1) HI earnings over the wage base in the year, 2) TC over the wage base in the year, 3) HI or TC earnings over the wage base in a later year combined with adjustments for AWI and HI scaled factors, 4) estimated lower bounds for covered earnings in earlier years with adjustments for AWI and HI scaled factors. In some cases, the program will choose the results from data source 4) over other sources if that results in a higher covered earnings amount. Note that comparing TC to HI in years 1994 and later, we observed that HI tends to be slightly higher than TC. When using TC to estimate covered earnings prior to 1994, we apply an adjustment factor to the TC earnings to account for this. TC, as used in the program, includes deferred compensation contributions but not distributions.

The program computes two estimates of covered earnings for each applicable record in the sample and takes the one judged to be the best estimate. The two estimates are “Covered_Earnings_1” and “Covered_Earnings_2.”

Covered_Earnings_1 is an estimate of covered earnings based on earnings in the current and later years. The program computes some intermediate values that will be used to estimate covered earnings.

For each worker *iii* in the sample and for each earnings year (*iyear*) from 1978 through SAMPLE_YEAR-1

$$HIplusAdjEarnings(iyear,iii) = \begin{cases} HIearnings(iyear,iii), & \text{if } iyear \geq 1994 \\ \max(HIearnings(iyear,iii), TC(iyear,iii)), & \text{if } 1978 \leq iyear \leq 1993 \end{cases}$$

For each worker *iii* in the sample and for each earnings year (*iyear*) from SAMPLE_YEAR-1 through 1951 (stepping backwards), a variable *ratioYearToUse* is created for each earnings year:

$$ratioYearToUse(iyear,iii) = \begin{cases} 0, & \text{if } earnings(iyear,iii) < wagebase(iyear) \\ iyear, & \text{if } HIplusadjearnings(iyear,iii) > wagebase(iyear) \\ iyear + 1, & \text{if } ratioYearToUse(iyear + 1) = iyear + 1 \\ \text{earliest } iyear \text{ for which } HIplusadjearnings(iyear,iii) > wagebase(iyear) \\ -1, & \text{if none of the above conditions are met} \end{cases}$$

This will yield a value other than “-1” if an individual has HI or TC earnings over the wage base in any year 1978 or later.

Then, for each worker *iii* in the sample with at least one year of earnings at the wage base, for each earnings year (*iyear*) 1951 to SAMPLE_YEAR-1:

$$ratio(iyear, iii) = \begin{cases} 0, & \text{if } ratioYearToUse(iyear, iii) \leq iyear \\ \frac{HIplusAdjEarnings(ratioYearToUse(iyear, iii))}{averageWage(ratioYearToUse(iyear, iii) * HIscaledFactor(iage))}, & \text{otherwise} \end{cases}$$

where $iage = iyear - yearOfBirth(iii)$

If there is a year, 1978 or later, with HI or TC earnings over the wage base, then Covered_Earnings_1 in that year will be set to the *HIplusAdjEarnings* value computed above. Otherwise, if the ratio is above zero, Covered_Earnings_1 will be set to the product of the ratio times the *HIscaledFactor* times the *averageWage* for the year, or to the wage base + \$1 if higher. If neither of these methods applies, then the variable is set to the wage base + \$1 and may be overwritten by Covered_Earnings_2 below.

Covered_Earnings_2 is an estimate of covered earnings in the current year based on estimates for earlier years. To set values for Covered_Earnings_2(year,iii) the program starts by setting the 1951 estimate to the taxable earnings value and using AWI increases and changes to HI scaled factors to estimate covered earnings in following years. For each consecutive year, the estimate will be the higher of the HI scaled factor and cumulative AWI increase applied to the prior year Covered_Earnings_2, or the taxable earnings of the current year.

The program then decides whether to use the value for Covered_Earnings_1 or Covered_Earnings_2 for the phase 1 estimate of covered earnings. Generally, Covered_Earnings_2 will be used if higher than Covered_Earnings_1, and if the Covered_Earnings_1 estimate is based on earnings at least 2 years different than the earnings year in question. In general, Covered_Earnings_2 ends up being used less often overall.

Phase 2

The program uses the covered earnings from Phase 1 to compute taxable ratios by trust fund, year, and sex. Taxable ratio targets are developed for OASI and DI outside of the Awards program. These target values are produced by age and sex for years 1954-1993 for OASI and years 1971-1993 for DI for those aged 20-64 in those years. The program also takes weighted averages of these single age targets to obtain annual goals in aggregate by sex and trust fund.

The program reads in the taxable ratio targets from input files. Below is a summary of the process used to generate these taxable ratio targets:

- 1) Obtain data from the CWHS for years 2004-2018 of taxable and covered earnings by age and sex for individuals fully insured as of the end of each year, and compute taxable ratios for each cell
- 2) In a side FORTRAN program, for each year 1953-1993, match to the aggregate taxable ratio from the Economics group, separately revising each of year 2004-2018 CWHS data for this purpose. From this exercise, the resulting output is 15 sets of individual taxable ratios modeled to fit the aggregate 1953-1993 taxable ratios. Average these 15 years' worth of data to establish individual age/sex taxable ratios for each year 1953-1993.

- 3) By age and sex for OASI, adjust the taxable ratios in step 2 by a) a constant factor to align from Economic group's taxable ratio for each year 2004-2018 to that of the data in step 1, and b) a second constant factor by birth cohort to align from the 2004-2018 CWHS data developed in step 1 to the taxable ratio for the 2017 initial entitlement sample; this is done for birth cohorts 1947-1955 for the 2017 sample.
- 4) By age and sex for DI, adjust DI taxable ratios from the levels in step 2 by the relative percentage closer to 1.000 of the taxable ratios between the CWHS data and that of the 2017 sample at that age/sex. This attempts to reflect the relative difference of taxable ratios for the smaller subset of DI cases (usually higher, that is, less earnings over the taxable maximum)
- 5) Aggregate these taxable ratio targets by year, sex, and Trust Fund, using the number of individuals in the 2017 sample at the appropriate age as the weights to derive one taxable ratio by trust fund, year, and sex.

Phase 3

For this final phase, the program starts an iterative process to match the overall targeted taxable ratio for that year, sex, and Trust Fund. If, for a given combination, the computed taxable ratio is within epsilon (.001 for OASI, .002 for DI) of the taxable ratio target then the covered earnings for that combination are accepted and taxable ratios by single age result from the analysis. If the computed taxable ratio is not within epsilon of the target, then all records with corresponding covered earnings over the wage base are either increased or decreased to move the computed taxable ratio closer to the taxable ratio target, and the two ratios are compared again. This is repeated until the computed taxable ratio is within epsilon of the taxable ratio target. If, in rare instances, all covered earnings are less than or equal to the wage base then no more iterations are done and those covered earnings are accepted.

Appendix 4.2-2

This appendix provides examples to help understand the calculations described in the model documentation of the AWARDS subprocess. **These examples do not necessarily reflect values actually used in the projections.**

Example 1.1: (OASI-Women with increasing economy-wide covered worker rates)—same method applies for men

Task: In projecting the 2017 sample of newly entitled female beneficiaries to represent newly entitled female beneficiaries in 2035, an adjustment to the earnings histories for those women age 30-34 is needed to reflect higher covered worker rates expected for women in this age group.

This example illustrates the calculation of the projected covered worker rate for women who are age 30-34 in the projection period. We will be comparing the group of women age 30-34 in the base period with its counterpart group of women age 30-34 in the projection period.

Information given:

- Newly entitled retired female beneficiaries represented in the 2017 sample are age 30-34 in the base period, 1977-1989, and the counterpart group of women retiring in 2035 is age 30-34 in the projection period, 1995-2007.
- Based on the 2017 sample, the covered worker rate for women age 30-34 in the base period = 71.05%.
- Fully insured rate for women age 62 in 2016 (SampleYear -1) = 89.33%
- Fully insured rate for women age 62 in 2035 = 91.55%
- Fully insured adjustment factor for women = 40%
- Unadjusted economy-wide covered worker rate in year 2016 for women age 30-34 = 67.38% in the base period 1977-1989.
- Economy-wide covered worker rate in year 2034 for women age 30-34 = 79.32% in the projection period 1995-2007.

Calculations:

1. Adjusted economy wide covered worker rate for women in the base period = Unadjusted economy-wide covered worker rate for women in the base period * (1+ (Fully insured rate for women in 2035/Fully insured rate for women in 2016 - 1) * fully insured adjustment factor) =68.05%
2. The potential difference in the economy-wide covered worker rate for women age 30-34 in the projection period is 100.0% - 79.32% or 20.68%.

3. The potential difference in the adjusted economy-wide covered worker rate for women age 30-34 in the base period is 100.0% -68.05% or 31.95%.
4. The ratio from *steps 2 and 3* is 64.73%.
5. The potential difference in the sample covered worker rate for the women age 30-34 in the base period is 100.0% - 71.05% or 28.95%.
6. The ratio from *step 4* is multiplied by the potential difference in the sample's covered worker rate for women age 30-34 in the base period to yield 18.74% (64.73% * 28.95%).
7. The amount in *step 6* (18.74%) would be subtracted from 1 to yield the sample's covered worker rate for women who are age 30-34 in the projection period (81.26%).

Example 1.2: (OASI-Men with decreasing economy-wide covered worker rates)—same method applies for women

Task: In projecting the 2017 sample of newly entitled male beneficiaries to represent newly entitled male beneficiaries in 2050, an adjustment to the earnings histories for those men age 40-44 is needed to reflect lower covered worker rates expected for men in this age group.

This example illustrates the calculation of the projected covered worker rate for men who are age 40-44 in the projection period. We will be comparing the group of men age 40-44 in the base period with its counterpart group of men age 40-44 in the projection period.

Information given:

- Newly entitled retired male beneficiaries represented in the 2017 sample are age 40-44 in the base period, 1987-1999, and the counterpart group of men retiring in 2050 is age 40-44 in the projection period, 2020-2032.
- Based on the 2017 sample, the covered worker rate for men age 40-44 in the base period = 86.92%.
- Fully insured rate for men age 62 in 2016 (Sample Year - 1) = 95.63%
- Fully insured rate for men age 62 in 2050 = 95.26%
- Fully insured adjustment factor for men = 100 %
- Unadjusted economy-wide covered worker rate in year 2016 for men age 40-44 = 85.27% in the base period 1987-1999.
- Economy-wide covered worker rate in year 2049 for men age 40-44 = 84.81% in the projection period 2020-2032.

Calculations:

1. Adjusted economy wide covered worker rate in the base period =
Unadjusted economy-wide covered worker rate in the base period *

$(1 + (\text{Fully insured rate 2050} / \text{Fully insured rate 2016} - 1) * \text{fully insured adjustment factor for men}) = 84.94\%$

2. The economy-wide covered worker rate for men age 40-44 in the projection period is 84.81%.
3. The ratio from *steps 2 and 1* is $.8481 / .8494$ or 99.85 %.
4. The ratio from *step 3* is multiplied by the sample's covered worker rate for men age 40-44 in the base period to yield 86.79% ($99.85 \% * 86.92\% = 86.79\%$).

The amount in *step 4* (86.79%) would be the sample covered worker rate for men who are age 40-44 in the projection period.

Example 1.3: (DI-Men with decreasing economy-wide covered worker rates) – same method applies for women

Task: In projecting the 2017 sample of newly entitled male DI beneficiaries age 60-64 to represent newly entitled male beneficiaries in 2050, an adjustment to the earnings histories for those men age 40-44 is needed to reflect lower covered worker rates expected for men in this age group.

This example illustrates the calculation of the projected covered worker rate for men who are newly entitled at age 60-64 in 2050 and are age 40-44 in the projection period. We will be comparing the group of men age 40-44 in the base period with its counterpart group of men age 40-44 in the projection period. The formulas are similar to Example 1.2 in relating the age-group of earnings to a specific entitlement age-group, but with no fully insured adjustment to economy-wide covered worker rates in the base year.

Information given:

- Newly entitled male DI beneficiaries who are age 60-64 represented in the 2017 sample are age 40-44 in the base period, 1993-2001, and the counterpart group of male DI beneficiaries who are age 60-64 in 2050 is age 40-44 in the projection period, 2026-2034.
- Based on the 2017 sample, the covered worker rate for male DI beneficiaries age 40-44 in the base period = 92.09%.
- The economy-wide covered worker rate in year 2016 for men age 40-44 in the base period 1993-2001 is 87.11%.
- The economy-wide covered worker rate in year 2049 for men age 40-44 in the projection period 2026-2034 is 85.5%.

Calculations:

1. The economy wide covered worker rate for men age 40-44 in the base period is 87.11%
2. The economy wide covered worker rate for men age 40-44 in the projection period is 85.5%.
3. The sample covered worker rate for male DI beneficiaries age 40-44 is 92.09%.
4. The ratio from *steps 2 and 1* is 98.15 %.
5. The ratio from *step 4* is multiplied by the sample's covered worker rate for men age 40-44 in the base period to yield 90.39% (98.15 % * 92.09%).

Example 2:

Task: In projecting the 2017 sample of newly entitled male OASI beneficiaries to represent newly entitled male OASI beneficiaries in 2050, an adjustment to the earnings histories for those men age 30 is needed to reflect higher covered worker rates expected for men in this age group. To achieve this target, the desired numbers of records with zero reported earnings in this age group are randomly selected and assigned earnings.

This example illustrates the calculation of earnings to be assigned to a randomly chosen newly entitled retired male record with zero taxable earnings in the base year.

Information given:

- Newly entitled retired male beneficiaries age 66 represented in the 2017 sample are age 30 in the base period 1981.
- Based on the 2017 sample, a male record, $r = 30$, has been randomly selected to replace his zero taxable earnings reported in the base year 1981 at age 30 with an amount based on his career earnings pattern. His year of birth is 1951.
- The Average Indexed Earnings for this record, AIE(30), is computed to be \$20,417. Note: This value is calculated by (1) using the record's annual taxable earnings reported each year through 2016, (2) converting them to 2016 year dollars, and then (3) summing the highest 35 years of earnings and dividing by 35.
- The Average Indexed Earnings for a hypothetical worker, AIE(w) whose year of birth is 1951 is \$67,770. This value is calculated as above given the hypothetical worker earned the *average taxable earnings* in each of the base years for men retiring at age 66 in the 2017 sample.
- The sample Average Taxable Earnings of men at age 30 in 1981 is ${}^s\text{ATE}(\text{OASI, male, 1981, 1951}) = \$17,388$.

Calculations:

1. The ratio of the Average Indexed Earnings for record number 30, AIE(30) to the Average Indexed Earnings of a hypothetical male worker

born in 1951 and retiring at age 66, AIE(w) is $\$20,417/\$67,770$ or 0.3017.

2. The amount in *step 1* (0.3017) would be multiplied by ${}^S\text{ATE}(\text{OASI, male, 1981, 1951})$, which is given as \$17,388. This yields the amount of earnings assigned to record number. Thus, $\text{Earnings}(30, 1981) = .3017 * \$17,388$ which equals \$5,246.

Note that, at this stage of the process, the average taxable earnings are modified to reflect the higher covered worker rates expected in the economy-wide labor force, but have not accounted for the dispersion effect. The next example 2.1 will illustrate the dispersion effect.

Example 2.1:

Task: In the example 2, an adjustment was made to the earnings to reflect higher covered worker rates for men in that age group. A further step is to reflect the dispersion effect by giving the different weights to the earnings below or above the median earnings in the same age group.

This example illustrates the calculation of earnings to be further modified for dispersion effect after the earnings were assigned to a newly entitled retired male record with zero taxable earnings in the base year.

Information given:

- Newly entitled retired male beneficiaries age 66 represented in the 2017 sample are age 30 in the base period 1981. His year of birth is 1951.
- Worker retires at age 66 in 2050 (year of birth 1984). Age 30, earnings would occur in 2014
- Earnings for record #30 for 1981 = \$5,246.
- Median earnings for men age 30 in 1981 = \$15,293.
- Dispersion factor for earnings below median (DFB,male,1981,1951) = 0.6587. This reflects dispersion from 1981 to 2010 (the last year of measured dispersion) since the projected worker born in 1984 would be age 30 in 2014, later than 2010.
- Dispersion factor for earnings above median (DFA,male,1981,1951) = 0.8473. Again this reflects dispersion from 1981 to 2010.

Calculations:

1. The earnings for this record number #30 after being assigned earnings (from example 2) is \$5,246.
2. Compare the earnings (\$5,246) of this record with median earnings (\$15,293) from the same age group in the base earnings year 1981. The dispersion factor for earnings below median (DFB) is chosen.

3. The amount in *step 1* (\$5,246) would be multiplied by dispersion factor (DFB,male,1981,1951), which is given as 0.6587. This yields the amount of earnings adjusted to record number 44897. Thus, $Earnings(30, 1981) = .6587 * \$5,246$ which equals \$3,455.

Note that, at this stage of the process, the average taxable earnings have been computed using projected earnings after dispersion and after adjustment for changes in the wage base. Adjustments to earnings for the earnings experience in the CWHS have not yet been applied. See the earlier section “Earnings Experience in the CWHS” and example 4 below for a description of this adjustment process.

Example 3:

Task: The AWARDS subprocess estimates projected values of Average Taxable Earnings by age and sex using the values¹¹ in the 2018 CWHS file supplied by the Economic subprocess as the base year on which to build our projections.

This example illustrates the calculation of the projected Average Taxable Earnings of the CWHS in 2022 for 42 year old women, $CWHSATE_{42,female}(2022)$. We will be using the number of female covered workers age 42 and the total taxable earnings for women age 42 as given the in 2018 CWHS data.

Information given:

- The average taxable earnings calculated from last 5 historical years in the CWHS for women age 42 are:

$$\begin{aligned} ATE_cwhs(2014) &= 38,682.48 \\ ATE_cwhs(2015) &= 40,693.53 \\ ATE_cwhs(2016) &= 41,259.98 \\ ATE_cwhs(2017) &= 43,158.24 \\ ATE_cwhs(2018) &= 44,478.19 \end{aligned}$$

- The economy-wide average taxable earnings for women age 42 (after adjustment for bias factor = 1.0018 for TR2021) are:

$$\begin{aligned} ATE_econ(2014) &= 37,119.14 \\ ATE_econ(2015) &= 38,205.67 \\ ATE_econ(2016) &= 38,751.94 \\ ATE_econ(2017) &= 40,234.46 \\ ATE_econ(2018) &= 41,502.87 \end{aligned}$$

¹¹ Because not all earnings are posted for the most recent years for a given CWHS file, adjustment factors, based on historical trends, are applied by the Economic subprocess to complete these earnings. For the 2018 CWHS, adjustment factors were applied to data in years 2014 through 2018.

$ATE_econ(2019) = 43,136.68$
 $ATE_econ(2020) = 43,963.34$
 $ATE_econ(2021) = 46,261.22$
 $ATE_econ(2022) = 47,958.43$

Note: The above ATE values reflect an adjustment from the aggregate values supplied by the Economics group for all ages, to make them consistent with the historical CWHS ATE values for ages 15-80.

Calculations:

1. The normalized ATE for women age 42 in 2022, using 2014-2018 CWHS data brought forward to 2022 ATE levels. The calculation is shown in 2 steps:

Step a

$$ATE_cwhs(2014)*ATE_econ(2022)/ATE_econ(2014) + ATE_cwhs(2015)*ATE_econ(2022)/ATE_econ(2015) + \dots + ATE_cwhs(2018)*ATE_econ(2022)/ATE_econ(2018)$$

Step b

Sum of Step a (\$254,962), then take the average (\$254,962 / 5 = \$50,992)

2. Repeat the same steps in (1) to calculate the normalized ATE for each age (15 to 80). Multiply the normalized ATE by number of econ-wide covered workers by age and sex in year 2022. This gives us the taxable earnings. Sum up the taxable earnings for all ages and sex (both male and female). Then divide the total taxable earnings (age and sex combined) by total number of economy-wide covered workers in 2022 (age and sex combined). The resulting ATE for 2022 using economy-wide covered workers and 2014-2018 CWHS data brought forward to 2022 ATE levels is \$47,698.
3. The value from *step 1* (\$50,992) is multiplied by a final multiplicative adjustment factor to match the aggregate ATE level produced by the Economic subprocess. The adjustment is a ratio of economy-wide ATE to CWHS projected aggregate average taxable earnings (\$47,958 / \$47,698 from step 2 above) yielding for women age 42 in 2022 an ATE value of \$51,270.33.

Example 3.1:

This example illustrates the calculation of the projected Average Taxable Earnings in 2005 for 40-year old women who retire at age 65 in 2030. We will be using the

historical and projected ATE and distribution of retired workers in the projection year to calculate the weighted CWHS ATE for that year.

Information given:

- The historical ATE from CWHS sample for women age 40 in 1992, who retire at age 65 in sample year 2017, is listed as follows:

$$\text{ATE}(\text{female}, 1992, 40) = 18,968$$

- The projected ATE, using historical CWHS sample data as a guide and projected covered worker data from the Economics group for age 40 in 2005, who retire at age 65 in sample year 2030, is listed as follows:

$$\text{ATE}(\text{female}, 2005, 40) = 29,826$$

- Average wage index in 1992 = \$22,935.42
- Average wage index in 2005 = \$36,952.94
- Sample average taxable earnings for age 40 in 1992 = \$19,157

Calculations:

1. The ratio of AWI (1992) to AWI (2005) = .6207 (\$22,935/\$36,953)
2. Calculate the projected ATE (CWHS sample) for women age 40 in 2005 (in 1992 dollars). Multiply the projected ATE by the ratio in step (1) = \$18,512 (\$29,826*.6207)
3. The adjusted sample ATE in 2005 (in 1992 dollar) = sample ATE in 1992 * projected ATE (CWHS sample) in 2005 in 1992 dollars from step (2) / historical ATE (CWHS sample) in 1992 = $19,157 * 18,512 / 18,968 = \$18,696$.

Example 4:

Task: In projecting the 2017 sample of newly entitled female beneficiaries to represent newly entitled female beneficiaries in 2030, the projected Average Taxable Earnings of women age 40 (year of birth 1952) in the sample (ATE_t)¹² for year $t = 2005$ must be adjusted by an amount, $\delta(2005)$, to meet a targeted Average Taxable Earnings(ATE_t') for 2005.

This example illustrates the calculation of $\delta(2005)$ for the female cohort retiring at age 65 in the projection year 2030. The value $\delta(2005)$ is the dollar amount in which the average annual earnings levels are adjusted for women age 40 in the year 2005 and

¹² The average taxable earnings have been computed using projected earnings adjusted for changes in the wage base and for changes in covered worker rates.

retiring in 2030. We will be comparing this group of women age 40 in the projection 2005 year with its counterpart of women age 40 in the base year 1992.

Information given:

- A cohort of newly entitled retired female beneficiaries retiring at ages 65 represented in the 2017 sample is age 40 in the base year, 1992, and the counterpart group of women retiring in 2030 is age 40 in the year 2005.
- Based on the 2017 sample, the average taxable earnings for women age 40 in the base year 1992 is \$18,696.
- For a sample projected to be retiring in 2030, the average taxable earnings (ATE_f) for women age 40 in the year 2005 is \$18,941¹³, after applying adjustments to the records' earning levels for changes in the wage base, covered worker rates, and dispersion effect.
- The average taxable earnings (ATE_f') of future sample (2030) for women age 40 in 2005 as shown in Example 3.1 is \$18,696.

Calculations:

- The difference in ATE_f' (\$18,696) and ATE_f (\$18,941) yields the $\delta(2005)$ value $-\$245$ ($\$18,696 - \$18,941$).

Example 5:

Task: In projecting the 2017 sample of newly entitled female OAB beneficiaries to represent newly entitled female OAB beneficiaries in 2030, for year $t = 2005$, $\delta(2005)$ is negative indicating an adjustment to earnings histories is needed to reflect lower average taxable earnings for women age 40 for the year 2005.

This example illustrates the calculation of the ratio (2005) in projection year 2005 for the women retiring at age 65 in the projection year 2030. The value, ratio (2005), is the adjustment ratio that will be applied to women age 40 projected covered earnings in 2005 in order to achieve the targeted Average Taxable Earnings of this cohort for 2005.

Information given:

- Earnings in the year 2005 for the newly entitled female beneficiaries retiring at age 65 in 2030 is the counterpart corresponding to earnings in the base year 1992 for the newly entitled female beneficiaries retiring at age 65 in the 2017 sample.
- The targeted average taxable earnings ATE_{2005}' for the year 2005 is \$18,696 (shown in Example 4).

¹³ Amount is in 1992 dollars, 'sample year dollars'.

- For newly entitled women retiring at age 65 in 2030, the average taxable earnings (ATE_{2005}) for the year 2005 is \$18,941¹⁴ (shown in Example 4).
- $\delta(2005)$, the difference in the targeted average taxable earnings ATE_{2005} ' and ATE_{2005} , is calculated to be -\$245 (\$18,696 - \$18,941).
- α is a subtractive constant to the ratio(2005). For women age 40 (1952 birth cohort in the 2017 sample) OABs, when $\delta(t)$ is negative, the constant α (birth cohort 1952, age 40 in 1992) is .003.

Calculations:

- The ratio(2005) multiplied to the covered earnings in 2005 for women age 40 retiring in 2030 is $1 + \delta(2005) / ATE_{2005} + \alpha = (1 + (-\$245 / \$18,941) - 0.003)$, or .9841.

Example 6:

Task: The AWARDS subprocess calculates the Average Indexed Monthly Earnings (AIME) of each beneficiary in the sample. The AIME values are then divided into 30 intervals.

This example illustrates the division of a possible AIME value into intervals.

Information given:

- An OAB beneficiary retired at age 64 in 2017
- This OAB beneficiary is record #150000 in the sample ($r = 150000$)
- The AIME for this individual is \$3,000
- The initial eligibility year is 2015, the year the individual turned age 62.
- The length of each interval (bp_n) in 1979 dollars is given in Equation 4.2.3. The length of each interval in 2015 dollars is given by the equation

$$bp_n(r) = bp_n * AWI(2013) / AWI(1977)$$

where bp_n is the length of interval n in 1979 dollars

- The average wage index (AWI) for year 2013 is \$44,888.16
- The AWI for year 1977 is \$9,779.44
- When converting the intervals from 1979 dollars to 2015 dollars, there is a 2-year lag in AWI values.
- $AIME_n(150000)$ is the AIME value in interval n for Record #150000

¹⁴ Amount is in 1992 dollars, 'sample year dollars'.

Calculations:

- The AIME for this individual (\$3,000) is compared to the indexed intervals. It falls within the 14th interval.
- The AIME₁₅ is the residual of \$3,000 subtracting the cumulative indexed bend points up to 13th interval (\$2,685.15). The AIME for this individual in 14th interval is \$314.85
- AIME_n (150000) for interval 1 through 13 equals bp_n (150000) for the corresponding intervals, such that PAP_n = AIME_n / bp_n = 1 for these intervals
- AIME₁₄(150000) = \$348.71, such that PAP₁₄ = \$314.85 / \$459.01 = 0.6859
- AIME_n (150000) for interval 15 through 30 equals 0, such that PAP_n = 0 for these intervals.
- The following table details these results.

n	bp _n in 1979 dollars	bp _n (r) in 2015 dollars	$\sum_{k=1}^n$ bp _n (r) in 2015 dollars	AIME _n (r) in 2015 dollars	PAP _n in 2015 dollars
1	\$45	\$206.55	\$206.55	\$206.55	1
2	45	206.55	413.10	206.55	1
3	45	206.55	619.65	206.55	1
4	45	206.55	826.20	206.55	1
5	45	206.55	1,032.75	206.55	1
6	45	206.55	1,239.30	206.55	1
7	45	206.55	1,445.85	206.55	1
8	45	206.55	1,652.40	206.55	1
9	45	206.55	1,858.95	206.55	1
10	45	206.55	2,065.50	206.55	1
11	45	206.55	2,272.05	206.55	1
12	45	206.55	2,478.60	206.55	1
13	45	206.55	2,685.15	206.55	1
14	100	459.01	3,144.16	314.85	0.6859
15	100	459.01	3,603.17	0	0
16	100	459.01	4,062.18	0	0
17	100	459.01	4,521.19	0	0
18	100	459.01	4,980.20	0	0
19	200	918.01	5,898.21	0	0
20	200	918.01	6,816.22	0	0

21	200	918.01	7,734.23	0	0
22	200	918.01	8,652.24	0	0
23	200	918.01	9,570.25	0	0
24	200	918.01	10,488.26	0	0
25	200	918.01	11,406.27	0	0
26	200	918.01	12,324.28	0	0
27	200	918.01	13,242.29	0	0
28	200	918.01	14,160.30	0	0
29	1000	4,590.05	18,750.50	0	0
30	1000	4,590.05	23,340.40	0	0

Appendix 4.2-3 Shuttling Method

The Awards program works with a sample of workers newly entitled in the sample year of 2016. The sample contains the age-sex distribution of newly entitled worker beneficiaries in the sample year. The beneficiaries area (subprocess 3.3) projects that the age-sex distribution of newly entitled worker beneficiaries will vary throughout the long-range period. In past years, the “Cost” area (subprocess 4.3) adjusted the Awards output to align it with the age-sex distribution from subprocess 3.3. Starting with the 2017 Trustees Report, the alignment adjustment occurs in the Awards program. This alignment adjustment is referred to as the “shuttling method” as it “shuttles” some workers retiring at one age in the sample to different ages in the projection years.

The shuttling method consists of determining an array for each projection year, which determines what proportion of retirees at each age will “shuttle” to retirement at older ages, or in some rare instances, to retirement at younger ages. For example, it allocates the proportion of sample year 2017 age 62 retirees into later retirement ages 63 through 70 in the projection year. The same process applies to allocate the proportion of age 63 retirees into revised retirement ages 64 through 70, and so on, through age 70. Shuttling backward to the earlier retirement age is rare but possible. For example, when appropriate, the method would allocate the proportion of sample year 2017 age 66 retirees into earlier retirement ages 62 through 65. For a specific age, then, the final PAPs equal a weighted average of these shuttled PAPs, with the PAPs for revised retirement ages potentially reflecting additional earnings when the revised retirement age is later.

The frequency of earnings in the additional years is set to match projected covered worker rates by age group/sex/projection year, using randomization to achieve this match. Earnings amounts are based on cohort average taxable earnings and the individual’s relative earnings level. PAPs values are computed for each shuttling year and averaged using similar weighting factors, as previously existed in the Cost program before the 2017 Trustees Report, to obtain shuttled PAPs values.

Example 7:

Task: The AWARDS subprocess illustrates the process in determining the matrix of shuttling at age 62 to 70 in the 2017 sample and how the matrix is used in providing adjusted PAPs values by single retirement age.

Information given:

Consider the following example. In this example, the projection year is 2024, for male retired workers.

We start with the age distribution from the OASI retired worker beneficiary and the distribution from Awards in the sample year 2017.

The *oasi_age_dist_samp* vector (initial sex-age distribution in 2017 from sample) is as follows.

	62	63	64	65	66	67	68	69	70
Male	0.2708	0.1117	0.0621	0.1041	0.3175	0.0429	0.0224	0.0170	0.0515

The *oasi_age_dist_rsb* vector in sample year 2017, is as follows.

	62	63	64	65	66	67	68	69	70
Male	0.2662	0.1134	0.0561	0.0955	0.2986	0.0591	0.0254	0.0159	0.0698

This alignment, applicable to beneficiary age distributions in projection years, is processed by the following steps.

- a) Take the difference between two distributions (*oasi_age_dist_rsb* vector – *oasi_age_dist_samp* vector) in the sample year.

	62	63	64	65	66	67	68	69	70
Male	-0.0046	0.0017	-0.0060	-0.0086	-0.0189	0.0162	0.0030	-0.0011	0.0183

The original projected OASI beneficiary distribution (*oasi_age_dist_rsb*) in year 2024 is:

	62	63	64	65	66	67	68	69	70
Male	0.1692	0.1058	0.0724	0.1041	0.1691	0.1977	0.0445	0.0355	0.1017

- b) To align the projected OASI beneficiary distribution (*oasi_age_dist_rsb*) with the

Awards sample distribution (*oasi_age_dist_samp*), we subtract the differences in Step a) from the *oasi_age_dist_samp* vector. The aligned age-sex distribution (*oasi_age_dist_rsb*) for projection year 2024 is as follows.

	62	63	64	65	66	67	68	69	70
Male	0.1738	0.1040	0.0784	0.1127	0.1880	0.1815	0.0415	0.0366	0.0835

The matrix *oads*, computed in this subprocess (4.2) is as follows. An explanation of how this matrix is generated appears below.

<i>ageentRSB</i> \ <i>ageentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
64	0.0000	0.0784	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0784
65	0.0000	0.0263	0.0621	0.0243	0.0000	0.0000	0.0000	0.0000	0.0000	0.1127
66	0.0000	0.0000	0.0000	0.0798	0.1082	0.0000	0.0000	0.0000	0.0000	0.1880
67	0.0000	0.0000	0.0000	0.0000	0.1815	0.0000	0.0000	0.0000	0.0000	0.1815
68	0.0000	0.0000	0.0000	0.0000	0.0278	0.0137	0.0000	0.0000	0.0000	0.0415
69	0.0000	0.0000	0.0000	0.0000	0.0000	0.0292	0.0074	0.0000	0.0000	0.0366
70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0150	0.0170	0.0515	0.0835
Total	0.2708	0.1117	0.0621	0.1041	0.3175	0.0429	0.0224	0.0170	0.0515	1.0000

Note that the column total is *oasi_age_dist_samp* and the row total is *oasi_age_dist_rsb*. The *oads* matrix is determined using these row and column sum constraints. The nonzero entries of the *oads* matrix zigzag down and to the right. To demonstrate the process of determining the values for the elements of the matrix *oads*, the irrelevant cells are zeroed out for simplicity.

1. Starting at the upper left hand corner, the lesser of *oasi_age_dist_samp* and *oasi_age_dist_rsb* is placed there. So $oads(62,62) = 0.1738$.

<i>ageentRSB</i> \ <i>ageentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738

2. Move one spot down to $oads(63,62)$. By the column sum constraint, we are forced to have $oads(63,62) = 0.2708 - 0.1738 = 0.0970$ to meet age 62 column sum constraint. Since the difference (0.0970) is less than the row sum constraint (0.1040) for age 63, we want the row sum to equal 0.1040, the difference will be set for $oads(63,63) = 0.1040 - 0.0970 = 0.0070$

<i>ageentRSB</i> \ <i>ageentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
Total	0.2708	0.1117	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

3. With the row sum constraint for age 63 met, we move one spot down to $oads(64,63)$. Since $oasi_age_dist_rsb(64) = 0.0784 < 0.1117$ ($oasi_age_dist_samp(63)$), we are forced to have the entry $oads(64,63) = 0.0784$

<i>ageentRSB</i> \ <i>ageentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
64	0.0000	0.0784	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0784
Total	0.2708	0.1117	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

4. The row sum constraint for age 64 is now met, we move one spot down to $oads(65,63)$. Since $0.1117 - 0.0070 - 0.0784 = 0.0263$, which is less than 0.1127 ($oasi_age_dist_rsb(65)$), we are forced to have $oads(65,63) = 0.0263$ to meet the age 63 column sum constraint.

<i>ageentRSB</i> \ <i>ageentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
64	0.0000	0.0784	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0784
65	0.0000	0.0263	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1127
Total	0.2708	0.1117	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

5. The column sum constraint for age 63 is now met, we move one spot to the right $oads(65,64)$. Since $oasi_age_dist_samp(64) = 0.0621 < 0.1127$ ($oasi_age_dist_rsb(65)$), the entry $oads(65,64) = 0.0621$ to meet the age 64 column sum constraint.

<i>ageentRSB</i> \ <i>ageentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
64	0.0000	0.0784	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0784
65	0.0000	0.0263	0.0621	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1127
Total	0.2708	0.1117	0.0621	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

6. The column sum constraint for age 64 is met, we move one spot to the right $oads(65,65)$. Since $oasi_age_dist_rsb(65) = 0.1127 > 0.1041$ ($oasi_age_dist_samp(65)$), we are forced to have the entry $oads(65,65) = 0.1127 - 0.0263 - 0.0621 = 0.0243$ to meet the age 64 row sum constraint.

<i>ageentRSB \ ageentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
64	0.0000	0.0784	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0784
65	0.0000	0.0263	0.0621	0.0243	0.0000	0.0000	0.0000	0.0000	0.0000	0.1127
Total	0.2708	0.1117	0.0621	0.1041	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

7. The row sum constraint for age 65 is now met, but not the column constraint for age 65, we move down one spot to $oads(66,65)$. Since $oasi_age_dist_samp(65) = 0.1041 < 0.1880$ ($oasi_age_dist_rsb(66)$), the entry $oads(65,66) = 0.1041 - 0.0243 = 0.0798$ to meet the age 64 column sum constraint.

<i>ageentRSB \ ageentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
64	0.0000	0.0784	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0784
65	0.0000	0.0263	0.0621	0.0243	0.0000	0.0000	0.0000	0.0000	0.0000	0.1127
66	0.0000	0.0000	0.0000	0.0798	0.0000	0.0000	0.0000	0.0000	0.0000	0.1880
Total	0.2708	0.1117	0.0621	0.1041	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

8. The column sum constraint for age 65 is met, but not the row constraint for age 66, we move one spot to the right to $oads(66,66)$. Since $oasi_age_dist_rsb(66) = 0.1880 < 0.3175$ ($oasi_age_dist_samp(65)$), the entry $oads(66,66) = 0.1880 - 0.0798 = 0.1082$ is forced to meet the age 66 row sum constraint.

<i>ageentRSB \ ageentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
64	0.0000	0.0784	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0784
65	0.0000	0.0263	0.0621	0.0243	0.0000	0.0000	0.0000	0.0000	0.0000	0.1127
66	0.0000	0.0000	0.0000	0.0798	0.1082	0.0000	0.0000	0.0000	0.0000	0.1880
Total	0.2708	0.1117	0.0621	0.1041	0.3175	0.0000	0.0000	0.0000	0.0000	0.0000

9. The row sum constraint for age 66 is now met, we move one spot down to $oads(67,66)$. Since $oasi_age_dist_rsb(67) = 0.1815 < 0.3175$ ($oasi_age_dist_samp(66) - 0.1082$), the entry is forced to meet the row constraint $oads(66,66) = 0.1815$

<i>ageentRSB \ ageentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
64	0.0000	0.0784	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0784
65	0.0000	0.0263	0.0621	0.0243	0.0000	0.0000	0.0000	0.0000	0.0000	0.1127
66	0.0000	0.0000	0.0000	0.0798	0.1082	0.0000	0.0000	0.0000	0.0000	0.1880
67	0.0000	0.0000	0.0000	0.0000	0.1815	0.0000	0.0000	0.0000	0.0000	0.1815
Total	0.2708	0.1117	0.0621	0.1041	0.3175	0.0000	0.0000	0.0000	0.0000	0.0000

10. The row sum constraint for age 67 is now met, but not the column sum constraint for age 66. We move down one spot to $oads(68, 66)$. The entry $oads(68, 66) = 0.3175 - 0.1082 - 0.1815 = 0.0278$ is forced to meet the age 66 column sum constraint.

<i>ageentRSB \ ageentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
64	0.0000	0.0784	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0784
65	0.0000	0.0263	0.0621	0.0243	0.0000	0.0000	0.0000	0.0000	0.0000	0.1127
66	0.0000	0.0000	0.0000	0.0798	0.1082	0.0000	0.0000	0.0000	0.0000	0.1880
67	0.0000	0.0000	0.0000	0.0000	0.1815	0.0000	0.0000	0.0000	0.0000	0.1815
68	0.0000	0.0000	0.0000	0.0000	0.0278	0.0000	0.0000	0.0000	0.0000	0.0415
Total	0.2708	0.1117	0.0621	0.1041	0.3175	0.0000	0.0000	0.0000	0.0000	0.0000

11. The column sum constraint for age 66 is now met, but not the row sum constraint for age 68. We move one spot to the right $oads(68, 67)$. By setting $oads(68, 67) = 0.0415 - 0.0278 = 0.0137$ to meet the row sum constraint 0.0415.

<i>ageentRSB \ ageentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
64	0.0000	0.0784	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0784
65	0.0000	0.0263	0.0621	0.0243	0.0000	0.0000	0.0000	0.0000	0.0000	0.1127
66	0.0000	0.0000	0.0000	0.0798	0.1082	0.0000	0.0000	0.0000	0.0000	0.1880
67	0.0000	0.0000	0.0000	0.0000	0.1815	0.0000	0.0000	0.0000	0.0000	0.1815
68	0.0000	0.0000	0.0000	0.0000	0.0278	0.0137	0.0000	0.0000	0.0000	0.0415
Total	0.2708	0.1117	0.0621	0.1041	0.3175	0.0429	0.0000	0.0000	0.0000	0.0000

12. Now the age 68 row sum constraint is met, we move one spot down to $oads(69, 67)$ to meet the column sum constraint (0.0429) by setting $oads(69, 67) = 0.0429 - 0.0137 = 0.0292$

<i>ageentRSB \ agentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
64	0.0000	0.0784	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0784
65	0.0000	0.0263	0.0621	0.0243	0.0000	0.0000	0.0000	0.0000	0.0000	0.1127
66	0.0000	0.0000	0.0000	0.0798	0.1082	0.0000	0.0000	0.0000	0.0000	0.1880
67	0.0000	0.0000	0.0000	0.0000	0.1815	0.0000	0.0000	0.0000	0.0000	0.1815
68	0.0000	0.0000	0.0000	0.0000	0.0278	0.0137	0.0000	0.0000	0.0000	0.0415
69	0.0000	0.0000	0.0000	0.0000	0.0000	0.0292	0.0000	0.0000	0.0000	0.0366
Total	0.2708	0.1117	0.0621	0.1041	0.3175	0.0429	0.0000	0.0000	0.0000	0.0000

13. The column sum constraint for age 67 is now met, we move one spot to the right $oads(69,68)$ to meet the row constraint for age 69 by setting $oads(69,68) = 0.0366 - 0.0292 = 0.0074$

<i>ageentRSB \ agentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
64	0.0000	0.0784	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0784
65	0.0000	0.0263	0.0621	0.0243	0.0000	0.0000	0.0000	0.0000	0.0000	0.1127
66	0.0000	0.0000	0.0000	0.0798	0.1082	0.0000	0.0000	0.0000	0.0000	0.1880
67	0.0000	0.0000	0.0000	0.0000	0.1815	0.0000	0.0000	0.0000	0.0000	0.1815
68	0.0000	0.0000	0.0000	0.0000	0.0278	0.0137	0.0000	0.0000	0.0000	0.0415
69	0.0000	0.0000	0.0000	0.0000	0.0000	0.0292	0.0074	0.0000	0.0000	0.0366
Total	0.2708	0.1117	0.0621	0.1041	0.3175	0.0429	0.0224	0.0000	0.0000	0.0000

14. The row sum constraint for age 69 is now met, we move one spot down to $oads(70,68)$ to meet the column sum constraint (0.0224) by setting $oads(70,68) = 0.0224 - 0.0074 = 0.0150$

<i>ageentRSB \ agentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
64	0.0000	0.0784	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0784
65	0.0000	0.0263	0.0621	0.0243	0.0000	0.0000	0.0000	0.0000	0.0000	0.1127
66	0.0000	0.0000	0.0000	0.0798	0.1082	0.0000	0.0000	0.0000	0.0000	0.1880
67	0.0000	0.0000	0.0000	0.0000	0.1815	0.0000	0.0000	0.0000	0.0000	0.1815
68	0.0000	0.0000	0.0000	0.0000	0.0278	0.0137	0.0000	0.0000	0.0000	0.0415
69	0.0000	0.0000	0.0000	0.0000	0.0000	0.0292	0.0074	0.0000	0.0000	0.0366
70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0150	0.0000	0.0000	0.0835
Total	0.2708	0.1117	0.0621	0.1041	0.3175	0.0429	0.0224	0.0000	0.0000	0.0000

15. The column sum constraint is now met, we move one spot to the right $oads(70,69)$ to

meet the row constraint (0.0835). Since $oasi_age_dist_samp(69) = 0.0170 < oasi_age_dist_samp(70) = 0.0835$, we are forced to have $oads(70,69) = 0.0170$ to meet the column sum constraint for age 69.

<i>ageentRSB \ ageentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
64	0.0000	0.0784	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0784
65	0.0000	0.0263	0.0621	0.0243	0.0000	0.0000	0.0000	0.0000	0.0000	0.1127
66	0.0000	0.0000	0.0000	0.0798	0.1082	0.0000	0.0000	0.0000	0.0000	0.1880
67	0.0000	0.0000	0.0000	0.0000	0.1815	0.0000	0.0000	0.0000	0.0000	0.1815
68	0.0000	0.0000	0.0000	0.0000	0.0278	0.0137	0.0000	0.0000	0.0000	0.0415
69	0.0000	0.0000	0.0000	0.0000	0.0000	0.0292	0.0074	0.0000	0.0000	0.0366
70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0150	0.0170	0.0000	0.0835
Total	0.2708	0.1117	0.0621	0.1041	0.3175	0.0429	0.0224	0.0170	0.0000	0.0000

16. The column sum constraint for age 69 is now met, we move one spot to the right $oads(70,70)$ to meet the row sum constraint (0.0835) by setting the last entry $oads(70,70) = 0.0835 - 0.0150 - 0.0170 = 0.0515$

<i>ageentRSB \ ageentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.1738	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1738
63	0.0970	0.0070	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1040
64	0.0000	0.0784	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0784
65	0.0000	0.0263	0.0621	0.0243	0.0000	0.0000	0.0000	0.0000	0.0000	0.1127
66	0.0000	0.0000	0.0000	0.0798	0.1082	0.0000	0.0000	0.0000	0.0000	0.1880
67	0.0000	0.0000	0.0000	0.0000	0.1815	0.0000	0.0000	0.0000	0.0000	0.1815
68	0.0000	0.0000	0.0000	0.0000	0.0278	0.0137	0.0000	0.0000	0.0000	0.0415
69	0.0000	0.0000	0.0000	0.0000	0.0000	0.0292	0.0074	0.0000	0.0000	0.0366
70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0150	0.0170	0.0515	0.0835
Total	0.2708	0.1117	0.0621	0.1041	0.3175	0.0429	0.0224	0.0170	0.0515	1.0000

To obtain the normalized shuttle weight w matrix, normalize the rows from step (16) by dividing by the row sum.

<i>ageentRSB \ agentAWD</i>	62	63	64	65	66	67	68	69	70
62	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
63	0.9327	0.0673	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
64	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
65	0.0000	0.2334	0.5510	0.2156	0.0000	0.0000	0.0000	0.0000	0.0000
66	0.0000	0.0000	0.0000	0.4245	0.5775	0.0000	0.0000	0.0000	0.0000
67	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
68	0.0000	0.0000	0.0000	0.0000	0.6699	0.3301	0.0000	0.0000	0.0000
69	0.0000	0.0000	0.0000	0.0000	0.0000	0.7978	0.2022	0.0000	0.0000
70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1796	0.2036	0.6168

With these normalized weights, final PAPs are computed for each age (row) as a weighted average of the PAPs, shuttled and non-shuttled, for the applicable individual ages with non-zero values in each row. Going down the row of the above matrix, then:

- Age 62 final PAPs = age 62 original (unshuttled) PAPs
- Age 63 final PAPs = .9327 * age 62 PAPs shuttled to age 63 + .0673*age 63 original PAPs
- Age 64 final PAPs = age 63 PAPs shuttled to age 64
- Age 65 final PAPs = 0.2334 * age 63 PAPs shuttled to age 65 + 0.5510 * age 64 PAPs shuttled to age 65 + 0.2156 * age 65 original PAPs
- Age 66 final PAPs = + .4245 * age 65 PAPs shuttled to age 66 + .5775 * age 66 original PAPs
- Age 67 final PAPs = age 66 original PAPs
- Age 68 final PAPs = .6699 * age 66 PAPs shuttled to age 68 + .3301 * age 67 PAPs shuttled to age 68
- Age 69 final PAPs = .7978 * age 67 PAPs shuttled to age 69 + .2022 * age 68 PAPs shuttled to age 69
- Age 70 final PAPs = .1796 * age 68 PAPs shuttled to age 70 + .2036 *age 69 PAPs shuttled to age 70 + .6168 * age 70 original PAPs

When PAPs are shuttled from a given age to age + x, all sample cases at the given original age have their AIME recalculated for retirement at age + x, with potential x additional earnings years included.

4.3. Cost

4.3.a. Overview

The COST subprocess projects the trust fund operations for each year of the long-range 75-year period. The COST subprocess projects the income and cost for each trust fund (OASI and DI). The two components of non-interest income are payroll tax contributions and taxation of benefits.¹ The other component of income is interest earned on the trust fund asset reserves. The three components of cost are scheduled benefits, administrative expenses, and the railroad interchange. Each of these components is projected for each trust fund (OASI and DI). The end-of-year assets is computed by taking the beginning-of-year assets (*ASSETS*), adding payroll contributions (*CONTRIB*), taxation of benefits (*TAXBEN*), and interest income (*INT*), and subtracting scheduled benefits (*BEN*), administrative expenses (*ADM*), and the railroad interchange (*RR*).

Equations 4.3.1 through 4.3.6 outline this overall structure and sequence.

$$CONTRIB = CONTRIB(\cdot) \quad (4.3.1)$$

$$BEN = BEN(\cdot) \quad (4.3.2)$$

$$TAXBEN = TAXBEN(\cdot) \quad (4.3.3)$$

$$ADM = ADM(\cdot) \quad (4.3.4)$$

$$RR = RR(\cdot) \quad (4.3.5)$$

$$INT = INT(\cdot) \quad (4.3.6)$$

$$ASSETS_{EOY} = ASSETS_{BOY} + CONTRIB + TAXBEN + INT - BEN - ADM - RR$$

The COST subprocess produces annual values that help assess the financial status of the OASI, DI, and combined funds. These include the annual income rate (*ANN_INC_RT*), annual cost rate (*ANN_COST_RT*), and trust fund ratio (*TFR*) as outlined below.

$$ANN_INC_RT = ANN_INC_RT(\cdot) \quad (4.3.7)$$

$$ANN_COST_RT = ANN_COST_RT(\cdot) \quad (4.3.8)$$

$$TFR = TFR(\cdot) \quad (4.3.9)$$

The COST subprocess also produces summarized values. These values are computed for the entire 75-year projection periods, as well as 25- and 50-year periods. These include

¹ As noted throughout the Trustees Report a third component of non-interest income is reimbursements from the General Fund of the Treasury. In the cost program such reimbursements are simply treated as payroll tax contributions.

the actuarial balance (ACT_BAL), unfunded obligation (UNF_OBL), summarized income rate ($SUMM_INC_RT$), summarized cost rate ($SUMM_COST_RT$), and closed group unfunded obligation ($CLOSEDGRP_UNFOBL$).

$$ACT_BAL = ACT_BAL(\cdot) \quad (4.3.10)$$

$$UNF_OBL = UNF_OBL(\cdot) \quad (4.3.11)$$

$$SUMM_INC_RT = SUMM_INC_RT(\cdot) \quad (4.3.12)$$

$$SUMM_COST_RT = SUMM_COST_RT(\cdot) \quad (4.3.13)$$

$$CLOSEDGRP_UNFOBL = CLOSEDGRP_UNFOBL(\cdot) \quad (4.3.14)$$

The following notation is used throughout this documentation:

- ni represents the first year of the projection period-2021 for the 2021 TR
- $ni+74$ represents the final year of the projection period-2095 for the 2021 TR
- nf represents the last year the cost program will project-2100 for the 2021 TR
- $nim1$ is equal to $ni-1$
- $nim2$ is equal to $ni-2$
- ns is equal to $ni+9$
- $nbase$, the year of the awards sample, is equal to 2017

4.3.b. *Input Data*

Data received as input from the short-range office are presented first. Then data from long range and all other sources are identified separately for each equation.

Short-range OCACT Data

- 1) Estimates for the first ten years of the projection period for the first six equations (4.3.1 through 4.3.6) mentioned above, including bond holdings by Trust Fund as of 12/31/($ni+10$).
- 2) Assets at the beginning of year ni .

All of this information is updated annually.

Long-range OCACT and other Data

i. Equation 4.3.1 – Tax Contributions (CONTRIB)

Economics-Process 2

- 3) Projected effective taxable payroll for years $nim1$ through nf , updated yearly

Other

- 4) Projected employee/employer payroll tax rate, by trust fund and year, for years 1981 through nf , updated as needed (e.g., as required due to legislative changes)

ii. Equation 4.3.2 – Scheduled Benefits (BEN)

Demography-Process 1

- 5) Projected number of married and divorced people in the Social Security area population by age for end of years *nim2* through 2100, updated yearly

Economics-Process 2

- 6) Historical COLA for years 1975 through *nim2*, with *nim2* updated yearly, years *ni-8* through *ni-2* used in SOSI.
- 7) Projected cost of living adjustment (COLA) for years *nim1* through *nf*, updated yearly
- 8) Historical SSA average wage index for years 1951 through *nim2*, with *nim2* updated yearly, years 1977 through *ni-2* used in SOSI
- 9) Projected percent increases in the average wage index for years *nim1* through *nf*, updated yearly

Beneficiaries-Process 3

- 10) Projected number of disabled workers newly awarded by sex, attained age, and duration from entitlement (0 through 9 and 10+) for years *ni* through *nf*, updated yearly from subprocess 3.2
- 11) Projected number of disabled workers in current-pay status by sex, age in current-pay, and duration of disability (0 through 9 and 10+) for years *nim1* through *nf*, updated yearly from subprocess 3.2
- 12) Projected number of retired worker beneficiaries in current-pay status by sex, age in current-pay, and age at entitlement for years *nim1* through *nf*, updated yearly from subprocess 3.3
- 13) Projected number of auxiliary beneficiaries (by benefit category) of retired-worker, deceased-worker, and disabled-worker beneficiaries for years *nim1* through *nf*, updated yearly from subprocesses 3.2 and 3.3
- 14) Projected number of disability insurance beneficiaries who convert to retirement insurance status upon the attainment of normal retirement age by age in current pay, for years *nim1* through *nf*, updated yearly from subprocess 3.2 and 3.3
- 15) Retired Workers 65+ by sex, and marital status (single, married, widowed, and divorced) for years *nim1* through *nf*
- 16) Retired Workers 62+ by sex, and marital status (single, married, widowed, and divorced) for years *nim1* through *nf*
- 17) Male widowed retired workers by age band (62-64, 65-69, 70-74, 75-79, 80-84, 85-89, 90-95, 95+) for years *nim1* through *nf*
- 18) Female widowed retired workers by age band (62-64, 65-69, 70-74, 75-79, 80-84, 85-89, 90-95, 95+) for years *nim1* through *nf*

Other

- 19) Adjustment factor, applied to DI worker retroactive benefit payments to align to historical data, updated yearly
- 20) Total (aggregate) PIA and MBA, not actuarially reduced, of DI male and

female workers in current payment status for years 2000 through *nim1*, with *nim1* updated yearly from the Table 1-A Supplement, years 2010 through *nim1* used in SOSI

- 21) Total (aggregate) PIA and MBA, actuarially reduced, of DI male and female workers in current payment status for years 2000 through *nim1*, with *nim1* updated yearly from the Table 1-A Supplement, years 2010 through *nim1* used in SOSI
- 22) Total (aggregate) PIA and MBA, not actuarially reduced, of newly awarded DI male and female workers for years 2000 through *nim1*, updated yearly from the Table 1-A Supplement, years 2010 through *nim1* used in SOSI
- 23) Cumulative distribution of AIME dollars for newly entitled retired-worker beneficiaries by age (62 through 70) and sex, for years *nim1* through *nf*, updated yearly from subprocess 4.2
- 24) Cumulative distribution of AIME dollars for newly entitled disabled-worker beneficiaries by age (20 through 65) and sex, for years *nim1* through *nf*, updated yearly from subprocess 4.2. Ages 15 through 19 are assumed to have the same distribution of dollars as does age 20. Future age 66 disabled workers are assumed to have the same distribution of dollars as age 65 workers have
- 25) Starting average PIA matrix for retired-worker benefits for the year *nim1*, by age at entitlement, age in current-pay and sex, updated yearly
- 26) Starting average PIA matrix for disabled-worker benefits, for the year *nim1*, by age in current-pay, duration and sex, updated yearly
- 27) Starting average PIA matrix for beneficiaries who convert from disabled worker to retirement worker status for the year *nim1*, by age in current-pay and sex, updated yearly
- 28) Starting average MBA matrix for retired-worker benefits for the year *nim1*, by age at entitlement, age in current-pay and sex, updated yearly
- 29) Starting average MBA matrix for beneficiaries who convert from disabled worker to retirement worker status for the year *nim1*, by age in current-pay and sex, updated yearly
- 30) Benefit relationships between worker and auxiliary benefits (linkages) for the year *nim1*, for all benefit categories and worker account holders of both sexes, updated yearly from *qlink21.xls*
- 31) Benefit relationships between workers and aged spouses for years *nim1* through *nf* with the effect of the ‘Bipartisan Budget Act of 2015’ on “claiming strategies” taken into account
- 32) Retroactive payment loading factors for auxiliary beneficiary categories for all years, for each benefit category and both sexes, updated yearly
- 33) Initial and ultimate post entitlement factors for retired workers by sex and duration updated yearly. Factors are read in such that there is a twenty-year linear phase-in from initial factors to ultimate factors.
- 34) Initial and ultimate post entitlement factors for disabled workers by sex and duration updated yearly. Factors are read in such that there is a twenty-year linear phase-in from initial factors to ultimate factors.
- 35) Initial and ultimate post entitlement factors for DI conversion workers by sex

and duration updated yearly. Factors are read in such that there is a twenty-year linear phase-in from initial factors to ultimate factors.

- 36) Initial Windfall Elimination Provision (WEP) factors by sex and age for attributed year, updated every 2 years
- 37) Ultimate WEP factors by sex read in as a percentage of the way from initial factors to one, updated every 2 years
- 38) Year in which ultimate WEP factor is reached by age at initial entitlement (62-70), updated every 2 years
- 39) Trendline by which WEP factors are phased-in from initial value to ultimate value, updated yearly
- 40) Workers' Compensation cumulative factors by duration that adjust benefits to account for decreasing offsets (i.e. – Workers Comp offsets decrease as duration increases), updated yearly
- 41) Workers' Compensation reduction factors (used in retroactive category) to reflect offsets starting and stopping in the year of DI entitlement, updated yearly
- 42) Workers' Compensation parameter to account for offsets that begin and end in the year of entitlement, updated yearly
- 43) Dual entitlement regression coefficients for the 12 dual entitlement equations (3 each for number of widows, number of widowers and the average excess amount for widows, and, 1 each for number of wives, number of husbands and the average excess amount for wives), updated yearly
- 44) Dual entitlement widower excess amount as a percent of widow excess amount, by age band and year, updated yearly
- 45) Dual entitlement husband excess amount as a percent of wife excess amount, by year, updated yearly
- 46) Dual entitlement average excess amounts and percentages of exposure population for December, year n_i-1 for wives, husbands, widows, and widowers, updated yearly
- 47) Average retired worker PIA in the last year of the projection period, by age band and sex, updated yearly
- 48) Target values for ratios relating to the twelve dual entitlement categories, updated yearly
- 49) Number of years in which the difference between the results from the regression coefficients and targeted values are phased in for the twelve dual entitlement categories, updated yearly
- 50) Historical adjustment factor for each of the twelve dual-entitlement categories, updated yearly
- 51) Adjustment factors for average retired and disabled worker benefit amounts (PIA and MBA) in current-pay at durations 0 through 5+, by sex and age, updated yearly
- 52) Number of months retroactive benefits are received by a worker who is paid retroactively in their year of entitlement, by trust fund, with retired workers further broken out by age less-than normal retirement age (NRA) or above NRA
- 53) Adjustment factors for average retired worker retroactive benefit amounts by

- sex, updated yearly
- 54) Adjustment factors for disabled worker retroactive benefit amounts in current-pay at durations 0 through 5+, by sex and age, updated yearly
- 55) Retired worker actuarial reduction factors and delayed retirement credit levels based on year of birth, based on current law (generally not updated)
- 56) Adjustment factor applied to all newly entitled DI workers to account for shifting distribution of awards by DI adjudicative level, updated yearly

iii. *Equation 4.3.3 – Taxation of Benefits*

Trust Fund Operations and Actuarial Status

- 57) Taxation of benefits as a percentage of scheduled benefits by trust fund for years $nim1$ through nf , updated yearly from subprocess #4.1

iv. *Equation 4.3.4 – Administrative Expenses*

Economics-Process 2

- 58) Average wage indexes for years $nim1$ through nf , updated yearly
- 59) Ultimate value of productivity factor for the period ni through nf updated yearly

Beneficiaries-Process 3

- 60) Total number of beneficiaries in current-pay status by trust fund for years $nim1$ through nf , updated yearly

v. *Equation 4.3.5 – Railroad Interchange*

Economics-Process 2

- 61) Increase in the average wage index for years $nim1$ through nf , updated yearly
- 62) Ultimate value of productivity factor for the period ni through nf updated yearly

Trust Fund Operations and Actuarial Status

- 63) Taxation of benefits as a percent of the amount of benefits scheduled to be paid, by trust fund for years $nim1$ through nf , updated yearly (use same factors as in equation 4.3.3)

Other input data

- 64) Nominal annual yield rate on the combined OASDI trust fund for year $nim1$
- 65) Regression coefficients to project annual prescribed interest rates, related to railroad interchange, updated annually
- 66) Ratio of railroad retirement OASI and DI average benefits to overall OASI and DI average benefits, updated yearly
- 67) Number of railroad beneficiaries (retirement and disability) for December of year $nim3$, updated yearly
- 68) Average taxable earnings in railroad employment for year $nim2$, updated

- yearly
- 69) Expected railroad new awards as a percent of the average of historical employment data, updated yearly
 - 70) Historical data on average railroad employment, 1960 through $nim2$
 - 71) Average worker benefit by sex and trust fund for December $nim1$ and December $nim2$, updated yearly
 - 72) Auxiliary loading factor by trust fund, updated yearly using 10 years of historical financial interchange benefit data
 - 73) Railroad initial mortality rate by trust fund for year $nim2$, updated yearly using 10 years of historical financial interchange benefit data
 - 74) Railroad mortality improvement rate by trust fund, updated yearly using 10 years of historical financial interchange benefit data
 - 75) Fiscal Year Railroad transfer amount in millions of dollars for year $nim2$
 - 76) Short-Range estimates for railroad administrative costs, military service adjustments, and prescribed interest rates.

vi. *Equation 4.3.6 – Interest Income*

Economics-Process 2

- 77) Annual increase in the CPI for years ni through nf , updated yearly

Trustees assumptions

- 78) Ultimate real interest rate, updated annually

Other input data

- 79) Factors for exposure to interest rate for benefits, payroll, and taxation of benefits, updated yearly
- 80) Factors for exposure to railroad interchange and administrative expenses, updated periodically
- 81) Annual effective interest rates for years $ns+1$ through nf , by trust fund, updated yearly

vii. *Equations 4.3.7 through 4.3.13 – Annual Values and Summarized Values*

All inputs for equations 4.3.7 through 4.3.13 are estimated internally in the Cost program.

viii. *Equation 4.3.14 – Closed Group Unfunded Obligation*

Demographics-Process 1

- 82) Single year population and mortality rate data for years 1941 through 2101, updated yearly

Economics-Process 2

- 83) Historical and projected single-year COLA data and average wage indexing series (AWI) data for years 1975 through 2100 (for COLA) and 1951 through 2100 (AWI), updated yearly as applicable

- 84) Historical and projected number of covered workers by single year of age 0-99 and 100+ from year ni-23 through 2100, updated yearly
- 85) Ultimate assumed annual average wage increase, wg_ult, updated yearly

Beneficiaries-Process 3

- 86) Total projected disabled workers by age for years nim1 to 2100, updated yearly
- 87) Total projected aged spouses, divorced aged spouses, surviving aged spouses and divorced surviving aged spouses by sex, single year of age (up to 95+) and for years nim1 to 2100.

Awards-Process 4

- 88) Projected number of workers and total taxable earnings by single year of age (15-80) and sex from nim1 to 2100, updated yearly

Other

- 89) Total count of beneficiaries in 20 of the 28 beneficiary categories (excluding retired workers, disabled workers, aged spouses (married, divorced, and dually entitled excess), and aged widow(er)s (married, divorced, and dually entitled excess) from the December 2020 Master Beneficiary Record (MBR)²—updated yearly
- 90) Total benefits paid in 20 of the 28 beneficiary categories (excluding retired workers, disabled workers, aged spouses (married, divorced, and dually entitled excess), and aged widow(er)s (married, divorced, and dually entitled excess) from the December 2020 MBR—updated yearly
- 91) Consumer Price Index data from 1951-1974 from Bureau of Labor Statistics (not updated)
- 92) Number of covered workers and average taxable earnings by single year of age 1-14 for years 1998-2017 from 1 percent Continuous Work History Sample (CWHS), updated yearly to include year ni-4
- 93) Number of covered workers and average taxable earnings by single year of age 81-99 for years 1998-2017 from 1 percent CWHS, updated yearly to include year ni-4
- 94) Distribution of assumed age differentials between aged spouses and workers ranging from -12 years to 15 years seniority for the worker by sex of the beneficiary, age of the beneficiary 62-74, and marital status obtained from the December 2018 MBR—updated every 3 years
- 95) Distribution of assumed age differentials between widow(er)s and workers ranging from -6 years to 12 years seniority for the man (generally not updated)
- 96) Factors to apply to the 95+ “in current pay” counts of retired workers, aged spouses, surviving aged spouses, divorced spouses, and dually entitled

² For disabled adult children of deceased workers and lump-sum beneficiaries, data were extracted from a 1- percent sample of the December 2020 MBR, mainframe dataset ACT.TAPEL.CAN1220. For the other 18 auxiliary beneficiary categories, data was extracted from the 100 percent December 2020 MBR, mainframe dataset ACT.TAPEH.MBR100.D2012.CANSORT.

spouses expanding the single age counts through 119, updated yearly

4.3.c. *Development of Output*

i. *Equation 4.3.1 – Payroll Tax Contributions (CONTRIB)*

It would be natural to estimate the payroll tax contributions by trust fund by multiplying the applicable employer/employee tax rate by effective taxable payroll. However, tax contributions are reported on a cash basis. That is, tax contribution amounts are attributed to the year in which they are actually received by the trust funds, while taxable payroll is attributed to the year in which earnings are paid. In other words, the lag between the time the tax liability is incurred and when the taxes are actually collected must be reflected. If *lag* represents the proportion of incurred payroll taxes estimated to be received by the trust fund (*tf*) in year *yr*, then tax contributions (*CONTRIB*) are given by the formula

$$\begin{aligned} \text{CONTRIB}(tf, yr) = & \text{lag} \times \text{tax rate}(tf, yr) \times \text{payroll}(yr) \\ & + (1 - \text{lag}) \times \text{tax rate}(tf, yr - 1) \times \text{payroll}(yr - 1) \end{aligned}$$

for $yr \geq ns$.

The value of *lag* is estimated from the combined OASI and DI tax contributions estimated to be collected in the final year of the short-range period, *ns*, and is given by

$$\text{lag} = \frac{\sum_{tf=1}^2 \text{CONTRIB}(tf, ns) - \sum_{tf=1}^2 (\text{taxrate}(tf, ns - 1) \times \text{payroll}(ns - 1))}{\sum_{tf=1}^2 (\text{taxrate}(tf, ns) \times \text{payroll}(ns) - \text{taxrate}(tf, ns - 1) \times \text{payroll}(ns - 1))}$$

For the first ten years of the long-range period, tax contributions are set equal to those provided by the short-range office. The same value of *lag* is used for all years, and both trust funds, thereafter.

ii. *Equation 4.3.2 – Scheduled Benefits (BEN)*

(1) Disabled-Worker Benefits

Disabled Worker Beneficiary Matrix

The number of disabled-worker beneficiaries for a given year and sex is provided from subprocess 3.2. For each projection year, two matrices are provided – one for men and one for women. The structure of each matrix is as follows:

- 11 columns. The columns are indexed by duration of disability (0-9 and 10+).

- 52 rows. These rows correspond to the age in current pay, ages 15 through 66.

The COST subprocess, however, only uses 10 durations (0-8 and 9+), and 47 ages (ages 20 through 66). This requires a manipulation of the matrix of DI beneficiaries in current-pay status from subprocess 3.2. For ages in current pay greater than or equal to 30, the duration 9 and 10+ columns of this matrix are added to give the total number of duration 9+ beneficiaries. For ages (*ag*) between 20 and 30 inclusive, the number of beneficiaries in current-pay aged *ag* and duration *ag-20* is the value provided by the DISABILITY subprocess added to the number of people in current pay aged *ag-j* and duration *ag-20* for $j=1,\dots,5$. (For example, the number of people aged 20 of duration 0 is combined with the number of people aged 15, 16, 17, 18 and 19 of duration 0; the number of people aged 21 of duration 1 is combined with the number of beneficiaries in current-payment status aged 16, 17, 18, 19, and 20 of duration 1, and so on. In other words, the five nonzero diagonals of the matrix provided by the DISABILITY subprocess are “combined with” the diagonal directly below it and then zeroed out.)

Building the Average PIA Matrix for Disabled Workers

In each projection year, the COST subprocess produces an average PIA matrix for each sex. Each matrix is a 47 by 10 matrix whose entries are the average PIA amounts of disabled worker beneficiaries whose age in current pay is indexed by the rows (ages 20 through 66) and whose duration of disability is indexed by the columns (durations 0 through 8 and 9+).

The 100 percent Master Beneficiary Record (100% MBR) extract is processed by a side model. The final product of the side model is two matrices of average PIA levels, one for men and one for women, for December *nim1* (2020 for the 2021 TR).

For a given year of the projection period, a new average PIA matrix is obtained by moving the average PIA matrix from the previous year one year forward. The next few paragraphs describe this procedure.

In general, for each age in current-pay, the age and duration are incremented by 1 and the previous PIA amount is given a cost of living adjustment. In addition, the beneficiaries are given a workers’ compensation adjustment and a post-entitlement adjustment. For each duration $j = 0, 1, \dots, 7$, and 8+ and sex, let the workers’ compensation offset factor be denoted $wkcomp(yr, sx, dur)$. We have, for durations 0 through 8, that

$$avgmba(yr, sx, ag, dur) = avgmba(yr - 1, sx, ag - 1, dur - 1) \times (1 + COLA(yr)) \\ \times (1 + wkcomp(yr, sx, dur)) \times PEadj(yr, sex, dur).$$

A more careful explanation of the factors, $wkcomp(yr, sx, dur)$ and

$PEadj(yr, sex, dur)$, is given later in this document. See the section titled *Average PIAs and MBAs for Disabled-Worker Beneficiaries*, below. To move duration 8 average PIAs to duration 9+ average PIAs, both average PIAs are given a cost of living adjustment and a post-entitlement adjustment (see section “Post-Entitlement Adjustments”). The resulting duration 9+ average PIA is the weighted average of the adjusted prior year duration 8 and 9+ average PIAs, weighted by the prior year’s numbers of beneficiaries in current-pay status for durations 8 and 9+ respectively.

The only column that does not follow this procedure is the duration 0 column. The duration 0 column corresponds to newly entitled disabled-worker beneficiaries. The following sections describe how average PIAs are determined for this group of beneficiaries.

Average PIAs for Newly Entitled Disabled-Worker Beneficiaries

The potential AIME percentage values for newly entitled disabled-worker benefits (DPAPs) are obtained from the AWARDS subprocess. The two bendpoints of the PIA formula, BP1 and BP2, are indexed by the increase in the average wage index. In 1979 dollars, the values of BP1 and BP2 are \$180 and \$1,085 respectively. The AIME dollars between 0 and BP1 are divided into four intervals (each of length \$45 in 1979 dollars). The AIME dollars between BP1 and BP2 are divided into fourteen intervals (nine of length \$45 and five of length \$100, in 1979 dollars). Twelve additional intervals are added beyond BP2 (ten of length \$200 and two of length \$1,000, in 1979 dollars).

To determine the average PIA for newly entitled beneficiaries, the DPAP values for each of the thirty intervals of AIME dollars are multiplied by the dollar amounts attributable to each interval (the length of the interval) and by the associated PIA factors. The distribution of prior year disability onset and current year disability onset is taken into consideration. It is assumed that this distribution is 6 months for prior year disability onset and 6 months for current year disability onset. In the formulas below, $j = 1$ signifies current year disability onset and $j = 2$ signifies prior year disability onset.

Let:

- $Wage_Idx_j(sx, ag, yr) = \frac{avgwg(yr - \max(ag - 60, 1 + j))}{avgwg(1977)}$ for $j=1,2$.
- $Cum_COLA_1(ag, yr) = \begin{cases} (1 + COLA(yr - 1)) \times (1 + COLA(yr)) & ag < 64 \\ \prod_{k=62}^{ag} (1 + COLA(yr - (k - 62))) & 64 \leq ag \leq 66. \end{cases}$
- $Cum_COLA_2(ag, yr) = \begin{cases} 1 + COLA(yr) & ag < 63 \\ \prod_{k=63}^{ag} (1 + COLA(yr - (k - 62))) & 63 \leq ag \leq 66. \end{cases}$

- $w_j = \frac{6}{12} = \frac{1}{2}, j=1,2.$
- PIA_factor_i represent the PIA factor for interval i (equal to 0.90 for intervals $i=1, \dots, 4$, 0.32 for intervals $i=5, \dots, 18$, and 0.15 for intervals $i=19, \dots, 30$).
- $AIME_dollars_i$ represent the length of interval i , expressed in 1979 dollars.
- $dpap_i(yr, sx, ag)$ represent the DPAP value for newly entitled disabled workers in year yr whose sex is sx and age is ag .

To take into account the workers' compensation offset to disability benefits, administrative data is reviewed, from which a factor is developed and applied to the average award benefit. We now describe how this factor, $facm2p(yr, sx)$ is computed. The table 1-A supplement, for each month in a given historical year, contains total award PIA and MBA data for disabled workers, by sex, for beneficiaries both non-actuarially reduced and actuarially reduced. Let $totmba_DIB_nar(yr, sx)$ and $totpia_DIB_nar(yr, sx)$ be the total annual MBA and PIA respectively for DIBs that are not actuarially reduced as found in the table 1-A data. In the historical period 2000- $nim1$ we define $facm2p(yr, sx)$ to be the ratio of the total MBA to the total PIA for those not actuarially reduced. In other words,

$$facm2p(yr, sx) = \frac{totmba_DIB_nar(yr, sx)}{totpia_DIB_nar(yr, sx)}.$$

In the period ni through $ns+9$, $facm2p(yr, sx)$ is defined as follows. Let

$$y1 = facm2p(nim1, sx)$$

$$y2 = (1.0 - y1) / 3.0$$

$$facm2p(yr, sx) = \max(y1 - y2, \min(y1 + y2, facm2p(yr - 1, sx) \times \left(\frac{facm2p(yr - 1, sx)}{facm2p(yr - 11, sx)} \right)^{1/20})).$$

Projected values of $facm2p$ are therefore held within a delta of $y2$ from the last historical year of $facm2p$.

This value is further adjusted by the variable $facm2p_param$ to reflect the offset amounts that end within the first entitlement year. For the 2021 TR the data suggests this factor should be 0.39. As a result, for $yr = ni, \dots, ns+9$,

$$\begin{aligned} facm2p(yr, sx) &= x + (1 - x) \times 0.39 \\ &= 0.39 + 0.61 \times x. \end{aligned}$$

The factor reaches its ultimate value in years $ns+10$ and later.

Another factor used in the development of average PIAs for newly entitled disabled worker beneficiaries is $DIB_adjudication$. This factor adjusts for expected distributions of awards by disability adjudicative level. A side model shows that expected future distributions will have a higher share of disabled workers coming on the rolls at the Administrative Law Judge level than currently shown in the Awards sample. This leads to a 0.48% expected reduction in newly entitled DI worker benefits. Therefore, $DIB_adjudication = 0.9952$.

The preliminary average PIA for newly entitled disabled worker beneficiaries may now be defined. It is equal to

$$LR_awdpia(sx, ag, yr) = \sum_{i=1}^{30} PIA_factor_i \times AIME_dollars_i \times dpap_i(yr, sx, ag) \\ \times \left(\sum_{j=1}^2 w_j \times Wage_Idx_j(sx, ag, yr) \times Cum_COLA_j(ag, yr) \right) \\ \times facm2p(yr, sx) \times DIB_adjudication.$$

Once these average PIAs of newly entitled disabled-worker beneficiaries are computed, their values are filled into the average PIA matrices for duration 0 for the appropriate entitlement age.

Average PIAs and MBAs for Disabled-Worker Beneficiaries

An overall average PIA of newly entitled disabled worker beneficiaries for each sex and projection year is computed by taking the weighted average of $awdpia(sx, ag, yr)$, the weights being the number of disabled workers in current payment status of duration zero. This value is denoted $awdpia(sx, yr)$.

In addition, an overall average PIA and MBA for all disabled worker beneficiaries in current-payment status is computed by finding the weighted average of the average PIAs for each age in current-pay and duration with the number of people in current pay for each of these ages and durations. The average PIAs were already reduced by a workers' compensation offset factor, as briefly described above; a more careful description is given in this section. To get the average MBAs, we apply a factor that reflects the differences in average MBAs and PIAs for disabled workers, isolating only the trend in cases with an actuarial reduction. We also provide a relatively small reduction to reflect offsets starting and stopping in the year of DI entitlement that are not captured by the current method. There is an additional adjustment to the weighted average disabled worker PIA and MBA amounts in current pay applied by duration from entitlement to award in order to account for benefit level differences that mature by duration 5.

Workers' Compensation Offset Factors

For each duration $j = 1, \dots, 7$, and 8+ and sex we define a workers' compensation factor. This factor is applied to the average worker PIA matrix as mentioned above. It is denoted $wkcomp(yr, sx, dur)$. Let $facm2p_pct(dur)$ be defined as in the following table.

Duration	Cumulative product above set at x% of
----------	---------------------------------------

	way between original facm2p and 1
1	0.523588
2	0.699937
3	0.773501
4	0.818732
5	0.852705
6	0.871823
7	0.886439
8+	0.901639

Then $wkcomp(yr, sx, dur)$ is defined so that

$$facm2p_pct(dur) = facm2p(yr - dur, sx) \times \prod_{j=1}^{dur} (1 + wkcomp(yr, sx, j)).$$

This is an iterative process that first computes $wkcomp(yr, sx, 1)$ by solving the above equation with dur set equal to 1. The remaining factors for higher durations are then computed recursively using the above formula.

Trend in Average MBA to Average PIA

This trend is captured in a factor denoted $Fam2p(yr, sx)$. The table 1-A supplement as of the end of December of each historical year contains total in-current pay PIA and MBA data for disabled workers, by sex, for beneficiaries both non-actuarially reduced and actuarially reduced. Let $totmba_nar(yr, sx)$ and $totpia_nar(yr, sx)$ be the total MBA and PIA respectively for DIBs that are not actuarially reduced as found in the table 1-A data. Similarly, let $totmba_ar(yr, sx)$ and $totpia_ar(yr, sx)$ be the total MBA and PIA respectively for cases that are actuarially reduced. In the historical period 2000-*nim1* we define $fam2p(yr, sx)$ to be the ratio of the total MBA to the total annual PIA for those not actuarially reduced. In other words,

$$Fam2p(yr, sex) = \frac{totmba_ar(yr, sex) + totpia_nar(yr, sex)}{totpia_ar(yr, sex) + totpia_nar(yr, sex)}.$$

In the period ni through $ns+10$, $fam2p(yr, sx)$ is defined as follows:

$$y1 = fam2p(nim1, sex)$$

$$y2 = (1.0 - y1) / 3.0$$

$$fam2p(yr, sx) = \max(y1 - y2, \min(y1 + y2, fam2p(yr - 1, sx) \times \left(\frac{fam2p(yr - 1, sx)}{fam2p(yr - 11, sx)} \right)^{1/20})).$$

The factor reaches its ultimate value in years $ns+10$ and later.

More Workers' Compensation Offsets

As mentioned above, we also we provide a relatively small reduction to retroactive benefits to reflect offsets starting and stopping in the year of DI entitlement that are not captured by the current method. Based on historical administrative data, we set these factors by duration as follows:

Duration	Percentage Reduction
0	0.2311%
1	0.2095%
2	0.0697%
3	0.0316%
4	0.0076%
5+	0%

We define $wkcomp_red(dur)$ to be 1 minus these percentage reductions.

By law, disabled workers are no longer subject to the workers' compensation offset at the attainment of a defined age. For those born prior to 1951, the defined age is 65. For those born in 1951 and later, the defined age is the Normal Retirement Age (NRA).³ Therefore, all DI worker benefit levels are adjusted at either age 65 (for those born prior to 1951) or at conversion to a retired worker benefit (for those born 1951 and later) to eliminate the effect of the offset.

Adjustment to Average Benefit Levels by Duration

Average disabled worker PIA and MBAs are adjusted further at each duration by a factor, DI_RI_facs , designed to account for average benefit level changes that occur when beneficiaries come on the rolls retroactively. This is different than projected retroactive payments discussed elsewhere in the documentation. These adjustments mature at duration 5, when the vast majority of disabled workers have started receiving benefits.

Computation of Average MBA for DI Workers

The disabled worker PIA as presented in the average benefit matrix was already incremented by age and duration using a COLA and a workers' compensation adjustment. The average PIA by year, age and duration, is denoted $avgpia(yr, ag, sx, dur)$. The overall average MBA by year and sex is the weighted average of $avgpia(yr, ag, sx, dur) \times Fam2p(yr, sx, dur)$, the weights being the number of DI workers in current payment status by age, sex, and duration.

Post-Entitlement Adjustments

As cohorts of beneficiaries age, their average benefit level will likely change for reasons other than just the COLA increase. The two primary reasons for this are post-entitlement work, which could lead to a re-calculation of one's benefit, and a known correlation between greater lifetime earnings and lower mortality rates. The Cost process uses post-entitlement factors by sex and duration to account for the expected

³ The NRA was 65 for individuals born before 1938. It increased to 66 at the rate of 2 months per year for individuals born 1938-1943. Under current law, the NRA will increase to 67 for individuals born from 1955-1960, again at the rate of 2 months per year.

dynamic benefit levels. Consecutive-year comparisons of one percent December MBR data from 2009-2010 to 2018-2019 are used to calculate post-entitlement factors.

For disabled workers we calculate separate factors for those in current pay (ICP) who are younger than 50 and those ICP who are age 50 or older. We use separate factors for each sex and each duration (0-9⁺). Initial and Ultimate factors are calculated before being read into the cost program with initial factors set to the most recent 3-year historical average and ultimate factors at the most recent 10-year average.

All initial factors are phased-in linearly to the ultimate factors over the first 20 years of the projection period (reaching the ultimate values in $ni+19$).

(2) Retired-Worker Benefits

Retired-Worker Beneficiary Matrix

The number of retired-worker beneficiaries for a given year and sex is provided from subprocess 3.3. Two matrices are provided – one for men and one for women. The structure of each matrix is as follows:

- 10 columns. The first 9 columns are the age at entitlement, ages 62 through 70. The last column is the number of disabled workers who are projected to convert to retired-worker beneficiary status (DI conversions) at normal retirement age.
- 34 rows. These rows correspond to the age in current pay, ages 62 through 94 and ages 95+.

Note that the entries on the diagonal at ages 62 through 70 (where age in current-pay equals age at entitlement) are the number of new entitlements projected for that year.

Building the Average Benefit Matrices for Retired Workers

In each projection year, the COST subprocess produces four average benefit matrices. For each sex there are two matrices, an average monthly benefit amount (MBA) matrix and the average primary insurance amount (PIA) matrix. Each matrix has the same structure as the beneficiary matrices. In other words, each matrix is a 34 by 10 matrix whose entries are the average benefit amounts of retired worker beneficiaries whose age in current pay is indexed by the rows and whose age at initial entitlement is indexed by the columns. The final column simply gives the average benefits for DI conversions at the various ages in current pay.

The 100% MBR extract is processed by a side model. This side model computes a starting matrix for year $ni-1$. This starting matrix contains the four initial *benefit* matrices, constructed using the most recent data. Aggregate MBAs and PIAs for the starting matrix are calculated, by sex, by multiplying each cell in the benefit matrix

by its counterpart in the beneficiary matrix and then summing the total benefit. The aggregate MBAs and PIAs are compared to data in the Table 1-A supplement for year $ni-1$ (with appropriate exclusion of excess benefit for dually entitled workers) and all individual MBAs and PIAs in the starting benefit matrices are proportionally adjusted so that the new aggregate MBAs and PIAs match the values in the supplement.

For a given year of the projection period, the average benefit matrix is updated from its previous year's value incrementing each benefit amount (PIA or MBA) by one year of age and increasing it by a cost of living adjustment (COLA) and by the appropriate post entitlement factor (see section "Post-entitlement adjustments") for men and women. Adjusted age 94 benefits and age 95+ benefits are averaged, based on the respective number of beneficiaries in current pay in the prior year, to get the new average benefit for age 95+. DI conversion benefits are handled similarly, except the average conversion benefit for each age 66 through 67 is combined (as a weighted average) between the number of DI worker beneficiaries at age NRA-1 in the prior year (computed in subprocess 3.2) and the number of DI conversions of age NRA-1 already receiving benefits as a DI conversion case, if any (provided by subprocess 3.3).

The entries along the diagonal, the average benefits of newly entitled beneficiaries by age, must still be computed. The remainder of this section will explain how these average benefits are computed. Once these are computed, all entries are computed and the average benefit matrix for the year is complete.

Average Benefits for Newly Entitled Retired Worker Beneficiaries

The potential AIME percentage (OPAPs) values for newly entitled retired-worker benefits are obtained from subprocess 4.2⁴. Average benefits for newly entitled retired-worker beneficiaries are calculated by sex and single years of age 62 through 69, and ages 70+. The two bendpoints of the PIA formula, BP1 and BP2, are indexed by the increase in the average wage index. In 1979 dollars, the values of BP1 and BP2 are \$180 and \$1,085 respectively. The AIME dollars between 0 and BP1 are divided into four intervals (each of length \$45 in 1979 dollars). The AIME dollars between BP1 and BP2 are divided into eighteen intervals (thirteen of length \$45 and five of length \$100, in 1979 dollars). Twelve additional intervals are added beyond BP2 (ten of length \$200 and two of length \$1,000, in 1979 dollars).

To determine the average PIA for newly entitled beneficiaries, the OPAP values for each interval of AIME dollars are multiplied by the dollar amounts attributable to each interval (the length of the interval). More precisely, let

- PIA_factor_i be the PIA factor for subinterval i (equal to 0.90 for intervals $i=1, \dots, 4$, 0.32 for intervals $i=5, \dots, 18$, and 0.15 for intervals $i=19, \dots, 30$),

⁴ Prior to the 2017TR, the raw OPAP values from subprocess 4.2 needed to be altered by the cost subprocess in order to account for projected changes from the static age distribution for newly entitled retired workers used by Awards subprocess 4.2. The method for doing this was referred to as the "Shuttling Method". As of the 2017 TR, the shuttling method has been moved to subprocess 4.2.

- $AIME_dollars_i$ be the length of subinterval i ,
- $opap_i(yr, sx, ag)$ be the OPAP value from subprocess 4.2 for retired workers newly entitled in year yr whose sex is sx and whose age is ag .
 - $wff(yr, sx, age)$ be a reduction factor to account for the Windfall Elimination Provision.⁵
 - $Wage_Idx(ag, yr) = \frac{avgwg(yr - (ag - 62))}{avgwg(nbase - 2)}$
 - $COLA_Idx(ag, yr) = \prod_{k=62}^{ag} (1 \times COLA(yr - (k - 62)))$

Then the average PIA for these newly entitled retired worker beneficiaries is equal to

$$LR_awdpia(sx, ag, yr) = Wage_Idx(ag, yr) \times COLA_IDX(ag, yr) \times wff(yr, sx, age) \\ \times \sum_{i=1}^{30} PIA_factor_i \times AIME_dollars_i \times opap_i(yr, sx, ag).$$

This formula incorporates the fact that the PAP values are the estimated cumulative distribution of AIME dollars. The average award MBA for a worker beneficiary is then the average newly entitled PIA multiplied by the appropriate actuarial reduction factors and delayed retirement credits, $arfdrc(ag, yr)$, based on age at initial entitlement.

Once these average benefits of newly entitled retired-worker beneficiaries are computed, their values are filled into the appropriate average benefit matrices.

For summary purposes, the COST subprocess computes an average PIA and MBA for all male and female newly entitled retired-worker beneficiaries. These are just the respective weighted averages of the average PIAs and MBAs by age and sex, the weights being the number of newly entitled retired-worker beneficiaries. Similarly, average PIA and MBA for all retired worker beneficiaries in current pay are computed, by sex.

DI Conversions

Disabled-worker beneficiaries convert to retired-worker beneficiary status (called DI conversions) at normal retirement age (NRA). The average new DI conversion benefit for a given sex at age NRA is the weighted average of the average DI worker benefits from the prior year for that sex and age NRA-1, weighted by the number of people in current pay in each duration and then increased by the current year COLA and adjusted by the appropriate Post-Entitlement factor. The average DI conversion

⁵ The Windfall Elimination Provision (WEP) reduces the first PIA formula factor from 90% to as low as 40% for individuals who receive a pension based on specified categories of non-covered employment, primarily non-covered state and local government employees and federal workers receiving a pension under the Civil Service Retirement System. The cost process uses initial factors by sex and age, ultimate factors, years in which ultimate factors are reached and phase-in trend lines to the ultimate factor, all supplied by a side model.

benefit for a given sex and single age NRA+1 through 95+ is the average DI conversion benefit from the previous year for the same sex and age cohort increased by the current year COLA and adjusted by the appropriate Post-Entitlement factor. Both the average conversion benefit for each sex and single age NRA through 95+, and the number of people in current pay for these ages, are used in the computation of average retired worker benefits.

Post-Entitlement Adjustments

As discussed in the previous section, the Cost process uses post-entitlement factors by sex and duration to account for changes in benefit levels aside from the cost-of-living adjustment. For retired workers we calculate separate factors for those ICP who converted from DI status and those ICP who came on the rolls as a retired worker. We use separate factors for each sex and each duration (0-12+). Initial and ultimate factors are calculated before being read into the cost program with initial factors set to the most recent 3-year historical average and ultimate factors at a 10-year average. For women the ultimate post-entitlement factors are adjusted further to reflect the trend that female retired workers are starting to have earnings and benefit levels more similar to men. Therefore, female ultimate post-entitlement factors are calculated in the program as 90% of the male 10-year average plus 10% of the female 10-year average.

All initial factors are phased-in linearly to the ultimate factors over the first 20 years of the projection period (reaching the ultimate values in ni+19).

Adjustment to Average Benefit Levels by Duration

Average retired worker PIA and MBAs are adjusted at each duration by a factor, *OA_RI_fac*s, designed to account for average benefit level changes that occur when beneficiaries come on the rolls retroactively. This is different than projected retroactive payments discussed elsewhere in the documentation. These adjustments mature at duration 5, when the vast majority of retired workers have started receiving benefits.

(3) Annualizing Benefits

Scheduled benefits are calculated by trust fund and projection year. For each year, scheduled benefits for each trust fund are found by adding up the appropriate benefit categories.

This section applies to all benefit amounts except the “dual entitlement excess amount.” If a retired worker beneficiary is also entitled to auxiliary spouse or widow(er) benefits and these auxiliary benefits are greater, then the amount by which the auxiliary benefit exceeds the worker’s MBA is the dual entitlement excess amount. The four categories of excess amounts (dually entitled wives, widows, husbands, and widowers) are projected separately. More information is found in subsection (4).

The first step is to determine average benefits by category. A list of the beneficiary categories follows. An odd category number refers to the male account holder, while an even category number refers to the female account holder. As an example, for category 4, the aged married spouse is the aged married husband of the retired female worker.

Category # (cat)	<u>Beneficiary Type</u>
	<u>Old-Age Insurance Beneficiaries</u>
1 & 2	Retired worker (includes DI conversions)
3 & 4	Aged married spouse
5 & 6	Aged divorced spouse
9 & 10	Young spouse with child
11 & 12	Child < 18
13 & 14	Student child
15 & 16	Disabled adult child
	<u>Disability Insurance Beneficiaries</u>
17 & 18	Disabled worker
19 & 20	Aged married spouse
21 & 22	Aged divorced spouse
25 & 26	Young spouse with child
27 & 28	Young child
29 & 30	Student child
31 & 32	Disabled adult child
	<u>Survivors Insurance Beneficiaries</u>
33 & 34	Aged married widow
35 & 36	Aged divorced widow
39 & 40	Young married disabled widow
41 & 42	Young divorced disabled widow
43 & 44	Aged parent
45 & 46	Young married widow with child
47 & 48	Young divorced widow with child
49 & 50	Young child
51 & 52	Student child
53 & 54	Disabled adult child
55 & 56	Lump sum death benefit (\$255)

For the worker categories, the prior sections describe the computation of average benefit levels at the end of each year. For a specific auxiliary beneficiary category, the average monthly benefit at the end of each year (*avgben*) is determined by multiplying:

- The linkage factor (the assumed relationship between an auxiliary beneficiary's benefit level and the corresponding worker benefit level) by
- The relevant average PIA or the average monthly benefit of the primary account holder (the worker beneficiary account on which the auxiliary beneficiary is entitled to receive the benefit).

Starting with the 2016 Trustees Report, the linkage factor for aged spouse categories comes not from the qlink workbook, but from file ‘BASps’. This file is created by a side model with the purpose of accounting for the expected changes to average aged spouse benefit levels due to the Bipartisan Budget Act of 2015.

In order to annualize benefits for each beneficiary category, two values are used. The beginning-of-year average benefit equals the average monthly benefit in December of the prior year. The end-of-year benefit equals the monthly average benefit of the worker beneficiary for December of the current year without the cost of living adjustment (COLA). The average benefit by category for each month is found by taking a weighted average of the benefits at the beginning and end of the year, the weights being the fractions of the year the prior and current year’s beneficiaries have been exposed. If $cp(cat, yr)$ is the number of beneficiaries in category cat for year yr , and $avgben(cat, yr)$ is the average monthly benefit for category cat for year yr , then the amount of aggregate benefits paid in year yr is given by the formula:

$$\begin{aligned} & \text{AggBen}(yr, cat) \\ &= \sum_{i=0}^{11} \left[\frac{(12-i)}{12} \times cp(cat, yr-1) \times avgben(cat, yr-1) + \frac{i}{12} \times cp(cat, yr) \times avgben(cat, yr) \right]. \end{aligned}$$

For all beneficiary categories except for the lump-sum benefit, the aggregate benefit amount is increased by the retroactive payments that were projected to be paid during the year. See section (5), below.

(4) Dually Entitled Beneficiaries and Benefits

Number of Dually Entitled Beneficiaries

There are four primary categories of dually entitled beneficiaries. They are the dually entitled wives, widows, widowers, and husbands. To project the number of dually entitled beneficiaries for each category we combine a series of regression equations (1 each for wives and husbands and 3 each, by age bands, for widows and widowers) with two coefficients each, a slope of $a_1^{(k)}$ and a y-intercept of $b^{(k)}$, with a third factor, $c^{(k)}(yr)$ derived from a process we describe as “add factoring”, and a fourth factor, $d^{(k)}(yr)$ derived from an adjustment to the most recent historical data point:

$$\text{PctExp}^{(k)}(yr) = a_1^{(k)} \frac{\text{PIA}(yr, M) - \text{PIA}(yr, F)}{\text{PIA}(yr, M)} + b^{(k)} + c^{(k)}(yr) + d^{(k)}(yr)$$

($k=1,2,3,4,5,6,7,8$) project the percentage of the exposed population entitled to wife (1), widow aged 62-74 (2), widow aged 75-84 (3), widow aged 85+ (4), widower aged 62-74 (5), widower aged 75-84 (6), widower aged 85+ (7), and husband (8) benefits.

$\text{PIA}(yr, sex)$ is the average PIA of all retired worker beneficiaries in current pay by sex,

and $PctExp(yr)$ is the percentage of the entitled population in the category that is dually entitled. We use the “add factoring” method with variable $c^{(k)}(yr)$ to account for the expected future comparative work history changes that will affect dual entitlement populations.

To derive $c^{(k)}(yr)$, suppose that $ult^{(k)}$ is the value obtained from the regression equation without add-factoring in the final year of the projection period. Therefore

$$ult^{(k)} = a_1^{(k)} \frac{PIA(ni + 74, M) - PIA(ni + 74, F)}{PIA(ni + 74, M)} + b^{(k)}.$$

Let $targ^{(k)}$ be the target value we estimate for the final year of the projection period. Let $phaseyrs$ be the number of years it takes to fully phase in the target value. Then we have

$$c^{(k)}(yr) = \min(yr - 2019, phaseyrs) \times \frac{targ^{(k)} - ult^{(k)}}{phaseyrs}.$$

To derive $d^{(k)}(yr)$, suppose that $b1$ is the measured difference between the most recent historical dual-entitlement percentage and the corresponding post-add-factoring regression value. We phase out this adjustment linearly over 20 years, thus:

$$d^{(k)}(yr) = b1 \times (1.0 - \min(yr - 2020, 20.0) / 20.0).$$

The following table displays the coefficients, target values, and phase-in years for each type of beneficiary.

k	Type	$a_1^{(k)}$	$b^{(k)}$	Target Value $targ^{(k)}$	Add-factoring Phase-in Years $phaseyrs$
1	Wife	0.82064	0.0	0.195	34
2	Widow 62-74	0.18480	0.47775	0.520	31
3	Widow 75-84	0.82691	0.41523	0.590	30
4	Widow 85+	0.50758	0.59192	0.615	37
5	Widower 62-74	-0.36223	0.17228	0.10	30
6	Widower 75-84	-0.45981	0.23001	0.085	33
7	Widower 85+	-0.14138	0.10547	0.08	76
8	Husband	-0.05516	0.02435	0.0090	36

In the above equations, the average PIA of newly entitled retired worker beneficiaries by

sex has already been computed (see subsection (2) above).

Average Excess Amount for Dually Entitled Beneficiaries

The projection of the average excess amounts for two categories of dually entitled beneficiaries (wives and widows) is similar to that of the number of dually entitled beneficiaries. The structure of the equations used to project these amounts is similar to the equations used to project the percentage exposures and, as is the case with the percentage exposures, we use 1 regression for wives and 3 for widows (1 for each of 3 age bands).

The equations used to project the average excess amount each have two terms and two adjustment factors, similar to the process for the number of dually entitled beneficiaries. Each equation k has two coefficients, a slope of $a_1^{(k)}$ and a y-intercept of $b^{(k)}$, with a third factor, $c^{(k)}(yr)$ derived from add factoring, and a fourth factor, $d^{(k)}(yr)$ derived from an adjustment to the most recent historical data. A target value in the 75th year of the projection period is used in deriving the “add-factor” adjustment. The four equations

$$\text{AvgExcPct}^{(k)}(yr) = a_1^{(k)} \frac{\text{PIA}(yr,M) - \text{PIA}(yr,F)}{\text{PIA}(yr,M)} + b_1^{(k)} + c^{(k)}(yr) + d^{(k)}(yr)$$

($k = 1, 4$) project the average excess benefit amounts of wives (1), widows aged 62-74 (2), widows aged 75-84 (3), and widows aged 85+ (4). The derivations of $c^{(k)}(yr)$ and $d^{(k)}(yr)$ are completed in the same manner as for the number of dually entitled beneficiaries above.

The table below shows the regression coefficients and other relevant adjustments in the 2021 Trustees Report.

k	Type	$a_1^{(k)}$	$b^{(k)}$	Target Value (2095) in Nominal Dollars $targ^{(k)}$	Add-factoring Phase-in Years $phaseyrs$
1	Wife	0.47477	0.0	3,672.36	22
2	Widow 62-74	-0.54954	0.60479	10,190.25	40
3	Widow 75-84	0.99254	0.08848	9,443.60	22
4	Widow 85+	0.30784	0.36750	9,746.73	38

The average excess amount of widowers and husbands is estimated to be a fixed percentage of the average excess amounts of widows and wives, respectively. The average excess amount of husband beneficiaries is measured to be 79.22% of wives in 2020 and is assumed to linearly phase to 80% in the 13th year of the projection period (2033 for 2021TR). Widowers are broken down into the three age bands, such that the average excess amount for widowers aged 62-74 is based on the average excess amount

for widows aged 62-74, etc. Similar to husbands, the factor is linearly interpolated from the measured historical value in 2020 to an ultimate factor in 2033. Below are the initial and ultimate factors:

Type	Average Benefit Level Relative to Widow (2020)	Average Benefit Level Relative to Widow (2033)
Widowers 62-74	58.97%	60.5%
Widowers 75-84	53.77%	55.5%
Widowers 85+	45.92%	47.5%

Annualizing Excess Amounts

The process to annualize excess amounts is very similar to the process for annualizing auxiliary benefits.

For each dual entitlement category, the number of beneficiaries is simply $cp(yr, cat) = PctExp^{(cat)}(yr) \times ExposedPop^{(cat)}(yr)$. With this method, however, no linkage factor is used. Instead, the projected average excess amount, as explained above, is used. Therefore,

$$\begin{aligned}
 & AggExcess(yr, cat) \\
 &= \sum_{i=0}^{11} \left[\frac{(12-i)}{12} \times cp(cat, yr-1) \times AvgExcAmt^{(cat)}(yr-1) + \frac{i}{12} \times cp(cat, yr) \times AvgExcAmt^{(cat)}(yr) \right].
 \end{aligned}$$

(5) Retroactive Payments

Frequently, beneficiaries start receiving payments later than their actual entitlement date, such that they receive a “catch-up” lump-sum amount for the time delay, in addition to regular monthly benefits going forward. These lump-sum amounts, which we call retroactive payments, apply for all beneficiary categories (except for the one-time \$255 death benefit) but are more significant for disabled workers because of the frequent and sometimes lengthy time lag in getting an allowance on their application, as well as the 12 months of retroactivity allowed at the time of benefit filing.⁶ This section discusses how retroactive benefit payments are projected for all beneficiary categories.

Disabled Workers

The number of disabled-worker newly awarded beneficiaries for a given year and sex is provided from subprocess 3.2. For each projection year, two matrices are provided – one for men and one for women. The structure of each matrix is as follows:

⁶ In contrast, retired workers retiring before NRA have no months allowed at the time of benefit filing, while retired workers retiring after NRA have up to 6 months of retroactively allowed.

- 11 columns. The columns are indexed by duration of disability (0-9 and 10+).
- 52 rows. These rows correspond to the age in current pay, ages 15 through 67.

The COST subprocess, however, only uses 10 durations⁷ (0 through 9), and 48 ages (ages 20 through 67). This requires a manipulation of the matrix of newly awarded DI beneficiaries from subprocess 3.2. For ages in current pay greater than or equal to 30, the duration 9 and 10+ columns of this matrix are added to give the total number of duration 9 beneficiaries. For ages (*ag*) between 20 and 30 inclusive, the number of newly awarded beneficiaries aged *ag* and duration *ag-20* is the value provided by the DISABILITY subprocess added to the number of new awards aged *ag-j* and duration *ag-20* for *j=1,...,5*. (For example, the number of people aged 20 of duration 0 is combined with the number of people aged 15, 16, 17, 18 and 19 of duration 0; the number of people aged 21 of duration 1 is combined with the number of newly awarded beneficiaries aged 16, 17, 18, 19, and 20 of duration 1, and so on. In other words, the five nonzero diagonals of the matrix provided by the DISABILITY subprocess are “combined with” the diagonal directly below it and then zeroed out.)

Newly awarded disabled worker beneficiaries are denoted by year, sex, age and duration as *dibaw(year,sex,age,dur)*.

Let *dur* be a duration, 0 through 9. Define

- $Cum_COLA(dur) = \prod_{j=0}^{dur} (1 + COLA(yr - j))$.
- For $i = 0, \dots, 10$, $Num_Months(i) = \begin{cases} 2 & dur = 0 \\ 5 & dur > 0 \text{ and } i = 0 \\ 12 & dur > 0 \text{ and } 0 < i < dur \\ 6 & dur > 0 \text{ and } i = dur \end{cases}$

Then the aggregate retroactive payments for disabled workers, in millions, are defined to be

Retro_DIB(sex,yr)

$$= \sum_{age=20}^{67} \sum_{dur=0}^9 \sum_{i=0}^{dur} dibpia(yr, sex, \min(age, 66), dur) \times wkcomp_red(dur) \times \frac{dibaw(yr, sex, age, dur)}{10^6} \times \frac{Num_Months(i)}{Cum_COLA(dur)}$$

dibpia is further altered in this formula by the adjustment factors in array:

⁷ While the DI Awards file from subprocess 3.2 displays durations 0 through 9 and 10+, there are no new awards after duration 5 for the 2021 TR. Therefore, durations 6+ show zeros for all ages and years.

$DI_RI_facsex(sex, age, dur, 2)$ ⁸

These adjustments to *dibpia* are designed to account for the differences between average benefit levels for all beneficiaries and levels for those receiving retroactive credit.

Retro_DIB(sex, yr) is then further adjusted to account for differences between projected retroactive benefits and observed retroactive benefits paid in the historical period. This adjustment, denoted *ret_hist*, is calculated to be 0.8662 for the 2021TR. After this adjustment *Retro_DIB(sex, yr)* is simply added to the disabled worker benefit category by year and sex.

Retired Worker Beneficiaries

Retired worker beneficiaries are assumed to have, on average, 0.4 months of retroactive payments if they begin receiving benefits prior to their NRA and 1.5 months of retroactive payments if they begin receiving benefits at or after NRA. The number of months for the two age groups (denoted ‘retrom’ in the below equation) is determined annually by a side model. Hence

$$retro_OAB(sex, yr) = \sum_{age=62}^{70} \frac{retrom \times \frac{oabicmp(yr, sex, age, age)}{1000} \times oabmba(yr, sex, age, age)}{1 + COLA(yr)}$$

In the above formula, *oabicmp(yr, sex, age, age)* is the number of newly entitled beneficiaries at age equal to *age* (age in current pay equals age at entitlement equals *age*) and *oabmba(yr, sex, age, age)* is the corresponding average benefit.

oabmba(year, sex, age, age) in the above formula is further altered by the adjustment factors:

$$retrom_adj(sex) = \{0.993538, 1.045963\}$$

These adjustments to *oabmba* are designed to account for the small differences between average benefit levels for all beneficiaries and levels for those receiving retroactive credit.

The aggregate retroactive benefits for retired worker beneficiaries are simply added to the retired worker benefit category by and sex.

Auxiliary Beneficiary Categories

Retroactive payments for auxiliary beneficiaries (determined by a side model using

⁸ The fourth index of the 4-dimensional *DI_RI_facsex* array is a binary flag to denote the ICP duration adjustment factor for a subscript of 1 and the retro adjustment factor for a subscript of 2.

historical experience) are treated as a loading of the aggregate annual benefits by auxiliary category. That is, each auxiliary benefit category has a loading factor to represent retroactive payments on top of regular monthly benefits, and the aggregate annual benefits by category are increased by this loading factor.

(6) Aggregate Scheduled Benefits (*BEN*)

Aggregate benefits by trust fund, $BEN(tf, yr)$, are computed as follows. For each year of the 75-year long-range projection period, the aggregate benefits by category (including retroactive payments, as described above) are summed up to give total annual scheduled benefits by trust fund. In the short-range period, the long-range values are overridden by the values estimated by the short-range office. The difference between long-range scheduled benefits and short-range benefits in the 10th year of the short-range period is called the scheduled benefits adjustment. In the 10 years after the end of the short-range period, the long-range scheduled benefits are adjusted by linearly grading the scheduled benefits adjustment to zero. From the 20th year forward, the projection is the pure long-range value.

iii. **Equation 4.3.3 – Taxation of Benefits (*TAXBEN*)**

The short-range office provides taxation of benefits levels by trust fund in the short-range period. These implicitly give, for each year, an estimated taxation of benefits factor, by trust fund, equal to the estimated taxation of benefits as a percentage of benefits scheduled to be paid. The long-range office projects these factors independently for every year of the projection period, also by trust fund. (See subprocess 4.1.) The difference in the factors between the two offices at the end of the short-range period is phased out linearly over the next ten years. The long-range projection of taxation of benefits is estimated by multiplying the projected taxation of benefits factors by the benefits scheduled to be paid, by trust fund. If $taxben_factor(tf, yr)$ is the percentage of scheduled benefits for the year, by trust fund, estimated to be collected as taxation on benefits, then

$$TAXBEN(tf, yr) = taxben_factor(tf, yr) \times BEN(tf, yr)$$

for $yr \geq ns + 10$.

iv. **Equation 4.3.4 – Administrative Expenses (*ADM*)**

Administrative expenses are estimated separately by trust fund. In the short-range period, the short-range office provides the estimates of administrative expenses by trust fund. Thereafter, administrative expenses are computed by multiplying the previous year's administrative expenses by three factors: annual changes in total beneficiaries, annual changes in AWI, and one minus annual productivity growth. As a formula, if $ticp(tf, yr)$ is the total estimated number of beneficiaries in current-pay status by trust fund and year, $AWI(yr)$ is the average wage index in year yr , and $prod$ is the ultimate assumed annual growth in productivity, then

$$ADM(tf, yr) = ADM(tf, yr - 1) \times [ticp(tf, yr) / ticp(tf, yr - 1)] \times [AWI(yr) / AWI(yr - 1)] \times (1 - prod)$$

for $yr > ns$.

v. **Equation 4.3.5 – Railroad Interchange (RR)**

Railroad interchange is disaggregated by trust fund and projection year. The long-range office does a projection for each year in the 75-year period. In the short-range period (first 10 years of the 75-year projection period), the short-range office provides the estimates of railroad interchange by trust fund and the long-range projection is overridden in these years. Over the next five years of the projection period, the estimate of the railroad interchange is a linear interpolation between the short-range projection at the end of the short-range period and the long-range projection five years hence. During the final 60 years of the projection period, the projection is as estimated by the long-range office.

By trust fund, the total cashflow in year yr , $rr_cashflow(tf, yr)$, is broken down into two positive components; railroad benefits in year yr and railroad administrative expenses in year yr , and two negative components; railroad contributions in year yr and railroad taxation of benefits in year yr . A positive cashflow in this calculation represents a net cost to the trust fund. Cashflows are calculated on a fiscal year basis.

Projections of numbers of newly entitled retired workers are determined by analyzing the ratio of new entitlements to previous levels of railroad employment using 2001-2018 new entitlement data in the analysis. After initial entitlement, a mortality rate of 5.2% for 2019 is assumed based on analysis of recent Railroad Retirement Board financial interchange data, with mortality assumed to improve thereafter. For projections of newly entitled disabled workers a similar trend analysis based on prior employment is used. A “mortality” rate (deaths plus recoveries plus conversions to retired worker benefits) is determined using Railroad Retirement Board (RRB) data, with the rate assumed constant thereafter.

Assuming a 90/10 male/female split, the average benefit level for an OASI railroad worker is calculated as a ratio to the average OASI retired worker benefit. This ratio is constant throughout the projection period and is derived by comparing MBR and Railroad Board data. Additionally, a constant loading factor based on the same data is applied to aggregate worker benefits to determine the aggregate benefit amounts for auxiliary OASI beneficiaries. This same approach is used to determine similar constants for DI railroad benefits. The aggregate disabled worker railroad benefits (and beneficiaries) are estimated in the same way.

It is assumed that the ratio of OASI taxation of benefits to OASI benefits and DI taxation of benefits to DI benefits are both the same for railroad taxation of benefits. The railroad taxation of benefits is estimated by multiplying the railroad benefits by these ratios.

Administrative expenses for railroad are computed separately by trust fund. They are set at levels determined by short range in the short-range period. For years $ni+10$ to nf , they

are computed similarly to OASDI administrative expenses. Administrative expenses in yr-1 are multiplied by (a) the change in the total number of worker beneficiaries, (b) the annual change in average wage, and (c) one minus the ultimate productivity growth.

Railroad contributions are estimated, by trust fund, to be total railroad employment, multiplied by average railroad earnings, multiplied by the combined OASDI employer/employee tax rate. Average railroad earnings levels are assumed to grow with the increase in the average wage index, and railroad employment is assumed to decrease over time, both of which are in line with the Railroad Retirement Board’s own “most likely” projections.

The interchange amount is calculated both on a fiscal year basis and at the date of the actual transfer from SSA to the Railroad Retirement Board (usually in the first week of June in the following year). These interchange amounts can be represented by $RR_Transfer_FY(tf, yr)$ and $RR_Transfer_CY(tf, yr+1)$. $RR_Transfer_FY(tf, yr)$ is calculated by adding the cashflow amount to an interest amount, $rr_interest(tf, yr)$, which includes interest accrued by the cashflow components along with a reconciliation of interest amounts from the previous fiscal year calculated transfer amount and the current year June interchange. If $irate_presc(tf, year)$ is the prescribed interest rate used by the Railroad Retirement Board the interchange amount in June is:

$$RR_Transfer_CY(tf, yr) = RR_Transfer_FY(tf, yr - 1) \times (1.0 + (2.0 / 3.0) \times Irate_Presc(yf, yr))$$

vi. Equation 4.3.6 – Interest Income (INT)

In the short-range period, the projection of interest income by trust fund is provided by the short-range office. In each year of the short-range period, the annual yield rate is defined as the ratio of interest earned by a fund to the average level of assets held by the fund during the year.

The ultimate annual yield rate on each trust fund is equal to the nominal yield, which is the real interest rate increased for inflation. As a formula,

$$ultimate\ yield\ rate = (1 + real\ interest\ rate) \times (1 + inflation\ rate) - 1.$$

A side model is used to determine the year in which the ultimate rate is achieved and to get the yield rate for each year between the end of the short-range period (ns) and the ultimate year by trust fund. The primary input to this model is a daily bond transaction log from the short-range office which is used to determine a monthly exposure to interest for the various cash flow items in addition to the expected duration of time various types of bonds are held. This calculation, and the bond holdings in December of the last year of the short-range projection period from the log, are then used to model a monthly long-range version of a bond transaction log. This model ultimately produces annual effective yield rates for OASI and DI which are read directly into the main cost model.

The projection of interest income in a given year is the yield rate for that year multiplied

by the average level of assets. As a formula,

$$INT(tf, yr) = yield(tf, yr) \times avg_assets(tf, yr).$$

The amount of assets in a trust fund at the end of a given year is estimated from the level of assets at the beginning of the year by:

- Increasing the level for the tax contributions and taxation of benefits income received during the year (each exposed to the point in the year in which they are estimated to be received, on average), and
- Decreasing the level for scheduled benefits, railroad interchange, and administrative expenses paid during the year (each exposed to the point in the year in which they are estimated to be disbursed, on average).

For all years of the projection period, tax contributions are given an exposure of 0.519, taxation of benefits are given an exposure of 0.625, railroad interchange is given an exposure of 0.583, and administrative expenses are given an exposure of 0.5. For scheduled benefits, separate OASI and DI exposures are determined through a side model. The exposure, $ben_exp(yr)$, is larger than 0.5 in the early years of the projection period for both trust funds, whereas in later years OASI benefit exposures fall below 0.5 with DI remaining above 0.5. The reason is that in the past, benefits were always paid on the 3rd of each month. Now benefits are paid out throughout the month, based on the birth date of the beneficiary. The reason for the differences between trust funds is that benefits are paid on the third of the month (exempting check cycling⁹) for a higher proportion of DI beneficiaries due primarily to (1) concurrent receipt of SSI benefits, or (2) state payment of Medicare premiums. The average assets held by the trust funds for a given year is estimated by the formula

$$\begin{aligned} avg_assets(tf, yr) = & ASSETS_{BOY}(tf, yr) + 0.519 \times CONTRB(tf, yr) \\ & + 0.625 \times TAXBEN(tf, yr) - ben_exp(tf, yr) \times BEN(tf, yr) \\ & - 0.583 \times RR(tf, yr) - 0.5 \times ADM(tf, yr). \end{aligned}$$

Finally, the effective interest rate for the hypothetical combined OASDI fund is calculated as the sum of the magnitude of the individual fund interest amounts divided by the magnitude of the average level of assets of the individual funds:

$$\begin{aligned} irate(yr) = & (ABS(INT(OASI, yr)) + ABS(INT(DI, yr))) / \\ & (ABS(avg_assets(OASI, yr)) + ABS(avg_assets(DI, yr))) \end{aligned}$$

vii. Equations 4.3.7, 4.3.8 and 4.3.9 – Annual Values

⁹ Under check cycling many Social Security beneficiaries filing for benefits after April 1997 are paid on either the 2nd, 3rd, or 4th Wednesday of each month.

The annual income rate for a trust fund is computed as the sum of payroll tax contributions plus taxation of benefits as a percentage of taxable payroll.

$$ANN_INC_RT(tf, yr) = \frac{CONTRB(tf, yr) + TAXBEN(tf, yr)}{payroll(yr)}.$$

The annual cost rate for a trust fund is computed as the total cost of providing scheduled benefits from that fund as a percentage of taxable payroll. If

$$COST(tf, yr) = BEN(tf, yr) + RR(tf, yr) + ADM(tf, yr),$$

then

$$ANN_COST_RT(tf, yr) = \frac{COST(tf, yr)}{payroll(yr)}.$$

The trust fund ratio measures the amount of beginning of year assets that can be used to pay total cost. It is expressed as a percentage:

$$TFR(tf, yr) = \frac{ASSETS_{BOY}(tf, yr)}{COST(tf, yr)}.$$

viii. Equations 4.3.10, 4.3.11, 4.3.12, and 4.3.13 – Summarized Values

Present values of cash flows during the year are computed using the yield rate on the combined OASDI trust fund for that year. Each component of trust fund operations is exposed, with interest, to the point in the year in which, on average, it is received or disbursed. These exposure levels, $ben_exp(tf, yr)$, are the same as described above. These exposed levels are then discounted to January 1 of the year of the Trustees Report, ni . If $yield(j)$ is the annual yield rate on the combined OASDI trust funds for year j and $v(yr)$ is the discounting factor for the year, then

$$v(yr) = \prod_{j=ni}^{yr} \frac{1}{[1 + yield(j)]}.$$

For a given year, and trust fund,

$$PV_TAX(tf, yr) = (1 + 0.519 \times yield(yr)) \times TAX(tf, yr) \times v(yr),$$

$$PV_TAXBEN(tf, yr) = (1 + 0.625 \times yield(yr)) \times TAXBEN(tf, yr) \times v(yr),$$

$$PV_BEN(tf, yr) = (1 + ben_exp(tf, yr) \times yield(yr)) \times BEN(tf, yr) \times v(yr),$$

$$PV_RR(tf, yr) = (1 + 0.58\bar{3} \times yield(yr)) \times RR(tf, yr) \times v(yr),$$

and $PV_ADM(tf, yr) = (1 + 0.5 \times yield(yr)) \times ADM(tf, yr) \times v(yr).$

The target fund for a year is next year's cost. Its present value is computed as

$$PV_TARG(tf, yr) = [BEN(tf, yr + 1) + RR(tf, yr + 1) + ADM(tf, yr + 1)] \times v(yr),$$

Taxable payroll is exposed to the middle of the year when computing present values:

$$PV_PAYROLL(yr) = (1 + 0.5 \times yield(yr)) \times payroll(yr) \times v(yr).$$

We also define

$$PV_INC(tf, yr) = PV_TAX(tf, yr) + PV_TAXBEN(tf, yr)$$

and

$$PV_COST(tf, yr) = PV_BEN(tf, yr) + PV_RR(tf, yr) + PV_ADM(tf, yr).$$

Summarized rates are calculated using beginning of period assets and a target fund. Let yr_1 = the first year of the valuation period and yr_2 = the ending year of the valuation. Then the summarized income rate is:

$$SUMM_INC_RT(tf, yr_1, yr_2) = \frac{ASSETS_{BOY}(tf, yr_1) + \left(\sum_{j=yr_1}^{yr_2} PV_INC(tf, j) \right)}{\sum_{j=yr_1}^{yr_2} PV_PAYROLL(j)}.$$

The summarized cost rate is similarly computed:

$$SUMM_COST_RT(tf, yr_1, yr_2) = \frac{\left(\sum_{j=yr_1}^{yr_2} PV_COST(tf, j) \right) + PV_TARG(tf, yr_2)}{\sum_{j=yr_1}^{yr_2} PV_PAYROLL(j)}.$$

The 75-year actuarial balance is computed for a period beginning January 1 of the Trustees Report year, ni . It includes both beginning of period assets and a target fund. Therefore,

$$ACTBAL_{75yr}(tf) = SUMM_INC_RT(tf, ni, ni + 74) - SUMM_COST_RT(tf, ni, ni + 74).$$

In general, an actuarial balance may be computed for any given subperiod of the projection period. In general, actuarial balances for a subperiod beginning on January 1 of year ni and continuing through the end of year yr are computed using

$$ACTBAL_{ni, yr}(tf) = SUMM_INC_RT(tf, ni, yr) - SUMM_COST_RT(tf, ni, yr).$$

The unfunded obligation of a trust fund for a given period is the excess of the present value of the net cash deficits for each year of that period over the trust fund balance at the

beginning of the period. The unfunded obligation for the period beginning on January 1 of year ni and continuing through the end of year yr is computed using

$$UNF_OBL(tf, yr) = \sum_{j=ni}^{yr} [PV_COST(tf, j) - PV_INC(tf, j)] - ASSETS_{BOY}(tf, ni).$$

Note that the unfunded obligation excludes the target fund.

ix. Equation 4.3.14—Closed Group Unfunded Obligation

The closed group is defined as individuals who attain specified ages in the first year of the projection period (ni). The Statement of Social Insurance displays unfunded obligations for closed groups (1) attaining 15 or later in 2021, (2) attaining 62 or older in 2021, and (3) attaining 15 to 61 in 2021. For each year of the projection period, closed group calculations attribute a portion of the items in equations 4.3.1 through 4.3.6 to individuals falling in the defined closed group. The calculation of the closed-group unfunded obligation, then, uses the equation above but only considering the present values of cost and income attributable to the closed group.

The following information, developed elsewhere in the “Cost” process, is used for developing closed group unfunded obligation amounts:

- Total number of workers and total taxable earnings by single year of age 0-119 and sex, years 1951 through nf , updated yearly (years 1998 through nf used in the SOSI)
- Taxable payroll, years ni through nf , updated yearly
- Payroll tax income, years ni through nf , updated yearly
- Income from taxation of benefits, years ni through nf , updated yearly
- Scheduled benefits by beneficiary category, years ni through nf , updated yearly
- Railroad interchange, years ni through nf , updated yearly
- Administrative expenses, years ni through nf , updated yearly
- Yield rate on the combined OASDI trust funds, years ni through nf , updated yearly
- Population counts for all retired workers, spouses, divorced spouses, and widow(er)s by year, sex, and age 95-119 (read in as a percentage of 95+ counts)
- Distribution of assumed age differentials between aged spouses and workers ranging from -12 years to 15 years seniority for the worker by sex of the beneficiary, age of the beneficiary 62-74, and marital status
- Distribution of assumed age differentials between aged widow(er)s and deceased workers from -6 years to +12 years

It is important to note that, for dependent beneficiaries, the age of the worker, on whose account the benefits are based, determines whether that beneficiary would fall in the

closed group. For instance, if the closed group were defined as individuals attaining age 15 or later in 2021, a 3-year-old minor child receiving benefits in 2021 on the account of a retired worker aged 63 would be considered part of this closed group because the account holder was at least age 15 in 2021. The following describes how the various components of income and cost are allocated to the defined closed group in question:

Payroll Tax Contributions

Closed group taxable payroll is defined as the percentage of OASDI taxable payroll attributable to the closed group in question. An input file of *closed group payroll factors*, containing these percentages by year from 2021 through 2120, is used by the cost program to compute payroll tax contributions attributable to the closed group. For each year, the *closed group payroll factors* are determined as follows:

- The number of projected workers by single year of age (ages 0-119) and sex are multiplied by the associated average earnings by age/sex.
- Then, the portion of total taxable earnings attributable to the closed group is calculated.

For each year of the projection period, the number of workers and average taxable earnings by single year of age and sex are determined as follows:

- For ages 0-99, the number of projected workers comes directly from Economics group projections.
- For ages 100-119, the total age 100+ amounts from Economics are distributed among ages 100 through 119 for each projection year using mortality rates derived by looking back 10 years and multiplying these populations by 10 years of mortality increases by sex.
- For ages 0-14, the average taxable earnings are obtained by analyzing historical 1998-2017 data of the average earnings at each age relative to age 15 average earnings, and judgmentally assigning a ratio (to age 15 average earnings) for each age.
- For ages 15-80, the average taxable earnings by age and sex come directly from the AWARDS subprocess.
- For ages 81-99, the average taxable earnings are calculated as a weighted average of the average earnings for ages 76-80, weighted by the ratios of earnings between these ages and each age 81 through 99 for earnings years 1998 through 2017. This is similar to the method to calculate average earnings for ages 0 through 14, though here we are using five anchor years instead of one. For ages 100-119, average taxable earnings are set equal to the average earnings calculated for age 99.

Benefits

Methodologies for computing benefits attributable to the closed groups differ among benefit categories, as described below:

Retired Workers

For each age in current pay, the number of beneficiaries is multiplied by the corresponding average benefit amount across all ages of entitlement. The same applies for DI conversion cases. Retroactive benefits for the current year, by age, are then calculated. The closed-group factors for old-age benefits for each year are found by summing the benefit amounts attributable to the specified closed group, as a proportion of total retired worker benefits for all ages. This process is done separately by sex.

Disabled Workers

For each age from 20 to the year before normal retirement age, the program adds the products of the number of beneficiaries for each duration and the PIA for that duration. Retroactive benefits for the current year, by age, are then calculated. The closed-group factors for disability benefits are found by summing the total benefit amounts attributable to the closed group, as a proportion of total disabled worker benefits for all ages. This process is done separately by sex.

Aged Spouses and Divorced Aged Spouses

Closed Group calculations are done separately (although in the same manner) for aged wives, aged divorced wives, aged husbands and aged divorced husbands (combined), dually entitled aged wives, and dually entitled aged husbands. The number of aged spouse beneficiaries in each beneficiary category in current pay status (no dual entitlement) is provided by single year of age (up through 119). Then, for each single year of age, the program allocates total numbers of workers by age, from 12 years younger to 15 years older than the aged spouse using an assumed category-specific distribution. Next, for each age of worker in current pay, the number of workers is multiplied by the weighted average retired worker benefit for that age; this is done for all ages. The closed group factors, then, are obtained by determining the proportion of total benefit amounts attributable to the given closed group (based on the worker's age).

Aged Widows and Divorced Aged Widows

Closed Group calculations are done separately (although in the same manner) for aged widows, aged divorced widows, aged widowers, and aged divorced widowers. The number of aged widow(er) beneficiaries in current pay status (no dual entitlement) is provided by age from 60 to 119. For each single year of age, the program allocates total number of aged widows by age of the deceased husband (age the husband would have been if he had not died), from 6 years younger to 12 years older than the aged widow using an assumed distribution. The same distribution is used in reverse order for aged widowers leading to an age range of 12 years younger to 6 years older for the deceased wife. For each age of deceased spouses aged 119 or

younger, a real wage growth factor is applied to reflect ultimate real wage growth taken to the power of the number of years younger than age 119.

$$Benefitadj = (1 + wg_ult - 1.024)^{(119 - deceased\ worker\ age)}$$

This exponent is intended to reflect differences in average levels of benefits, with younger deceased spouses having higher benefits based on real wage growth. The closed group factors, then, are obtained by determining the proportion of total benefit amounts attributable to the closed group.

For aged widow(er)s in dual entitlement status (i.e. aged widow(er)s with a smaller worker benefit) we do similar calculations with the assumption that the age distributions are equal to the combined distribution of non-dually entitled aged widow(er)s and aged divorced widow(er)s.

Other Beneficiary Categories

For the 20 other dependent beneficiaries of retired workers, disabled workers, and deceased workers, an input file of closed group benefit factors is created, which represents the proportion of total (open-group) projected benefits in that category attributable to the given closed group age and year. This file is used by the cost program to compute amounts from each beneficiary category attributable to the closed group. The file, separately created for each closed group run, contains closed group benefit factors for ages 0 through 150 for each of the 20 beneficiary categories by sex of the account holder (worker). These input files are created by examining a recent sample of Master Beneficiary Record (MBR) data¹ for each of the beneficiary categories by age of the worker, and projecting future distributions by age of the worker based on population and, for survivor benefits, projected deaths by age. Then, adjustments are made for real wage growth to reflect different benefit levels by birth cohort.

Taxation of Benefits

Since taxation of benefits is related to benefits, the closed-group taxation of benefit amounts are computed by multiplying the total (open-group) taxation of benefit amounts by Trust Fund, by the corresponding total closed-group benefit factors by Trust Fund.

Administrative Expenses

Since administrative expenses are also assumed to be related to benefits, the closed-group administrative expense amounts are computed by multiplying the total (open-group) administrative expenses by Trust Fund, by the corresponding total closed-group benefit factors by Trust Fund.

Railroad Interchange

Since the railroad interchange has both a payroll tax and benefit component, each component is multiplied by its corresponding closed-group factor. That is, total payroll tax contributions arising from railroad interchange are multiplied by the closed group payroll factor discussed above in the “Tax on Contributions” section. Total railroad benefits, by Trust Fund, are multiplied by the aggregate closed-group benefit factors by Trust Fund. Closed-group railroad administrative expenses and closed-group railroad taxation of benefits are also estimated by applying aggregate closed group benefit factors by Trust Fund. The final amount is then the difference in the components (closed group railroad income less closed group railroad cost).